Basics of the Cost of Capital and the Valuation Ratio: Capital versus Consumption, Using Extended Equations

Hideyuki Kamiryo

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1. Introduction

This paper extends the equations in Kamiryo [2005b and 2005c], concentrating on the measurement of the cost of capital and the valuation ratios of capital and consumption. This will clarify the relationship between saving and consumption and also the relationship between consumption and compensation/wages in national disposable income. Taxes in GDP will be finally distributed to saving and consumption, where saving corresponds with net investment after depreciation.[1] Wages and rental (as the amount of capital services) must be estimated so that the sum of wages and rental is equal to the sum of saving and consumption, which is proved using equations in equilibrium.[2] When the cost of capital[3] in the government sector is significantly minus due to budget deficit (as seen in most EU and Asian countries), it lowers economic growth as a country. When saving is too much as in Singapore and Malaysia, the difference between saving and rental (or, consumption and wages) is enlarged. In any case, we need to pay more attention to a modified technology-golden rule, where the rate of saving

1) The literature shows net investment as gross investment less depreciation, where depreciation is given. My model directly uses net investment without setting depreciation and the depreciation rate as external. Since my endogenous model measures the rate of technological progress, an endogenous depreciation rate is, as a whole, connected with the growth rate of capital (or the ratio of net investment to capital). This idea is important in estimating capital when capital is not available. However, my model clarifies the relationship between the growth rate of capital (using net investment, hereafter "investment") and the depreciation rate under convergence. For example, if gross investment is zero (at minimum), the growth rate of capital equals the depreciation rate under convergence (this is stated in Appendix).

2) The current SNA assumes that gross operating surplus of the government sector equals depreciation or that rental is zero. In this case, we cannot measure the ratio of rental to capital for the government sector.

3) The cost of capital in this paper corresponds with the user costs of capital defined by Hall and Jorgenson (1967) and Paul Schreyer (OECD, 2004). However, the literature discusses this concept as external, where the market interest rate, the depreciation rate, and asset price change are used. My cost of capital is endogenous and takes into consideration the growth rate of output.

equals the relative share of profit under technological progress.

After discussing the cost of capital and the valuation ratio, this paper will present basics of the relationship between the current external balance (the balance of payment that excludes capital transfers, net), budget deficit, and the difference between saving and investment in the private sector. The literature does not clarify each sector's influence on the growth rate and the ratio of rental to capital. The relationship between the ratio of investment to output, *i*, and the growth rate under convergence (in the balanced growth-state) is not formulated yet. I will clarify this point using extended equations. Furthermore, if the rate of saving in the government sector is negatively significant, what is an opportunity cost of deficit?

2. Further extended equations under convergence

2.1 *Review of compensation/wages in national income using a function of consumption*

National accounts express the relationship between compensation of employed persons/wages (hereafter wages) and returns/profit/rental (hereafter rental) in output and saving and consumption. Wages in *GDP* turn to consumption in national disposable income (*NDI*). The relationship between *GDP* and *NDI* in national accounts is shown using capital consumption (D_{EP}), indirect taxes ($T_{AX(IND)}$) and subsidies (S_{UBS}): $GDP - D_{EP} - T_{AX(IND)} + S_{UBS} = NDI$. After depreciation and tax redistribution, wages and rental in *GDP* are absorbed into consumption and saving in *NDI*. In equilibrium, output as the supply-side is equal to income as the demand-side.

How can I measure/estimate this equal relationship? Modified wages and rental are estimated using a function of consumption (see Kamiryo [2005b]), where output=income[4] (hereafter, output, *Y*) integrates the rate of rental in production, *r*, and the discount rate, *rho*, in consumption. The value of (*rho*/*r*) is 1.0 when the rate of saving equals the relative share of rental and is determined so that the product of the capital-labor ratio and the ratio of rental to wages is fixed and equal to the relative share of rental, α. (see Kamiryo [2004c]). Wages and rental are estimated so as to satisfy the marginal productivities of capital and labor at maximum and the corresponding consumption at optimum. As a result, I am able to estimate rental and accordingly, the ratio of rental to capital and these are estimated simultaneously with wages

4) To show actual situations conservatively, I prefer *NDI* to "*NDI* + factor income abroad" (except for the case of the present valuation of wealth) and also for national saving, I prefer "the difference between saving and net investment" to "financial surplus/deficit," whose difference is capital transfers, net.

and accordingly, the relative share of labor, each by year and sector. The above is my starting point in this paper.

In the function of consumption, $(rho/r)(1\text{-}\alpha)$ or $(rho/r)(c)$, a discount rate of consumption, *rho*, is used for the present values of saving and consumption each as a flow, and r is the maximum rate of rental and used for the present values of rental and wages each as a flow. A basic equation is shown (see Eq. 11 in Kamiryo [2005b]):

$$(1-\alpha) = c/(rho/r) \text{ and } c = (rho/r)(1-\alpha), \tag{1}$$

Or, $rho(1-\alpha) = r(1-s)$, where c is the ratio of consumption to output: $c = C/Y$.

Suggested by the idea of Jan Tinbergen [1960], I extend Eq. 1 to Eq. 1-2, by introducing the capital-output ratio Ω^* and using the investment ratio, i, instead of c.

$$(rho/r)_{\Omega^*} = i/((1-\alpha)\Omega^*). \tag{1-2}$$

Eq. 1-2 is needed to know $\Omega^*(i)$ or the change in Ω^*, to the change in the investment ratio, i. As a result, Eq. 1-2 makes it possible to find $\beta^*(\Omega^*)$ through $\beta^*(i)$.

2.2 *The cost of capital and the valuation ration of capital using the rate of rental and the growth rate of output under convergence*

This section clarifies the characteristics of such variables as the capital-output ratio, Ω^*, the rate of rental, r^*, the growth rate of output, g_Y^*, and the growth rate of per capita output, g_y^*, each under optimum convergence, where the relationship between the rate of saving and the relative share of rental will be shown using $\theta \equiv i/s$.

Before starting, I will conclusively show related equations (see Eqs. 8 to 11 in Kamiryo [2005c]). This purpose is to directly confirm the relationship between the ratio of rental to capital and the growth rate. Note that each equation is expressed, setting the ratio of investment ratio to output as an independent variable, but each equation needs to fix other parameters.

$$\Omega^* = \frac{\beta^*_{\delta=\alpha} \cdot i\,(1-\alpha)}{i\,(1-\beta^*_{\delta=\alpha})(1+n)+n(1-\alpha)}. \tag{2}$$

$$r^* \equiv \frac{\alpha}{\Omega^*} = \alpha\left(\frac{i\,(1-\beta^*_{\delta=\alpha})(1+n)+n(1-\alpha)}{\beta^*_{\delta=\alpha} \cdot i\,(1-\alpha)}\right).^{5)} \tag{3}$$

Next, the rate of technological progress is expressed as $g_A^* = i\,(1-\beta^*_{\delta=\alpha})$, where

5) As a preliminary step to obtain $r^*(i)$, $\beta^*(i)$ must be solved since beta[*] is composed of four parameters: $\beta^* = \left(\frac{\Omega^* \cdot n(1-\alpha)}{(1-\alpha)+\Omega^*(1+n)}\right)\frac{1}{i} + \frac{\Omega^*(1+n)}{(1-\alpha)+\Omega^*(1+n)}$ holds from Eq. 22, β^* = ↗

$g_A(t) = i_A \cdot k(t)^{\alpha-\delta}$ and $\alpha = \delta$. Then, g_Y^* in discrete time is formulated by inserting $g_A^* = i\,(1-\beta^*_{\delta=\alpha})$ into a well-known $g_Y^* = \dfrac{g_A^*(1+n)}{1-\alpha} + n$.[6] (4)

Then, the relationship between the ratio of rental to capital and the growth rate of output under convergence is now derived by using $A = \dfrac{i(1-\beta^*_{\delta=\alpha})(1+n)}{1-\alpha}$ for both the above r^* in Eq. 3 and the above g_Y^* in Eq. 4.

Since $r^* = \left(\dfrac{\alpha}{i \cdot \beta^*_{\delta=a}}\right) \cdot (A+n)$ and $g_Y^* = A + n$ and,

$r^* = \left(\dfrac{\alpha}{i \cdot \beta^*_{\delta=\alpha}}\right) \cdot g_Y^*$ or $g_Y^* = \left(\dfrac{i \cdot \beta^*_{\delta=\alpha}}{\alpha}\right) \cdot r^*$ is derived. (5)

In Eq. 5, I define the coefficient of a modified-technology golden rule, α_{GOLDEN}, as $i \cdot \beta^*$ (hereafter, for simplicity, I abbreviate $\beta^*_{\delta=\alpha}$ as β^*).

Now, I will first explain the relationship between the conventional golden rule under no technology and mine under technological progress and then clarify the character of $\alpha_{GOLDEN} = i \cdot \beta^*$.

$\alpha_{GOLDEN} = i \cdot \beta^* = s \cdot \theta \cdot \beta^*$, where $\theta \equiv i / s$ and, (6)

if $\alpha = \alpha_{GOLDEN}$, $\left(\dfrac{\alpha}{i \cdot \beta^*}\right) = \left(\dfrac{\alpha}{s \cdot \theta \cdot \beta^*}\right) = 1.0$ holds, resulting in $r^* = g_Y^*$.

First, $\alpha / i \cdot \beta^* = \alpha / s \cdot \theta \cdot \beta^*$ presents a special/generalized condition for the $\alpha = s$ in the golden rule/age which Phelps [1961, 1965] finalized under no technology. I call $\alpha / i \cdot \beta^*$ "a coefficient of the golden rule." Using Eq. 6, the differences between generalized and conventional rules are indicated as follows:

1. If I assume no technological progress, $\beta^* = 1$ and $\alpha_{GOLDEN} = i$ hold.
2. If I assume that net investment is equal to saving, $\theta = 1$ and $\alpha_{GOLDEN} = s \cdot \beta^*$ hold.
3. If I assume that $\beta^* = 1$ and $\theta = 1$, $\alpha_{GOLDEN} = \alpha = s$ hold, reducing to the conventional

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$\dfrac{\Omega^*\left(n(1-\alpha)/i + (1+n)\right)}{(1-\alpha) + \Omega^*(1+n)}$, in Kamiryo [2004a]. The values of $i\,\beta^*$ and $i\,(1-\beta^*)$ are simply shown as: $\beta^* \cdot i = B \cdot i + A$ and $(1-\beta^*)\,i = (1-B)\,i - A$, where $A \equiv \dfrac{\Omega^* \cdot n(1-\alpha)}{(1-\alpha)+\Omega^*(1+n)}$ and $B \equiv \dfrac{\Omega^*(1+n)}{(1-\alpha)+\Omega^*(1+n)}$. Now using $\beta^* \cdot i = B \cdot i + A$, $r^*(i) = \alpha\left(\dfrac{\left((1-B)\,i - A\right)(1+n) + n(1-\alpha)}{(B \cdot i + A)(1-\alpha)}\right)$ is shown.

6) Similarly, $g_Y^*(i) = \dfrac{((1-B)\cdot i - A)(1+n)}{1-\alpha} + n$, where $g_A^* = (1-B)\,i - A = (1-\beta^*)\,i$.

golden rule.

In Eq.5, when $\beta^* \neq 1$ and $\theta \neq 1$, a modified technology-golden rule holds as follows:

1. If $\alpha > \alpha_{GOLDEN}$, $r^* > g_Y^*$ holds.
2. If $\alpha = \alpha_{GOLDEN}$, $r^* = g_Y^*$ holds.
3. $\alpha < \alpha_{GOLDEN}$, $r^* < g_Y^*$ holds.

The ratio of net investment to saving, $\theta = i/s$, is less than 1.0 when exports exceed imports, and also $\theta = i/s$ is more than 1.0 in the government sector when government budget shows deficit.

In Eq. 5, when $\alpha = s = i$ (as in the conventional golden rule) and $\beta^* \neq 1$, the modified technology-golden rule holds as follows:

1. If $\beta^* < 1$, $r^* > g_Y^*$ holds under a plus technology.
2. If $\beta^* = 1$, $r^* = g_Y^*$ holds under no technology.
3. If $\beta^* > 1$, $r^* < g_Y^*$ holds under a minus technology.

In the above cases, the relationship between the rate of optimum rental, r^*, and the growth rate of output under convergence, g_Y^*, is expressed as the endogenous cost of capital, $r^* - g_Y^*$. I use the terminology of the "endogenous" cost of capital to distinguish the cost of capital or the discount rate in the literature with mine. For example, Tobin James and William C. Brainard [1977, 244–245][7] indicate that the discount rate is not any observed interest rate on long-term bonds or other fixed-money-value obligations, and express the marginal efficiency of capital R (that corresponds with r^* in this paper) and the discount rate r_K (that corresponds with $r^* - g_Y^*$ in this paper), each in a continuous case, but each are not observable. However, an endogenous cost of capital here is formulated as,

$$g_Y^* = \left(\frac{i \cdot \beta^*_{\delta=\alpha}}{\alpha} \right) \cdot r^*, \quad (7)$$
[8]

7) According to Tobin James and William C. Brainard [1977, 245], the marginal efficiency of capital R is defined by the equation, $V = \int_0^\infty E(t) e^{-Rt} dt$, and the discount rate r_K is defined by the equation, $\overline{MV} = \int_0^\infty E(t) e^{-r_K t} dt$.

8) $r^*(i) - g_Y^*(i)$ is shown using $r^*(i) = \alpha \left(\frac{((1-B)\, i - A)(1+n) + n(1-\alpha)}{(B \cdot i + A)(1-\alpha)} \right)$ and $g_Y^*(i) = ((1-B)\, i - A) \frac{(1+n)}{1-\alpha} + n$. Note that $\Omega^* = \frac{i}{(rho/r)_\Omega (1-\alpha)}$ is equal to $\Omega^*(i) = \frac{(B \cdot i + A)(1-\alpha)}{((1-B)\, i - A)(1+n) + n(1-\alpha)}$ and, $\beta^*(\Omega^*)$ is equal to $\beta^*(i)$ if $\Omega^*(i)$ is introduced. The capital-output ratio is constant in $\beta^*(i)$ if $\Omega^*(i)$ is not introduced.

$$\text{or, } r^* - g_Y^* = g_Y^*\left(\frac{\alpha}{s\cdot\theta\cdot\beta^*} - 1\right).$$

The above endogenous cost of capital (hereafter, for simplicity, abbreviating "endogenous" as the cost of capital) must be plus. The required conditions are *beta** < 1 and $\alpha / i\cdot\beta^*$ > 1. This cost of capital also corresponds with Arrow's [1990, xviii] discount factor for utilities when the discount rate of consumers, *rho*, is equal to the endogenous ratio of rental to capital, r^*. I will further discuss the relationship between *rho* and r^* when I introduce newly the valuation ratio of "consumption" (see below).

Proposition 1: If the $\alpha_{GOLDEN} = i\cdot\beta^*$ is less than *alpha*, the cost of capital is plus.

This implies that the relationship between *alpha* and the ratio of net investment to output, *i*, determines a change in sign of the cost of capital, together with an effective range of *beta** that exists roughly between 0.7 and 0.95: If *alpha* = 0.12 and *beta** = 0.7 as in the private sector, *i* must be less than 0.17 = 0.12 ÷ 0.7. If *alpha* = 0.08 and *beta** = 0.95 as in the government sector, *i* must be less than 0.084 = 0.08 ÷ 0.95. This suggests that the ratio of net investment to output in the government sector should be less than 0.1.

Next, the valuation ratios of capital and consumption, $v_K \equiv V_K / K$ and $v_C \equiv V_C / C_{STOCK}$, will be discussed. First, the valuation ratio of capital, defined as the valuation value of capital, V_K, to capital, K, is shown:

$$v_K = \frac{g_Y^*\cdot\alpha / i\cdot\beta^*}{g_Y^*(\alpha / i\cdot\beta^* - 1)} \text{ and accordingly, } v_K = \frac{\alpha}{\alpha - i\cdot\beta^*},^{9)} \tag{8}$$

since $v_K \equiv \dfrac{V_K}{K} = \dfrac{r^*}{r^* - g_Y^*}$, by using $V_K = \dfrac{\Pi}{r^* - g_Y^*}$ [10] and $K = \dfrac{\Pi}{r^*}$.

The valuation ratio of capital is shown as a three dimensional graph by using Eq. 8.

Then, by reforming Eq. 8,

$$v_K = \frac{-\alpha / i}{\beta^* - \alpha / i} \text{ is derived.} \tag{9}$$

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9) $v_K(i) = \dfrac{-\alpha / B}{i - (\alpha - A)/B}$ holds using Eq. 8, $v_K(i) = \dfrac{\alpha}{\alpha - \beta^*\cdot i}$, where $\beta^*\cdot i = B\cdot i + A$, $A = \left(\dfrac{\Omega^*\cdot n(1-\alpha)}{(1-\alpha)+\Omega^*(1+n)}\right)$, and $B = \dfrac{\Omega^*(1+n)}{(1-\alpha)+\Omega^*(1+n)}$.

10) Note the difference between the discrete and continuous case (see Kyoury, Sarkis J., and Torrence D. Parsons [1981, 49–52]. For a usual example, in the discrete case, $P_0 = D_0\sum_{t=0}^{\infty}\left(\dfrac{1+g_D^*}{1+r^*}\right) = \dfrac{D_1}{r^* - g_D^*}$ holds. In the continuous case, $PV = \int_{t=0}^{\infty} D_t e^{-(r^* - g_D^*)} dt = D_0 / (r^* - g_D^*)$.

Eq. 9 is a hyperbolic function of *beta**: $v_K(\beta^*)$, where the vertical asymptote is shown as α / i and the curvature of this function is equal to α / i. Since the curvature is now independent of *beta**, Eq. 8 is used at the same time. Eq. 9 is useful to policy-makers in that the relationship between rental and net investment determines the vertical asymptote.

The above valuation ratio of capital in real assets, v_K, is compatible with Tobin's [1980, pp. 84–96] q_K for firms. The value of q_K is estimated using the market discount rate for the market valuation of capital, and the value of v_K is estimated using the endogenous cost of capital for the endogenous valuation of capital, by assuming that capital is equal to replacement/reproduction cost and the optimum rate of rental that expresses the marginal productivity/efficiency of capital is equal to an internal rate of return (IRR) under convergence.[11] Tobin [ibid., 90], after confirming that the above q_K is a weighted average of q_E for equity and q for borrowings, indicates that businesses can be modeled as if they are pure equity firms, citing the Modigliani-Miller theorem. The above v_K and q_K are the valuation ratios for aggregated real capital in national accounts and thus, I treat v_K as if it is pure equity (citing the above expression) or a weighted average: borrowings are only discussed in financial assets. The surplus of funds lying as the difference between real and financial assets will be discussed by taking into consideration the current external balance together with capital transfers.

Now in detail, I will discuss the character of the valuation ratio, v_K and lead to setting up a proposition. Eq. 8 indicates directly the relationship between the ratio of net investment and the valuation ratio. Similarly to Tobin's q_K, the valuation ratio v_K is required to be more than 1.0. The ranges of the valuation ratio are shown step by step using the following cases:

Case 1: If $i \cdot \beta^* < \alpha$, $v_K > 0$ holds.
Case 2: If $i \cdot \beta^* = \alpha$, v_K is impossible to get.
Case 3: If $i \cdot \beta^* = 0$ or $\beta^* = 0$, $v_K = 1$ holds.
Case 4: If $i \cdot \beta^*$ is less than α, v_K is more than 1.0.
Case 5: If $i \cdot \beta^* = 0.5\alpha$, $v_K = 2$ holds.
Case 6: If $i \cdot \beta^* < 0.5\alpha$, $v_K > 2$ holds.

11) According to Kyoury, Sarkis J., and Torrence D. Parsons [1981, 59], te difference between net present value (NPV) and internal rate of return (IRR) is shown as follows: "The IRR method is equivalent to the NPV method in every respect except for the discount factor. In the NPV calculation the discount factor is known and is equal to the cost of capital. In the IRR case the discount factor is unknown and it may be much higher than the cost of capital." Tobin James [1980, 89], for capital valuation uses the terminology of the IRR for replacement cost of capital, which corresponds with the optimum rate of rental r^* in my model.

The effective range of the valuation ratio exists in the above Cases 4 to 6. At the same time, the effective range of *beta** exists when *beta** is less than 1.0; e.g., 0.7 to 0.95. Tobin and Brainard [ibid., Table 2, 254] show that q_K roughly ranges between 1.5 to 2.5 for firms. My empirical results show similar tendencies, depending on different situations by country (see below). Note that if the optimum rate of rental is equal to the growth rate of output under convergence, the endogenous cost of capital is impossible to get. This is called the Petersburg paradox. I already indicated the existence and measurement of this paradox in Kamiryo [2004b], but this indication was made before my introduction of a function of consumption into the Cobb-Douglas production function.

Proposition 2: If the relative share of rental, *alpha*, equals the product of the ratio of net investment to output, *i*, and *beta**, the Petersburg paradox [David Durand, 1956] exists: $\alpha = \alpha_{GOLDEN} = i \cdot beta^*$, where the endogenous cost of capital is zero due to the ratio of rental to capital equals the growth rate of output under convergence: $r^* = g_Y^*$.

Proposition 3:[12] If the $\alpha_{GOLDEN} = i \cdot \beta^*$ is less than *alpha* within an effective range of *beta**, the valuation ratio $v_K = V / K$ will be higher and more stable in an effective range of the valuation ratio: $v_K > 1$.

Now back to Eq. 9, the vertical asymptote of the valuation ratio is shown by α / i: the higher α / i the more stable the valuation ratio is. It is interesting to confirm the relationship between the financial parameter $\theta = i / \alpha$ and the vertical asymptote.

Using $\frac{i}{\alpha} = \frac{i}{s} / \frac{\alpha}{s} = \theta / \frac{\alpha}{s}$,

$$\theta \equiv \frac{i}{s} = \frac{\alpha}{s} \cdot \frac{i}{\alpha} = \frac{\alpha}{s} / \frac{\alpha}{i} \text{ is confirmed.} \quad (10)$$

And thus,

$$s - i = s(1-\theta) = s\left(1 - \frac{\alpha}{s} / \frac{\alpha}{i}\right) \text{ is obtained.} \quad (11)$$

12) Here I will explain the relationship between capital stock and its valuation value. The above propositions hold regardless of whether the present value of capital, *K*, includes price change or not. The price change is evaluated by the asset market, using asset price indices. The alternative perpetual inventory method (PIM) in OECD (2001) takes into consideration this price change. The endogenous cost of capital in this paper uses the ratio of rental to capital, which does not include market evaluation. Therefore, the valuation ratio is slightly influenced by the initial capital stock *K* if price change is included in *K*. The initial capital stock in this paper is estimated using my own method (Kamiryo, 2004c), supplemented by a rough PIM for a whole capital (not dividing by the type of asset as in PIM in the literature).

Eq. 11 shows the relationship between $s - i$ and i/s. In Eq. 11, α/s is related to the conventional golden rule and α/i is the vertical asymptote of the valuation ratio of capital. When Eq. 11 is used for the total economy, Eq. 11 directly connects the balance of payment with the vertical asymptote of the valuation ratio. When Eq. 11 is used for the government sector, Eq. 11 connects budget deficit (and a minus government saving) with the vertical asymptote of the valuation ratio in the government sector, where $\theta_G \equiv \frac{i_G}{s_G} = \frac{\alpha_G}{s_G} / \frac{\alpha_G}{i_G}$ plays an important role, using the output of the government sector, Y_G, for the denominator of each parameter. In this case, if the output of the total economy, Y, is used instead of Y_G, government surplus/deficit to output, shown by $(s-i)_G$, is now directly comparable to the balance of payment, $s-i$. $(s-i)_G$[13] is shown using the ratio of government output to total output, Y_G/Y, as,

$$(s-i)_G = (s-i)_G \cdot Y_G / Y. \tag{12}$$

where $(s-i)_G = s_G(1-\theta_G) = s_G\left(1 - \frac{\alpha_G}{s_G} / \frac{\alpha_G}{i_G}\right)$.

Let me now summarize in detail the relationship between taxes and government expenditures (that excludes investment).[14]

1. $GDP = C + I_{PRI} + (I_G + E_G) + (S - I)$ holds in ex-ante equilibrium, where GDP shows supply and the RHS shows demand and also, government outlays are defined as the sum of investment, I_G, and expenditures, E_G.
2. $GDP - T = C + I_{PRI} + (I_G + E_G - T) + (S - I)$, where T is taxes and E_G is expenditures.
   $GDP - T - C - I_{PRI} = (I_G + E_G - T) + (S - I)$. Define $S_{PRI} \equiv GDP - T - C$.
3. $S_{PRI} - I_{PRI} = (I_G + E_G - T) + (S - I)$. Now define $S_G \equiv T - E_G$.
4. Then, $(S - I) = (S - I)_G + (S - I)_{PRI}$ holds. Note that if the current external balance is set equal to the RHS, this equation shows the relationship in equilibrium ex-ante. The equation also holds ex-post. Therefore, budget surplus/deficit is shown using taxes, expenditures, and investment:

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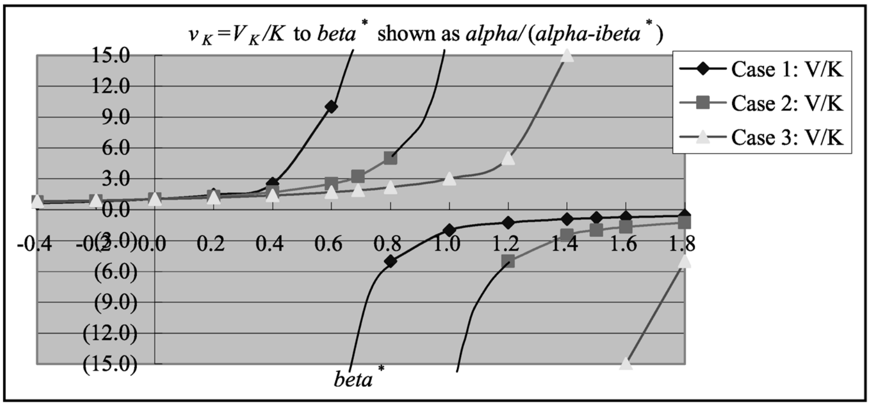
13) In the government sector, its saving as "taxes less expenditures" is independent of its investment. How can we determine the influences of saving on the growth rate and the rate of rental to capital? I treat S_G, similarly to I_G: if $S_G<0$, a minus S_G will offset the effect of I_G on the growth rate of output, g_Y^*, and the ratio of rental to capital. I express this idea as the opportunity cost of a minus government saving. Note that the higher I_G or $i_G = I_G/Y_G$, the higher g_Y^*.

14) I am thankful to Dr. Kazuhiro Igawa for his sincere advice and discussion. I pay attention to the saving in the government sector: $S_G \equiv T - E_G$, which is more important to the decrease in deficit than government expenditures. Bayley, M. G. [1971] uses the difference between government expenditures and taxes.

$$(S-I)_G = T - E_G - I_G.^{15)} \tag{13}$$

The literature shows Eq. 13 by divided it with GDP. For simplicity, assume that the current external balance is zero (or, $S = I$) and government expenditure, E_G, is replaced by government consumption, C_G. Then, $GDP = C_{PRI} + I_{PRI} + (I_G + C_G) = C + S$, where GDP is equal to national disposable income. Therefore, I use a symbol, Y, where $GDP = NDI$. Justified by this equality, I modify both wages and rental using the sum of consumption and saving and a function of consumption, $(rho/r)(c)$. When, I divide Eq. 13 by Y, $(s-i)_G = t_{AX} - e_G - i_G$ is shown.

Eq. 13 makes it possible to interpret the EU 3% rule for government deficit. This is discussed in empirical results soon below.



Note: The vetical asymptote of $v = V/K$ to $beta^*$, $alpha/i$, becomes higher if $alpha$ is higher than i. For Case 1, $\alpha = 0.1$ and $i = 0.15$, for Case 2, $\alpha = i = 0.15$, and for Case 3, $\alpha = 0.1$ and $i = 0.15$

Figure 1 $alpha_{GOLDEN} = i \cdot beta^*$ **and the valuation ratio,** $v_k = V_k / K$**, by** $beta^*$ **(1)**

2.3 *Comparison of the valuation ratio of consumption with the valuation ratio of capital*

In this section, first I will present the valuation ratio of consumption stock, $v_C \equiv V_C / C_{STOCK}$, and then compare it with the valuation ratio of capital above. By comparing both valuation ratios, I will find how each character of the two valuation ratios differs each other and also how the function of consumption works in the long-term between capital and consumption: if the situation is saving-oriented, the valuation ratio of consumption is higher than that of capital,

15) The relationship between taxes and government output is shown neglecting interest paid as follows: $T = (S-I)_G + E_G + I_G = S_G + C_G = Y_G$, which implies that taxes always equals government output. Also, when interest paid, net, of national debts, R_{DEBT}, is introduced, the relationship between budget and primary balance is shown as $(S-I)_{G(PRIMARY)} = (S-I)_G + R_{DEBT}$.

and vice versa.

Before starting, I will explain the circumstances lying behind the conventional (in the absence of technological progress) golden rule, by citing some of important statements in Joan Robinson [1962, 226]. For the modified technology golden rule, I integrate a function of consumption with the Cobb-Douglas production function. Robinson investigated a history of the golden rule citing R. F. Kahn [1959] and J. Desrousseaux [1961] besides herself and her pupils, and supposes the golden rule, but without technology and consumption/utility function.[16] She finalizes the proposition of the golden rule as, "When we think of the proposition in terms of the condition that the workers consume the whole wage and capitalists save the whole profit it appears somewhat mysterious. When we realize that it does not matter at all who does the saving so long as the rate of profit is equal to the rage of growth, it seems fairly obvious." For this justification, I used three kinds of saving ratios which are wholly replaced by the ratio of consumption to output [Kamiryo, 2005b]. Also, I stress here that wages and profits/rental are not given while consumption and saving are given in disposable income and that the golden rule will be completed through newly estimating wages and rental at output = income, Y.

Now, the valuation ratio of consumption is formulated by using the discount rate for consumers, *rho* or ρ, instead of $r = r^*$, but using $g_Y^* = g_C^*$ under convergence with a fixed *alpha*. For the valuation ratio of consumption, I need a new concept of consumption stock which is comparable with capital stock. However, this concept is only justified in that a final purpose is to compare the valuation ratio of consumption with the valuation ratio of capital.

Since $rho = r(rho/r) = r \cdot c/(1-\alpha)$ and $rho = \dfrac{\alpha \cdot (rho/r)}{i \cdot \beta^*} \cdot g_C^*$,

$$v_C = \frac{\alpha \cdot (rho/r)}{\alpha \cdot (rho/r) - i \cdot \beta^*}, \tag{14}$$

where the valuation ratio of consumption $V_C = \dfrac{C}{\rho - g_C^*}$ and

consumption stock (as a new concept) $C_{STOCK} = \dfrac{C}{\rho}$.

Similarly to v_K, the vertical asymptote and the curvature are obtained as,

16) Joan Robinson [1962, 226] states: "consumption per man employed is at the maximum when all profits are saved and all wages spent follows immediately when we combine a Keynesian theory of profits with a properly articulated neo-classical production function."

$$v_C = \frac{-(\alpha / i)(\rho / r)}{\beta^* - (\alpha / i)(\rho / r)}. \tag{15}$$

where $(\alpha / i)(\rho / r)$ shows both the vertical asymptote and the curvature.
The valuation ratio of consumption, v_C, differs from the valuation ratio of capital, v_K, by the introduction of (*rho*/*r*).

1. If (*rho*/*r*) < 1, $v_C > v_K$ holds (as seen in Asian countries).
2. If (*rho*/*r*) = 1, $v_C = v_K$ holds.
3. If (*rho*/*r*) > 1, $v_C < v_K$ holds (as seen in most EU countries).

Proposition 4: If (*rho*/*r*) = 1 or consumption is equal to wages (the ratio of consumption to output is equal to the ratio of wages to output), the valuation ratio of consumption equals the valuation ratio of capital. If (*rho*/*r*) < 1 or consumption is less than wages (the ratio of consumption to output is less than the ratio of wages to output), the valuation ratio of consumption is higher than the valuation ratio of capital, and vice versa.

Proposition 5: If an economy is much saving-oriented, $v_C > v_K$ holds and consumption is more stimulated in the long-term. If an economy is much consumption-oriented, $v_C < v_K$ holds and saving is more stimulated in the long-term.

The above Proposition 4 proves that optimum consumption in income and maximized rental in output are compatible with each other when the discount rate for income is equal to the rate of rental for output, where income equals output.

Next, in terms of the curvature and the asymptote, let me compare the valuation ratio of capital, v_K, with the valuation ratio of consumption, v_C. Both v_K and v_C have the same curvature of α / i. The curvature is milder when α is less than the ratio of net investment to output. However, the asymptote of v_K, α / i, is a little bit lower or higher than the asymptote of v_C, (*rho*/*r*)α / i (see Figures 2 and 3[17]):

Case 1: If (*rho*/*r*) < 1, the asymptote of v_K is lower than the asymptote of v_C.

Case 2: If (*rho*/*r*) > 1, the asymptote of v_K is higher than the asymptote of v_C.

What do the above two cases suggest? For the interpretation of the above differences due to the level of (*rho*/*r*), we need the character of *beta** under convergence. High technology firms and the private sector have much lower *beta** while uncompetitive firms and the government sector have much higher *beta**. Note that the *beta** of the government sector is higher than the

17) I showed these figures without expressing each exact vertical asymptote (by cutting the line of each valuation ratio just before and after each vertical asymptote), which needs much shorter interval for the values of *beta**.

beta* of the private sector. Therefore, I will extend the above two mild cases to four cases, apart from the effective ranges of *beta** (i.e., 0.7~0.95) as follows:

With *beta** <<1 and α/i >>1 under a stable/matching situation:

1. If the situation is too much saving-oriented under (*rho*/*r*)<<1, the valuation value of capital is lower than that of consumption, which in turn leads to a mitigation of too much saving-oriented.
2. If the situation is too much consumption-oriented under (*rho*/*r*)>>1, the valuation value of capital is higher than that of consumption, which in turn leads to a mitigation of too much consumption-oriented.

With *beta**>1 and α/i = 1 under an unstable/mismatching situation:

3. If the situation is consumption-oriented under (*rho*/*r*)>1, this combination is already mismatching, but it may be possible to recover from the mismatching. It is urgent to execute structural reform so as to decrease *beta**.

With *beta** > 1 and α/i < 1 under an unstable/mismatching situation:

4. If the situation is consumption-oriented under (*rho*/*r*)>1, this combination is extremely mismatching and may not lead to a shift from consumption-oriented to saving-oriented. It is urgent to decrease both *beta** and investment. A typical case is the government sector in Japan.

4. The cost of capital and the difference between saving and investment by sector

Does budget deficit aggravate the rate of rental to capital, the growth rate of output, and accordingly, the cost of capital? This section proves the above affirmative relationship, formulating equations under convergence.

First, the difference between saving and investment by sector is summarized:

$(S–I) = (S_{G)} – I_G) + (S_{PRI)} – I_{PRI})$, where $(S_{PRI} – I_{PRI}) = (S_{CORP} – I_{CORP}) + (S_H – I_H)$. (16)

For simplicity, $\Psi = (S - I)$ of the total economy is the balance of payment in the open economy, $\Psi_G = (S - I)_G = (S_G - I_G)$ is budget surplus/deficit, and $\Psi_{PRI} = (S - I)_{PRI} = (S_{PRI}–I_{PRI})$. When capital transfers, net, are deducted in each item, domestic saving is estimated.

1. $\Psi = \Psi_G + \Psi_{PRI}$ but, $\psi \neq \psi_G + \psi_{PRI}$, if each is defined as $\psi \equiv \Psi / Y$, $\psi_G \equiv \Psi_G / Y_G$, *and* $\psi_{PRI} \equiv \Psi_{PRI} / Y_{PRI}$. (17)
2. Instead, if $\psi_{G/Y} \equiv \Psi_G / Y$ *and* $\psi_{PRI/Y} \equiv \Psi_{PRI} / Y$, $\psi = \psi_{G/Y} + \psi_{PRI/Y}$ holds. (18)
3. Using Y_G/Y and Y_{PRI}/Y, $\psi_{G/Y} = (Y_G / Y)\psi_G$ and $\psi_{PRI/Y} = (Y_{PRI} / Y)\psi_{PRI}$, where Y_G/Y is the

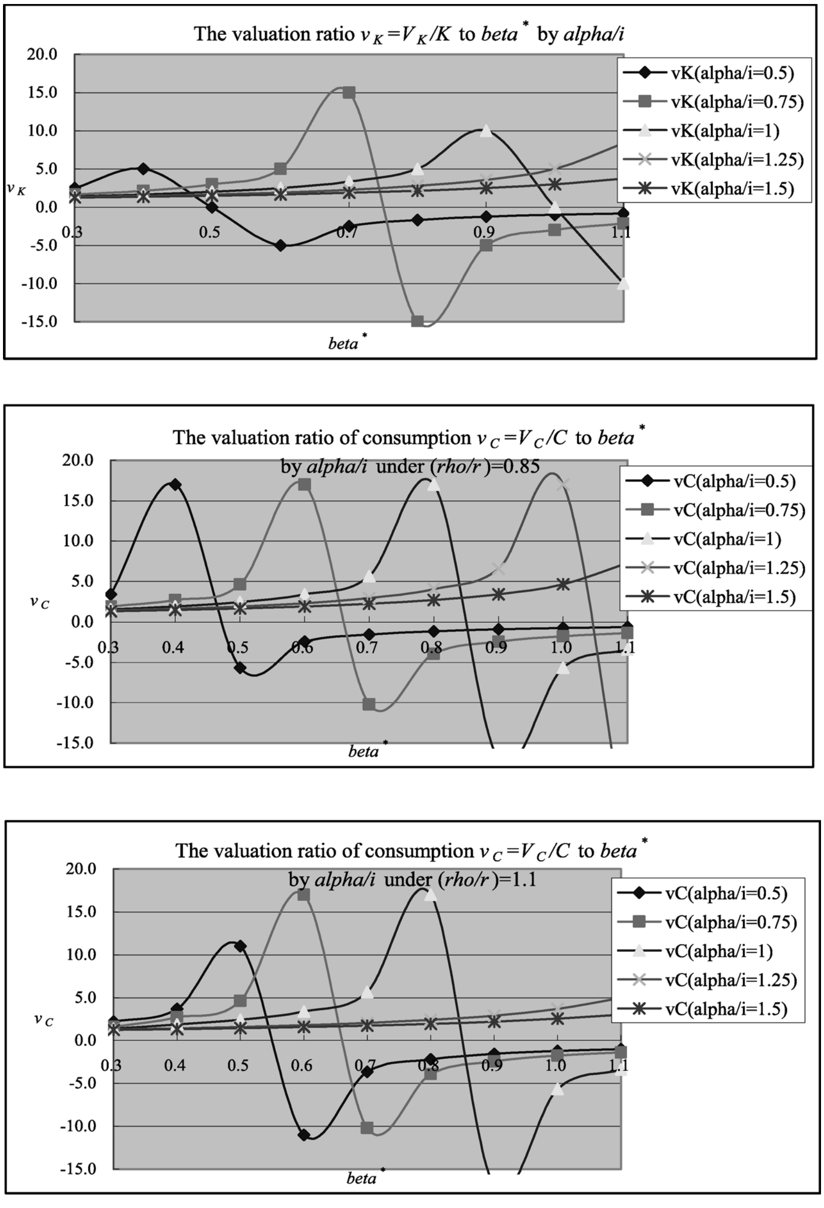


Figure 2 *The* valuation ratios of *K* and *C*, $v_K = V_K / K$ & $v_C = V_C / C$, to $beta^*$ by *alpha/i* (1)

output share of government sector and Y_{PEI}/Y is the output share of private sector.

For each sector, I will first use Eq. 17 and then for integration I will use Eq. 18. This is because in each sector I use the ratio of an item to the output of each sector.

Second, let me confirm the relationship between the rate of saving, *s*, and the ratio of net

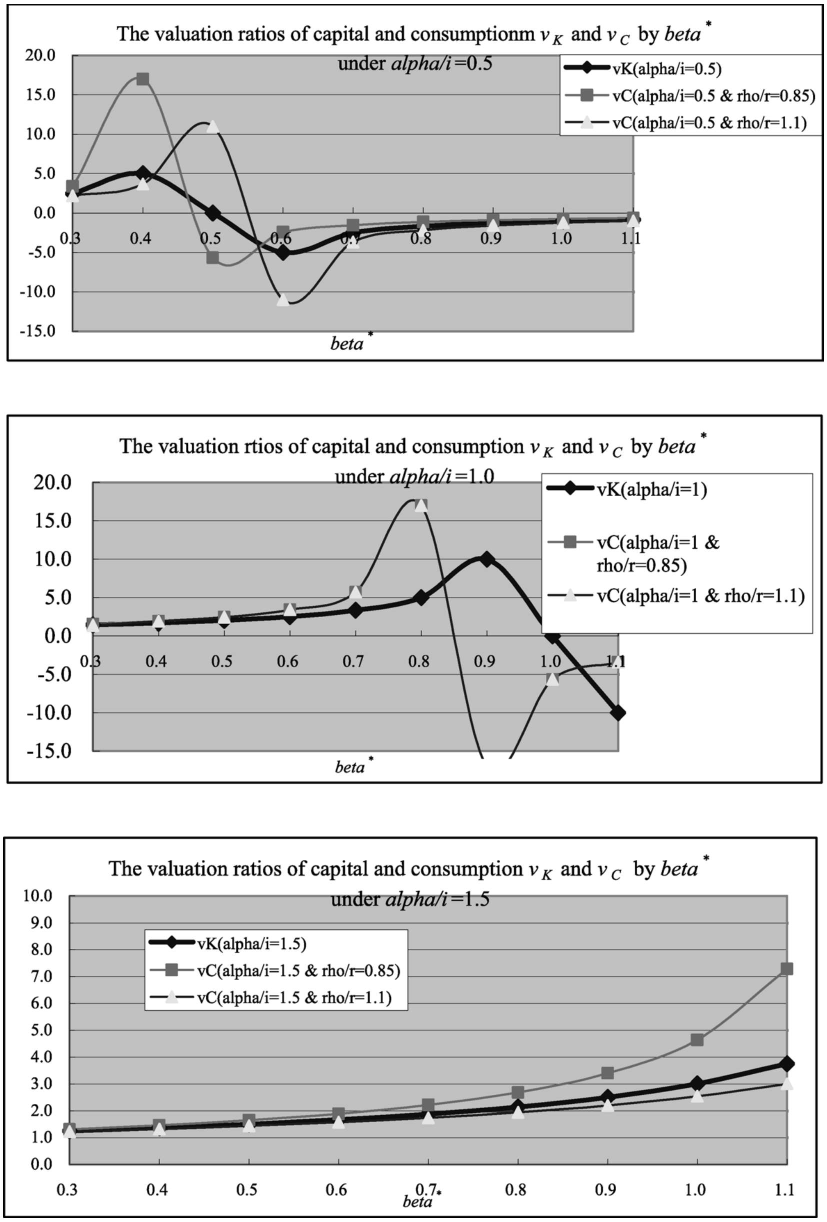


Figure 3 *The* valuation ratios of *K* and *C*, $v_K = V_K / K$ & $v_C = V_C / C$, to *beta by *alpha/i* (2)**

investment to output, i. The ratio of i to s was defined as θ: $\theta \equiv i / s$. And, $\psi \equiv s - i$ was shown as above. What is the relationship between ψ and θ? This is interestingly summarized as follows:

$$\psi = s(1-\theta)^{18)} \text{ or } s = \frac{\psi}{1-\theta} \text{ or,} \tag{19}$$

$$\psi = i\,(1/\theta - 1) \text{ or } i = \frac{\psi}{1/\theta - 1}. \tag{20}$$

Third, let me connect Eq. 20 each with the growth rate of output and the cost of capital, under convergence, where i is used.

Since $g_Y^* = \dfrac{i(1-\beta_{\delta=\alpha}^*)(1+n)}{1-\alpha} + n$ in Eq. 4,

$$g_Y^* = \frac{\psi(1-\beta_{\delta=\alpha}^*)(1+n)}{(1/\theta-1)(1-\alpha)} + n. \quad s \equiv \psi + i \text{ and } i \equiv s - \psi \tag{21}$$

Since $r^* - g_Y^* = g_Y^*\left(\dfrac{\alpha}{i \cdot \beta^*} - 1\right)$ in Eq. 7,

$$r^* - g_Y^* = g_Y^*\left(\frac{(1/\theta-1)\alpha}{\psi \cdot \beta^*} - 1\right). \tag{22}$$

Eqs. 21 and 22 show the case of the current external balance under convergence. The case of budget surplus/deficit is shown similarly as:

$$g_{Y(G)}^* = \frac{\psi_G(1-\beta_{\delta=\alpha(G)}^*)(1+n_G)}{(1/\theta_G-1)(1-\alpha_G)} + n_G. \tag{21-2}$$

$$r_G^* - g_{Y(G)}^* = g_{Y(G)}^*\left(\frac{(1/\theta_G-1)\alpha_G}{\psi_G \cdot \beta_G^*} - 1\right). \tag{22-2}$$

Likewise, the case of the private sector is shown as:

$$g_{Y(PRI)}^* = \frac{\psi_{PRI}(1-\beta_{\delta=\alpha(PRI)}^*)(1+n_{PRI})}{(1/\theta_{PRI}-1)(1-\alpha_{PRI})} + n_{PRI}. \tag{21-3}$$

$$r_{PRI}^* - g_{Y(PRI)}^* = g_{Y(PRI)}^*\left(\frac{(1/\theta_{PRI}-1)\alpha_{PRI}}{\psi_{PRI} \cdot \beta_{PRI}^*} - 1\right). \tag{22-3}$$

When Eqs. 21 to 22-3 are used for simulation using the change in the rate of saving, the relative share of rental, *alpha*, will change according to Eq. 1, $1-\alpha = c/(rho/r)$, where a utility coefficient,[19] (*rho*/*r*), will change with the change in saving. Note that budget deficit shows

18) For the current external balance and budget deficit, $s \equiv \psi + i$ and $i \equiv s - \psi$ are used a base. For example, in the case of the government sector, $s_G \equiv \psi_G + i_G = (T - C_G)/Y_G$ holds, where T is taxes and C_G is government expenditures that excludes investment. We need to treat each influence on the growth rate of output and the rate of rental to capital, separately using government expenditures and investment.

19) I do not intend to formulate/measure a utility function in this paper. The utility coefficient con- ↗

both $\psi_G < 0$ and $\theta_G < 0$, and if $\psi_G = 0$ and $\theta_G = 1$, Eq. 21-2 will be $g^*_{Y(G)} = n_G$ and the cost of capital is still minus by the growth rate of population or employed persons: $r^*_G - g^*_{Y(G)} = -n_G$.

The above equations suggest important policies for a solution of budget deficit in the government sector as follows:

1. Both investment and expenditures must be basically reduced under a given capital-output ratio so that the relative share of rental becomes plus.
2. For the improvement of deficit, ψ_G, the increase in saving (or, a plus rate of saving through the reduction of expenditures) is more essential than the reduction of investment alone.
3. Policy-makers must decrease the difference between saving and investment, where both levels must be reduced carefully so that the cost of capital becomes gradually plus.

5. Empirical results and findings in the cost of capital: capital versus consumption

This section briefly shows empirical results, in particular, for the relationship between the balance of payment, budget deficit and the difference between saving and investment in the private sector, together with each growth rate of output, the cost of capital, and the valuation ratio of capital under convergence, using Eqs. 7, 9, and 14. The raw data by country and by the government sector come from IFSY and GFSY, IMF. Tables 2-1, 2-2, and 2-3 show empirical results of twenty countries, 1996~2003, by sector.

The usefulness of these results depends on how suitably the values of (*rho*/*r*) by the level of the ratio of consumption to output, *c*, are estimated by country and sector. I recognize that each country has its own character of (*rho*/*r*). I distinguish the (*rho*/*r*) of consumption-oriented countries and that of saving-oriented countries such as Singapore and Malaysia, where, for example, 20% or more of employed persons' saving is deposited by the government until after retirement. A quadratic function of (*rho*/*r*)(*c*) is determined differently by the national taste or character. This function is easily determined when capital is given by country and sector (as in Japan). However, almost all countries do not publish capital or fixed assets since capital estimated using the perpetual inventory method by asset is not always trustworthy or consistent with a whole economy. Capital, I advocate, should be a value that is consistent with an economy as a whole. Kamiryo [2004c] presented a method for estimating

↘ nects rental or wages with consumption or saving, where output/income is the sum of consumption and saving.

Table 1 The ratio of i to Ω^*, i/Ω^*, and the ratio of g_y^* to Ω^*, g_y^*/Ω^*, by sector

AVERAGE 1996~2003	Japan	Korea	China	India	Brazil	Singapore	Malaysia	Indonesia	Thailand	Philippines
i/Ω^*	0.0230	0.1056	0.2011	0.2254	0.2129	0.2298	0.2716	0.4015	0.2552	0.2647
gy^*/Ω^*	0.0045	0.0360	0.0851	0.1366	0.1365	0.1159	0.1601	0.2882	0.1619	0.1740
$i_G/\Omega_G{}^*$	0.0469	0.0694	0.1387	(0.0107)	0.0651	0.0824	0.2322	0.1785	0.2547	0.1463
$gy_{(G)}{}^*/\Omega_G{}^*$	0.0041	0.0187	0.0337	(0.0351)	0.0018	0.0183	0.0777	0.0839	0.1284	0.0826
$i_{PRI}/\Omega_{PRI}{}^*$	0.0245	0.1100	0.2255	0.2381	0.2256	0.4514	0.2755	0.5895	0.1793	0.2981
$gy_{(PRI)}{}^*/\Omega_{\Pi P}$	0.0047	0.0373	0.1092	0.1506	0.1467	0.3143	0.1820	0.4716	0.1358	0.1987

AVERAGE	The U S	Canada	Russia	Australia	New Zealand	The U K	Sweden	Germany	France	Italy
i/Ω^*	0.0389	0.0523	0.1131	0.1100	0.1051	0.0329	0.0534	0.0614	0.0546	0.0617
gy^*/Ω^*	0.0079	0.0147	0.0613	0.0414	0.0444	0.0093	0.0231	0.0222	0.0181	0.0241
$i_G/\Omega_G{}^*$	0.0272	0.0723	0.2911	0.0647	0.0483	0.0113	0.0116	0.0331	0.0464	0.0429
$gy_{(G)}{}^*/\Omega_G{}^*$	0.0024	0.0313	0.1812	0.0294	0.0102	(0.0074)	0.0025	0.0188	0.0196	0.0148
$i_{PRI}/\Omega_{PRI}{}^*$	0.0400	0.0497	0.1009	0.1148	0.1101	0.0376	0.0572	0.0649	0.0557	0.0686
$gy_{(PRI)}{}^*/\Omega_{\Pi P}$	0.0059	0.0126	0.0551	0.0401	0.0429	0.0098	0.0191	0.0209	0.0173	0.0269

capital under an assumption that the relative share of rental is fixed and given. This was because a function of consumption had not been well-finalized.

Capital and rental are consistently estimated at the same time in an economy. Capital is estimated using the capital-labor ratio, k, and rental is estimated using the function of consumption, $(rho/r)(c)$, and accordingly, the relative share of rental α. Capital and rental are basically connected with $k = (\alpha/(1-\alpha))/(r/w)$. This equation is used either when capital is fixed or rental is fixed externally. When we estimate capital and rental simultaneously, it is important for us to know beforehand some characteristics and tendencies of (1) the ratio of rental to capital, (2) the capital-labor ratio, and (3) the capital-output ratio. The above factor each has its own lower/upper limit which is related to other factors as follows:

1. The ratio of rental to capital, r, has its upper limitation. The value of r cannot be far from those in other countries and also too far away from the central bank interest rate due to the existence of financial assets-neutrality.
2. The values of (rho/r) of "the private sector" are similar by country due to international competition/globalization, where we easily find a criterion of (rho/r) by the ratio of consumption to output.

The capital-labor ratio, k:

1. The capital-labor ratio, k, increases steadily by year if economic growth is stable. First set the value of the ratio of r to the wage rate, (r/w), constant by year and see the tendency of the capital-labor ratio: (1) If the capital-labor ratio decreases, adjust (r/w) so that the capital-labor ratio is maintained constant year by year. In this case, economic growth is expected to

be high. (2) If the capital-labor ratio does not decrease, economic growth is expected to be unstable or shocked.

2. Assume a stage that r decreases gradually by year, then (r/w) will be lower by year, assuming that the wage rate is constant. In this case, if the wage rate increases, (r/w) will be much lower by year. Then, the capital-labor ratio will rise rapidly, which increases the capital-output ratio rapidly so that the tendency of the capital-output ratio must be examined carefully (see next).

The capital-output ratio, Ω:

1. The level of the capital-output ratio, Ω, helps to decide/finalize the level of the capital-labor ratio. Because the capital-labor ratio does not include the level of technology, A, and the rate of technological progress, while the capital-output ratio does: $\Omega = k^{1-\alpha} / A$. In other words, the capital-output ratio is a better criterion than the capital-labor when it is measured endogenously.

2. The capital-output ratio has its upper limit at 3 ~ 4. As I indicated in Kamiryo [IARIW, 2004b], Club DD countries cannot raise this ratio above 1, Club DA countries such as China and India raise this ratio from 1 to 2 or 3, while Club AA (advanced) countries must maintain a constant level below 3 ~ 4.

The relative share of rental, α:

1. The relative share of rental, α, is the product of the capital-output ratio and the ratio of rental to capital: $\alpha = \Omega \cdot r$. As pointed out by Solow [1958], the value of α usually remains relatively constant (except for financial crisis and/or changing stages). From the relationship of $\alpha = \Omega \cdot r$, I indicate that the value of α usually falls between 8 to 15%.

2. When α changes unnaturally in the above range, I review the contents of the utility coefficient, (rho/r), and national taste hidden in $1 - \alpha = c / (rho / r)$, together with the utility coefficient to capital, $(rho/r)_{\Omega}$, in the function of consumption to capital, $(1-\alpha) = \dfrac{i}{(rho / r)_{\Omega^*} \cdot \Omega^*}$.

3. Finally, all in all, I will estimate capital and rental by country and by year, taking into consideration the exchange rate, CPI, and the Central Bank interest rate. After repeating the above processes by country, estimation of capital and rental will approach better result, where "measurable" is the beginning of fact findings.

Next, let me first summarize the relationship between the balance of payment or the current balance, budget deficit, and the difference between saving and investment in the private sector. Among twenty countries after 1996, the balance of payment is plus in many countries, except for India, Brazil, Philippines, the US, and the UK. However, budget deficit is minus in almost all countries, except for Singapore, Canada, Russia, New Zealand, and Sweden (but not in 2003). Nevertheless, all the countries show each a plus difference between saving and investment in the private sector, except for Philippines, the US, Australia, New Zealand, and the UK (though not always). Extreme cases are: (1) Japan has maintained a plus balance of payment, but with an extreme budget deficit, and (2) the US and the UK (not always) show each a minus balance of payment and a minus deficit.

Does budget deficit lower the growth rate? Or, does budget deficit significantly aggravate the cost of capital in the government sector? Both are facts and proved using equations above. Democratic countries spend government saving for expenditures with excessive investment, beyond its limit in order to win elections. Budget deficit makes the growth rate of output of the government sector extremely minus: for example, this rate is minus 10%, which implies that national assets decreases by that amount, resulting in an assets-deflation. There are several cases for the same limit of 3% ($s_G – i_G$) EU rule: if $s_G = 0$, $i_G = -3\%$. The countries show a plus rate of government saving are Korea, China, Singapore, Malaysia, Indonesia (nearly), Thailand, Canada, Russia, Australia (nearly), New Zealand, and Sweden. These countries can recover from deficit rather easily. Extreme cases on average are: Japan, 40% (of government output), India, 70%, Brazil, 30%, Philippines, 20%, the US, 10%, the UK, 5%, Germany, 8%, France, 10%, and Italy, 6%. Among these countries, Japan, the US, and the UK have aggravated each level.

The growth rates of output of all the government sectors show plus under convergence, except for some periods after 1996. However, the ratio of rental to capital shows minus in many countries, except for Korea, China, Singapore, Malaysia, Indonesia, Thailand, the US, Canada, Australia, New Zealand, and Sweden. Or, Japan, India, Brazil, Philippines, Germany, France, and Italy need to turn the cost of capital each from a minus to a plus. In this respect, the cost of capital is a key criterion for improving deficit.

A country uses a minus cost of capital for people and earns a plus cost of capital in the private sector. However, each country has its responsibility to maintain a minimum growth rate of output. It is definitely true that an extreme deficit aggravates the growth rate of output as a whole. This is explained, using the mild case ("final" consumption instead of "actual" consumption) of Japan government sector, 1996~2003, as follows:

1. The ratio of investment to output has been still kept extremely high (from 28.77% to

12.66%). As a result, the growth rate of output under convergence is plus (from 5.85% to 2.15%), but the government share to total output has decreased (from 17.15% to 14.19%). It is true that government output to some extent increases with an extremely high level of

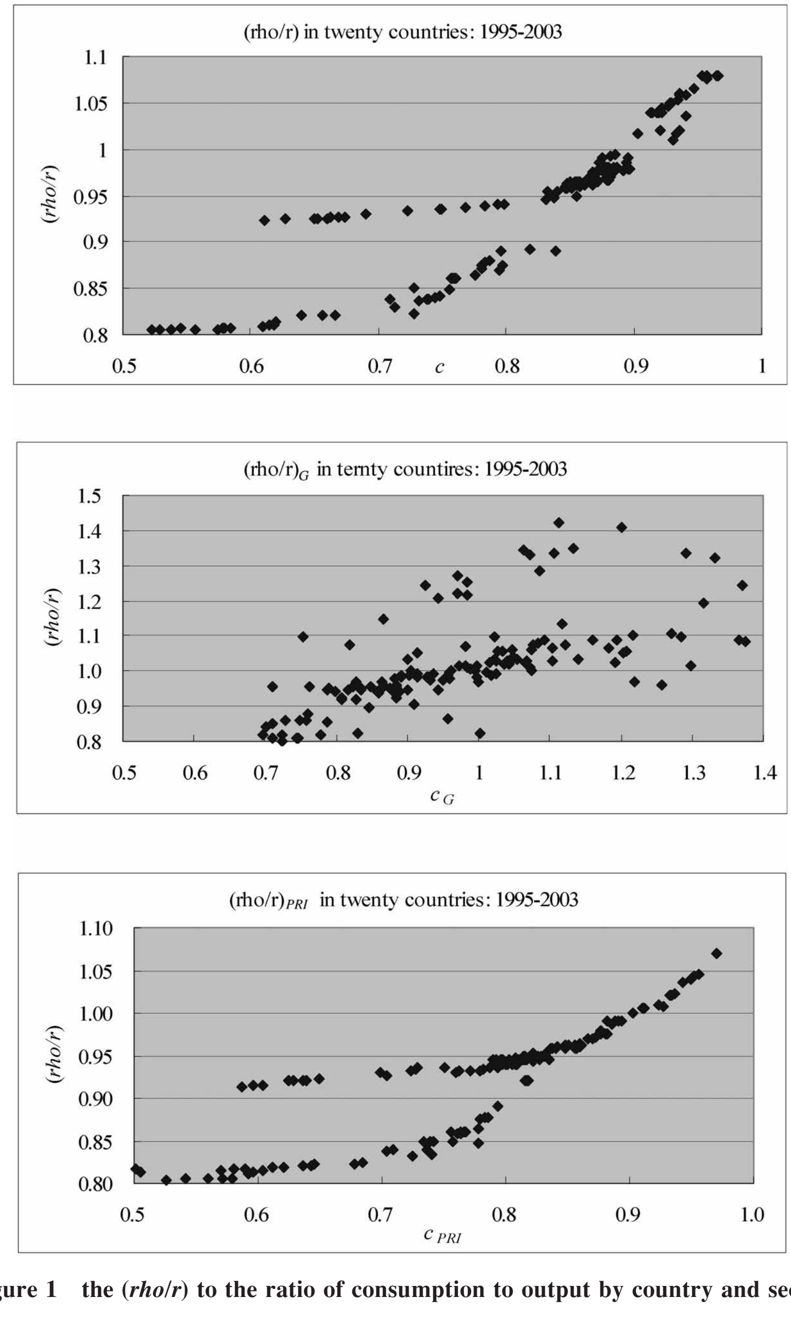


Figure 1 the (*rho*/*r*) to the ratio of consumption to output by country and sector

Table 2-1 The growth rate of output, g_Y^*, and the cost of capital, $r^* - g_Y^*$, by country

	Japan	Korea	China	India	Brazil	Singapore	Malaysia	Indonesia	Thailand	Philippines
The growth rate of output under convergence, g_Y^*, for the total economy										
1996	0.0334	0.1179	0.1528	0.1233	0.1072	0.2787	0.3029	0.1876	0.1989	0.1438
1997	0.0293	0.0966	0.1434	0.1098	0.1079	0.2795	0.3080	0.1684	0.1451	0.1518
1998	0.0233	0.0484	0.1401	0.1047	0.1033	0.2364	0.1604	0.1416	0.0690	0.1203
1999	0.0209	0.0682	0.1405	0.1033	0.0893	0.2058	0.1223	0.1004	0.0601	0.0938
2000	0.0210	0.0787	0.1393	0.0976	0.0878	0.1747	0.1587	0.1150	0.0686	0.1006
2001	0.0196	0.0694	0.1402	0.0908	0.0841	0.1555	0.1439	0.1143	0.0742	0.0799
2002	0.0159	0.0674	0.1376	0.0938	0.0699	0.1256	0.1280	0.0994	0.0742	0.0742
2003	0.0148	0.0632	0.1345	0.0926	0.0548	0.1085	0.1121	0.1051	0.0808	0.0741
AVERAGE	0.0223	0.0762	0.1410	0.1020	0.0880	0.1956	0.1795	0.1290	0.0964	0.1048
The cost of capital, $r^*-g_Y^*$, for the total economy										
1996	0.0022	(0.0026)	0.0576	0.1067	0.1107	0.3880	0.2251	0.1118	0.0188	0.0905
1997	0.0035	0.0065	0.0649	0.0864	0.0884	0.3339	0.2142	0.0816	0.0675	0.0493
1998	0.0065	0.0427	0.0599	0.0647	0.0779	0.3389	0.3458	0.1892	0.1607	0.0702
1999	0.0085	0.0413	0.0506	0.0522	0.0541	0.2874	0.3766	0.1409	0.1289	0.0832
2000	0.0085	0.0258	0.0458	0.0506	0.0434	0.2507	0.3649	0.1463	0.1085	0.0537
2001	0.0095	0.0166	0.0432	0.0520	0.0399	0.2570	0.3198	0.1308	0.0874	0.0578
2002	0.0157	0.0095	0.0403	0.0368	0.0417	0.2658	0.2878	0.1119	0.0936	0.0536
2003	0.0171	0.0127	0.0360	0.0179	0.0543	0.3195	0.2862	0.0551	0.0920	0.0488
AVERAGE	0.0089	0.0191	0.0498	0.0584	0.0638	0.3052	0.3026	0.1209	0.0947	0.0634

	The U S	Canada	Russia	Australia	New Zealand	The U K	Sweden	Germany	France	Italy
The growth rate of output under convergence, g_Y^*, for the total economy										
1996	0.0304	0.0348	0.1040	0.0738	0.0835	0.0243	0.0330	0.0465	0.0375	0.0385
1997	0.0312	0.0411	0.0840	0.0764	0.0702	0.0231	0.0300	0.0435	0.0343	0.0386
1998	0.0332	0.0402	0.0674	0.0812	0.0650	0.0279	0.0333	0.0409	0.0355	0.0380
1999	0.0345	0.0407	0.0455	0.0809	0.0668	0.0264	0.0353	0.0474	0.0384	0.0401
2000	0.0351	0.0390	0.0662	0.0732	0.0571	0.0255	0.0376	0.0476	0.0425	0.0426
2001	0.0326	0.0395	0.0681	0.0644	0.0558	0.0249	0.0339	0.0411	0.0418	0.0418
2002	0.0287	0.0396	0.0594	0.0642	0.0602	0.0230	0.0320	0.0331	0.0394	0.0413
2003	0.0291	0.0390	0.0553	0.0720	0.0631	0.0222	0.0283	0.0320	0.0372	0.0385
AVERAGE	0.0318	0.0392	0.0687	0.0732	0.0652	0.0247	0.0329	0.0415	0.0383	0.0399
The cost of capital, $r^*-g_Y^*$, for the total economy										
1996	0.0149	0.0234	0.1257	0.0089	0.0170	0.0260	0.0552	0.0118	0.0177	0.0351
1997	0.0148	0.0146	0.0865	0.0136	0.0190	0.0249	0.0596	0.0129	0.0220	0.0292
1998	0.0131	0.0148	0.1081	0.0037	0.0193	0.0225	0.0532	0.0136	0.0200	0.0239
1999	0.0096	0.0149	0.1141	(0.0029)	0.0137	0.0226	0.0483	0.0106	0.0180	0.0160
2000	0.0085	0.0179	0.1213	0.0040	0.0236	0.0233	0.0461	0.0062	0.0142	0.0117
2001	0.0123	0.0186	0.0753	0.0082	0.0211	0.0231	0.0428	0.0142	0.0152	0.0182
2002	0.0156	0.0187	0.0489	0.0034	0.0152	0.0236	0.0460	0.0226	0.0166	0.0174
2003	0.0142	0.0194	0.0539	(0.0078)	0.0091	0.0231	0.0460	0.0242	0.0151	0.0148
AVERAGE	0.0129	0.0178	0.0917	0.0039	0.0173	0.0236	0.0496	0.0145	0.0173	0.0208

Note:

1. The cost of capital is defined as $r^*-g_Y^*$, which is used for the denominator of the valuation ratio of capital.
2. The value of r^*/g_Y^* is confirmed by the value of $c(s-\alpha)=\alpha/\alpha_{GOLDEN}=((\theta_{OPEN}\cdot\beta^*)\cdot s)$.
3. $s=\alpha$ under the golden rule only if $\theta_{OPEN}\cdot\beta^*=1.0$.

Table 2-2 The growth rate of output, $g_{Y(G)}{}^{*}$, and the cost of capital, $r_{(G)}{}^{*} - g_{Y(G)}{}^{*}$, in the government sector

	Japan	Korea	China	India	Brazil	Singapore	Malaysia	Indonesia	Thailand	Philippines
The growth rate of output under convergence, $g_{Y}{}^{*}{}_{G}$, in the government sector										
1996	0.0585	0.0873	0.0995	0.0173	(0.0571)	0.1927	0.1219	0.0740	0.1815	0.0921
1997	0.0512	0.0999	0.1027	0.0135	0.0262	0.1111	0.1543	0.1226	0.3092	0.1287
1998	(0.0158)	0.0817	0.1089	0.0715	0.0613	0.1778	0.1507	0.0708	0.2330	(0.0140)
1999	0.0963	0.0154	0.1082	0.0668	0.0379	0.0771	0.2215	0.0873	0.2534	0.0727
2000	0.0495	0.0066	0.1109	0.0258	0.0523	0.1002	0.1611	0.1869	(0.0140)	0.1178
2001	0.0199	0.0644	0.1122	(0.0067)	0.0433	(0.0425)	0.1959	0.0709	0.0836	(0.0929)
2002	0.0057	0.0172	0.1210	0.0622	0.1167	(0.0132)	0.1379	0.0779	(0.0396)	0.0666
2003	0.0215	0.0314	0.1207	(0.0217)	(0.0289)	(0.0124)	0.1376	0.0264	0.0529	(0.0105)
AVERAGE	0.0359	0.0505	0.1105	0.0286	0.0315	0.0738	0.1601	0.0896	0.1325	0.0450
The cost of capital, $r^{*}{}_{G}$-$g_{Y}{}^{*}{}_{G}$, in the government sector										
1996	(0.0613)	(0.0199)	(0.0279)	(1.1067)	(0.8124)	0.5038	0.0018	0.0932	0.0991	0.0121
1997	(0.0490)	(0.0366)	(0.0262)	(0.9149)	(1.2467)	0.3026	0.0612	(0.0602)	(0.1281)	(0.0574)
1998	(0.0032)	(0.1009)	(0.0369)	(1.2012)	(1.2630)	0.4157	(0.0965)	0.0081	(0.0549)	0.0001
1999	(0.1158)	0.0342	(0.0551)	(1.3938)	(0.9341)	0.2467	(0.1765)	0.0594	(0.1654)	(0.2564)
2000	(0.0701)	0.0643	(0.0669)	(1.4958)	(0.4728)	0.2223	(0.0996)	(0.1852)	0.0273	(0.4356)
2001	(0.0213)	0.0441	(0.0853)	(1.5266)	(0.0651)	(0.1439)	(0.1453)	(0.0429)	(0.0585)	0.1212
2002	(0.0073)	0.0750	(0.0462)	(1.7183)	0.0801	(0.1578)	(0.1719)	(0.0902)	0.0732	(0.2462)
2003	(0.0207)	(0.0296)	(0.0270)	(1.9597)	(0.2080)	0.0902	(0.1400)	0.0343	(0.0207)	(0.0390)
AVERAGE	(0.0436)	0.0038	(0.0464)	(1.4146)	(0.6152)	0.1849	(0.0958)	(0.0229)	(0.0285)	(0.1127)

	The U S	Canada	Russia	Australia	New Zealand	The U K	Sweden	Germany	France	Italy
The growth rate of output under convergence, $g_{Y}{}^{*}{}_{G}$, in the government sector										
1996	0.0100	0.0359	0.0233	0.0552	0.1050	0.0693	0.0346	0.0242	0.0443	0.0773
1997	0.0332	0.0544	0.1025	0.0346	0.0100	0.0117	0.0046	0.0072	0.0471	0.0941
1998	0.0206	0.0290	(0.0491)	0.0455	(0.0264)	0.0459	0.0241	0.0288	0.0179	0.0367
1999	0.0485	0.0622	0.0600	(0.0070)	0.0703	0.0203	0.0246	(0.0117)	0.0193	0.0128
2000	0.0488	0.0504	0.2029	0.0644	(0.0392)	(0.0026)	(0.0089)	0.0324	0.0236	(0.0014)
2001	(0.1082)	0.0459	0.1396	0.0053	0.0404	0.0100	(0.0220)	(0.0152)	0.0186	0.0208
2002	(0.0005)	0.0293	0.1520	0.0327	0.0366	(0.0617)	0.0379	0.0078	0.0203	(0.0374)
2003	0.0299	0.0392	0.0997	0.0397	0.0228	0.0236	(0.0931)	0.0232	0.0174	(0.0112)
AVERAGE	0.0103	0.0433	0.0914	0.0338	0.0274	0.0146	0.0002	0.0121	0.0261	0.0240
The cost of capital, $r^{*}{}_{G}$-$g_{Y}{}^{*}{}_{G}$, in the government sector										
1996	0.1181	0.0360	(0.7728)	(0.0760)	0.2455	0.0497	(0.1825)	(0.0692)	(0.1511)	(0.3314)
1997	0.1289	0.0845	(0.7266)	0.0271	0.3076	0.1379	0.0119	(0.0307)	(0.0887)	(0.0617)
1998	0.1626	0.1036	(0.2272)	0.1613	0.0812	0.1496	0.0682	(0.0464)	(0.0307)	(0.0527)
1999	0.1451	0.0389	(0.6269)	0.0305	0.0597	0.1671	0.2225	(0.0097)	(0.0054)	0.0090
2000	0.1577	0.0297	(0.0253)	0.0972	0.0470	0.2096	0.4223	0.0502	0.0079	(0.0319)
2001	0.2355	0.0006	0.3245	0.0746	0.0847	0.1934	0.2778	(0.0810)	0.0049	0.0494
2002	0.1410	0.0434	0.1238	(0.0771)	0.1359	0.2899	0.3139	(0.1064)	(0.0646)	0.0352
2003	0.0493	0.0512	0.1729	0.0052	0.2410	0.1373	0.1388	(0.1450)	(0.0857)	0.0110
AVERAGE	0.1423	0.0485	(0.2197)	0.0303	0.1503	0.1668	0.1591	(0.0548)	(0.0517)	(0.0466)

Note:

1. The cost of capital is defined as r^{*}-$g_{Y}{}^{*}$, which is used for the denominator of the valuation ratio of capital.
2. The value of $r^{*}/g_{Y}{}^{*}$ is confirmed by the value of $c\,(s-\alpha)=\alpha/\alpha_{GOLDEN}=((\theta_{OPEN}\cdot\beta^{*})\cdot s)$.
3. $s=\alpha$ under the golden rule only if $\theta_{OPEN}\cdot\beta^{*}=1.0$.

Table 2-3 The growth rate of output, $g_{Y(PRI)}^*$, and the cost of capital, $r_{(PRI)}^* - g_{Y(PRI)}^*$, in the private sector

	Japan	Korea	China	India	Brazil	Singapore	Malaysia	Indonesia	Thailand	Philippines
The growth rate of output under convergence, $g_{Y}^{*}{}_{PRI}$, in the private sector										
1996	0.0289	0.1226	0.1702	0.1363	0.1449	0.2824	0.3413	0.2840	0.2065	0.1555
1997	0.0270	0.0961	0.1567	0.1207	0.1241	0.3230	0.3198	0.2392	0.1082	0.1569
1998	0.0409	0.0448	0.1502	0.1082	0.1113	0.2413	0.1311	0.1928	0.0199	0.1518
1999	0.0031	0.0771	0.1519	0.1071	0.0987	0.2563	0.0679	0.1300	(0.0231)	0.0980
2000	0.0157	0.0907	0.1496	0.1047	0.0938	0.1974	0.1138	0.1120	0.1140	0.0975
2001	0.0176	0.0702	0.1506	0.0998	0.0908	0.2337	0.0606	0.1331	0.0554	0.1129
2002	0.0173	0.0747	0.1434	0.0967	0.0631	0.1751	0.0668	0.1123	0.1334	0.0754
2003	0.0086	0.0673	0.1392	0.1022	0.0674	0.1453	0.0469	0.1114	0.0860	0.0885
AVERAGE	0.0199	0.0804	0.1515	0.1095	0.0993	0.2318	0.1435	0.1643	0.0875	0.1171
The cost of capital, $r^*{}_{PRI} - g_{Y}^{*}{}_{PRI}$, in the private sector										
1996	0.0282	(0.0012)	0.0932	0.2250	0.2062	0.1310	0.1545	0.2432	(0.0835)	0.1206
1997	0.0300	0.0118	0.1010	0.1597	0.2282	0.1955	0.0930	0.2786	0.0226	0.0839
1998	0.0202	0.0555	0.0969	0.1386	0.2182	0.1429	0.2681	0.3770	0.1427	0.1018
1999	0.0612	0.0406	0.0954	0.1195	0.1350	0.1778	0.4210	0.1961	0.2061	0.1672
2000	0.0471	0.0184	0.0946	0.1052	0.0762	0.1161	0.3056	0.2078	0.1160	0.1484
2001	0.0363	0.0130	0.0992	0.0905	0.0419	0.3883	0.2043	0.1144	0.1344	0.0464
2002	0.0413	0.0005	0.0741	0.0820	0.0449	0.3729	0.2270	0.1180	0.0618	0.1101
2003	0.0501	0.0156	0.0590	0.0516	0.0534	0.3104	0.1970	0.0378	0.0839	0.0740
AVERAGE	0.0393	0.0193	0.0892	0.1215	0.1255	0.2294	0.2338	0.1966	0.0855	0.1065

	The U S	Canada	Russia	Australia	New Zealand	The U K	Sweden	Germany	France	Italy
The growth rate of output under convergence, $g_{Y}^{*}{}_{PRI}$, in the private sector										
1996	0.0330	0.0346	0.1041	0.0771	0.0797	0.0177	0.0326	0.0505	0.0362	0.0322
1997	0.0309	0.0388	0.0825	0.0839	0.0812	0.0246	0.0366	0.0501	0.0317	0.0287
1998	0.0347	0.0422	0.0724	0.0876	0.0818	0.0253	0.0356	0.0432	0.0391	0.0383
1999	0.0329	0.0369	0.0353	0.0969	0.0662	0.0273	0.0378	0.0580	0.0424	0.0479
2000	0.0335	0.0370	0.0368	0.0747	0.0731	0.0294	0.0484	0.0503	0.0464	0.0544
2001	0.0495	0.0383	0.0410	0.0739	0.0580	0.0268	0.0455	0.0509	0.0464	0.0471
2002	0.0316	0.0416	0.0279	0.0685	0.0637	0.0341	0.0308	0.0372	0.0433	0.0608
2003	0.0290	0.0389	0.0310	0.0767	0.0691	0.0221	0.0554	0.0336	0.0411	0.0497
AVERAGE	0.0344	0.0385	0.0539	0.0799	0.0716	0.0259	0.0403	0.0467	0.0408	0.0449
The cost of capital, $r^*{}_{PRI} - g_{Y}^{*}{}_{PRI}$, in the private sector										
1996	0.0032	0.0218	0.0966	0.0172	(0.0043)	0.0239	0.0851	0.0201	0.0409	0.0787
1997	0.0029	0.0065	0.1246	0.0093	(0.0128)	0.0117	0.0621	0.0160	0.0378	0.0436
1998	(0.0025)	0.0030	0.0357	(0.0169)	0.0053	0.0081	0.0501	0.0207	0.0254	0.0337
1999	(0.0034)	0.0128	0.0956	(0.0127)	0.0094	0.0058	0.0285	0.0097	0.0199	0.0179
2000	(0.0053)	0.0170	0.0962	(0.0063)	0.0136	0.0027	0.0033	0.0000	0.0138	0.0224
2001	(0.0124)	0.0213	0.0309	(0.0020)	0.0156	0.0058	0.0172	0.0222	0.0149	0.0105
2002	0.0037	0.0147	0.0224	0.0078	0.0044	(0.0039)	0.0251	0.0358	0.0263	0.0112
2003	0.0109	0.0150	0.0166	(0.0108)	(0.0106)	0.0135	0.0212	0.0456	0.0270	0.0144
AVERAGE	(0.0003)	0.0140	0.0648	(0.0018)	0.0026	0.0085	0.0366	0.0213	0.0258	0.0290

Note:

1. The cost of capital is defined as $r^* - g_Y^*$, which is used for the denominator of the valuation ratio of capital.
2. The value of r^*/g_Y^* is confirmed by the value of $c(s-\alpha) = \alpha/\alpha_{GOLDEN} = ((\theta_{OPEN} \cdot \beta^*) \cdot s)$.
3. $s = \alpha$ under the golden rule only if $\theta_{OPEN} \cdot \beta^* = 1.0$.

investment, but at the expense of a high level of the capital-output ratio (from 4.03 to 5.65). The actual growth rate of government output has been unstable: e.g., in 2002 it was −13.94%.

2. Budget deficit has increased from 33.34 % to 64.99 %, and the cost of capital has been minus (from −6.13% to −2.07%). The valuation value of government capital has decreased, calling in assets-deflation and yet, the capital-output ratio has passed the range of limit. In this sense, if government investment is partly replaced by private investment, the growth rate of total output will increase more definitely.

6. Conclusions

This paper shows the basics of the ratio of rental to capital, the growth rate of output, the cost of capital, the valuation ratio, each under convergence, and the arrangement for the differences between saving and investment in the total economy, the government sector, and the private sector, with the influence on the cost of capital and the valuation ratio by sector. The cost of capital in this paper differs from the user costs of capital in the literature in several points: (1) the cost of capital is endogenous under convergence while the user costs of capital is exogenous in the current situation, (2) the cost of capital is that after deducting the growth rate of output while the user costs of capital corresponds with the ratio of rental to capital in the current situation (without deducting the growth rate), and (3) the cost of capital is divided into the two sectors while the user costs of capital cannot since the government sector assumes that gross operating surplus equals depreciation (or rental is zero).

The difference between saving and net investment by sector is shown using the balance of payment = budget surplus/deficit + the difference between saving and net investment in the private sector. For the total economy and the private sector, I need each a function of consumption and a function of consumption to capital, together with a utility coefficient, (rho/r), and a utility coefficient to capital, $(rho/r)_{\Omega}$, which enables all the equations are completed under convergence. Variables are expressed using the ratio of investment to output each as an independent variable. By so doing, the difference between saving and net investment by sector is connected with the growth rate of output and the cost of capital. In these cases, a technology-golden rule (comparable to the golden rule of Phelps [1961, 65]) works in the total economy and the private sector.

For empirical analysis, this paper showed interesting results by country and by sector, applying the above functions to the data available in IFSY, IMF.[20)] Without the function of consumption and accordingly, the function of investment, my equations were not completed, where I find that the difference of national taste by country is significantly important in these functions. The difference of national taste by country or by club will be further discussed in IARIW, Finland, 2006. It is vital for an economy first to have *beta* closer to *beta**, through structural reform. However, even if *beta** is within reach the growth rate under convergence will be extremely low under extreme deficit, as shown in Japan. Empirical comparisons of the government sector by country will be discussed in Kamiryo [2006], Finland, IARIW.

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Appendix The relationship between gross and net investment and depreciation

This appendix discusses the relationship between gross and net investment and depreciation, using the ratio of gross investment to capital, the growth rate of capital, and the depreciation rate. The literature assumes that the depreciation rate is given under an exogenous growth model. When an endogenous growth model is used, what is the relationship between the growth rate of capital (using net investment after depreciation) and an endogenous depreciation rate? I will discuss this issue when the growth rate of capital is endogenously measured.

Saving is directly related to net investment after depreciation. The literature shows net investment after deducting depreciation using given depreciation or a depreciation rate.[21] I indicate that net investment is closely related to depreciation under both the current and convergence situations, and accordingly, an endogenous growth rate of capital (using net investment) is closely related to an endogenous rate of depreciation. For simplicity, it is reasonable to assume that the ratio of gross investment to capital is two times the growth rate of capital. In this case, a given/exogenous depreciation rate will be examined together with a discussion of whether the depreciation rate should reflect technology and the rate of technological progress under convergence.

First I discuss the above issue using net investment and depreciation. Second, I replace net investment with gross investment, where I expect the same results as the use of net investment. In both approaches, I introduce two cases: (1) gross investment is depreciated from the current period/year, and (2) gross investment is depreciated from the next period/year as in the literature, where I expect a negligible difference between the two cases. Let me start with the first case that uses net investment with definitions:

$I_{NET(t)}$ is net investment after deducting total depreciation that is the sum of the depreciation for the capital at the beginning of the period, $K_{(t\text{-}1)}$, and the depreciation for gross investment during the current period t. Net investment is shown as,

$$I_{NET(t)} = I_{GROSS(t)} - D_{EP(t)} = I_{GROSS(t)} - \delta_{DEP(t)}(I_{GROSS(t)} + K_{t-1}), \tag{A1}$$

where $D_{EP(t)} = \delta_{DEP(t)}(I_{GROSS(t)} + K_{t-1})$ and

21) D. W. Jorgenson and Z. Griliches [1967, Eq. 14 on page 277] show $K_{t+1} = I_t + (1-\delta)K_t$, where gross investment is not depreciated at the current year, and thus, $\delta = D_{EP} / K_t$.

$$I_{NET(t)} = I_{GROSS(t)}(1-\delta_{DEP(t)}) - \delta_{DEP(t)} \cdot K_{t-1}.$$

Then, I can connect Eq. A1 with the growth rate of capital (or, the growth rate of "net" investment to capital) as,

$$g_{K(t)} \equiv \frac{K_t - K_{t-1}}{K_{t-1}} = \frac{I_{NET(t)}}{K_{t-1}} = \frac{I_{GROSS(t)} - \delta_{DEP(t)}(I_{GROSS(t)} + K_{t-1})}{K_{t-1}}. \tag{A2-1}$$

$$\text{Or, } g_{K(t)} = \frac{I_{GROSS(t)}(1-\delta_{DEP(t)})}{K_{t-1}} - \delta_{DEP(t)}, \tag{A3-1}$$

where if $(I_{GROSS(t)}(1-\delta_{DEP(t)}))/K_{t-1} > \delta_{DEP(t)}$, $g_{K(t)} > \delta_{DEP(t)}$, if $(I_{GROSS(t)}(1-\delta_{DEP(t)}))/K_{t-1} = \delta_{DEP}(t)$, $g_{K(t)} = 0$, and, if $(I_{GROSS(t)}(1-\delta_{DEP(t)}))/K_{t-1} < \delta_{DEP(t)}$, $g_{K(t)} < \delta_{DEP(t)}$. Note by definition that depreciation includes the depreciation for gross investment in the current period and that total depreciation is larger than the depreciation for the initial capital.

Now let me go to the second case: When gross investment is depreciated in the next period (as in Jorgenson and Griliches [1967], the above equations will be:

$$g_{K(t)} \equiv \frac{K_t - K_{t-1}}{K_{t-1}} = \frac{I_{NET(t)}}{K_{t-1}} = \frac{I_{GROSS(t)} - \delta_{DEP(t)} \cdot K_{t-1}}{K_{t-1}}, \text{ or} \tag{A2-2}$$

$$g_{K(t)} = \frac{I_{GROSS(t)}}{K_{t-1}} - \delta_{DEP(t)}. \tag{A3-2}$$

Now assume that the minimum of the ratio of gross investment to capital, $I_{Gross(t)}/K_{t-1}$, is zero, the growth rate of investment to capital equals the depreciation rate. This assumption is justified when the disposal of assets/capital differs from investment. When the disposal is included in gross investment, $I_{Gross(t)}/K_{t-1}$ may be minus. Then, a minus growth rate of capital is more than the depreciation rate, but showing a minus value.

In both cases, a special condition of $I_{GROSS(t)} = D_{EP(t)}$ (or net investment=0) constitutes a base for investment since "saving" is not used for gross investment. Only when gross investment is more than depreciation, the difference between gross investment and depreciation is supplied by saving. In other words, net investment always matches saving. When gross investment is less than depreciation, net investment is negative and, as a result, saving will be negatively used or borrowings are collected.

In both cases, what is the relationship between the growth rate of capital and the depreciation rate? Reforming Eq. A3-1, the depreciation rate is defined as,

$$\delta_{DEP(t)} \equiv \frac{I_{GROSS(t)} - g_{K(t)} \cdot K_{t-1}}{I_{GROSS(t)} + K_{t-1}}, \tag{A4}$$

where assuming $I_{Gross(t)}/K_{t-1}=0$, $\delta_{DEP(t)}=g_{K(t)}$, $g_{K(t)}=\delta_{DEP(t)}=\dfrac{I_{GROSS(t)}}{I_{GROSS(t)}+2K_{t-1}}$.[22)]

(A5-1)

Eq. A5-1 indicates that the growth rate of capital using net investment is almost equal to the depreciation rate.[23)] When $\delta_{DEP(t)}\neq g_{K(t)}$, how can the depreciation rate is determined? This is discussed in the second approach below when the ratio of gross investment to capital is introduced.

The above result is more simply shown when gross investment is depreciated in the next period (as in Jorgenson and Griliches [1967]. Eq. A5-1 is more simply shown using Eq. A2-2 as:

Assuming $I_{Gross(t)}/K_{t-1}=0$, $\delta_{DEP(t)}=g_{K(t)}$, $g_{K(t)}=\delta_{DEP(t)}=\dfrac{I_{GROSS(t)}}{2K_{t-1}}$. (A5-2)

Since gross investment equals the sum of net investment plus depreciation, Eq. A5-2 shows that net investment equals depreciation. In the case of Eq. A5-1, if I assume that gross investment in its denominator is zero, the same result will be obtained. In short, net investment and depreciation are dual or supplementary to gross investment.

Second, let me introduce into the above two cases the ratio of gross investment to capital, $g_{K(GROSS)t}$. First, for the case that gross investment is depreciated at the current period,

$$g_{K(GROSS)t}\equiv\frac{I_{GROSS(t)}}{K_{t-1}}=\frac{I_{NET(t)}+\delta_{DEP(t)}(I_{GROSS(t)}+K_{t-1})}{K_{t-1}}. \qquad \text{(A6-1)}$$

Second, for the case that gross investment is depreciated at the next period,

$$g_{K(GROSS)t}\equiv\frac{I_{GROSS(t)}}{K_{t-1}}=\frac{I_{NET(t)}+\delta_{DEP(t)}\cdot K_{t-1}}{K_{t-1}}, \qquad \text{(A6-2)}$$

$\dfrac{g_{K(GROSS)t}}{g_{K(t)}}=\dfrac{I_{GROSS(t)}}{I_{NET(t)}}$ always holds. In the above Eq. A5-2,

$g_{K(GROSS)t}=2g_{K(t)}=2\delta_{DEP(t)}$ holds. (A7)

This result was already suggested in Eq. A5-2.

Finally, let me discuss an endogenous depreciation rate under convergence. In my endogenous growth model, the growth rate of output equals the growth rate of capital, both of

22) Compare with Eq. A5-2: If $\delta_{DEP(t)}=\dfrac{I_{GROSS(t)}}{2K_{t-1}}$ holds, it is more useful to the development of growth models.

23) E.g., if $I_G=3$, $g_K=0.08$, $K=15$, then $\delta=0.1$ holds. If $I_G=3$, $D_{EP}=1.8$, $K=15$, then $g_K=0.08$ and $\delta=0.1$ hold. Assuming that $g_K=\delta$, this rate is 0.0909.

which come from an endogenous rate of technological progress. Since we cannot take into consideration the ratio of gross investment to capital under convergence, is it possible for us to assume that the ratio of gross investment to capital is two times the growth rate of capital under convergence? If this assumption is acceptable, net investment will be equal to depreciation. Then, the growth rate of capital equals the depreciation rate under convergence, which in turn fully reflects technological progress. This depreciation rate is measured as a whole in my endogenous growth model and differs from the depreciation rate exogenously given in the literature. Yet, it is useful to compare both depreciation rates in terms of technological progress.

Proposition 6: If gross investment is known and depreciated at the next period, the growth rate of gross investment is the sum of the growth rate of net investment plus the depreciation rate.

Proposition 7: If the growth rate of capital is endogenously measured under convergence, the corresponding depreciation rate under convergence reflects technological progress since the growth rate of output/capital is measured by an endogenous rate of technological progress. Since we cannot measure the ratio of gross investment to capital under convergence, the growth rate of output/capital will be equal to the deprecation rate, using an assumption that the minimum limit of gross investment is zero.

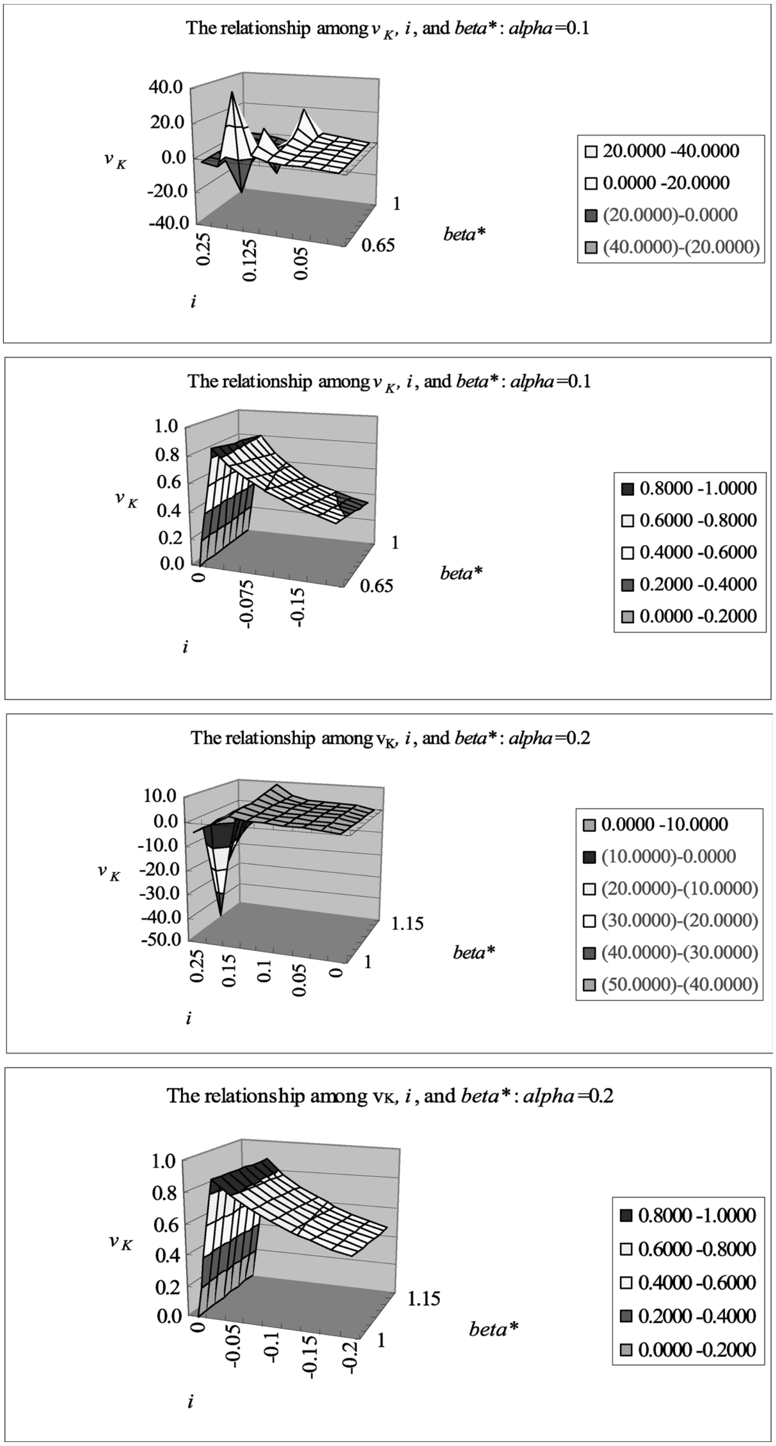

Figure A-1 The valuation ratio of capital: with *i*, *beta, and *alpha***

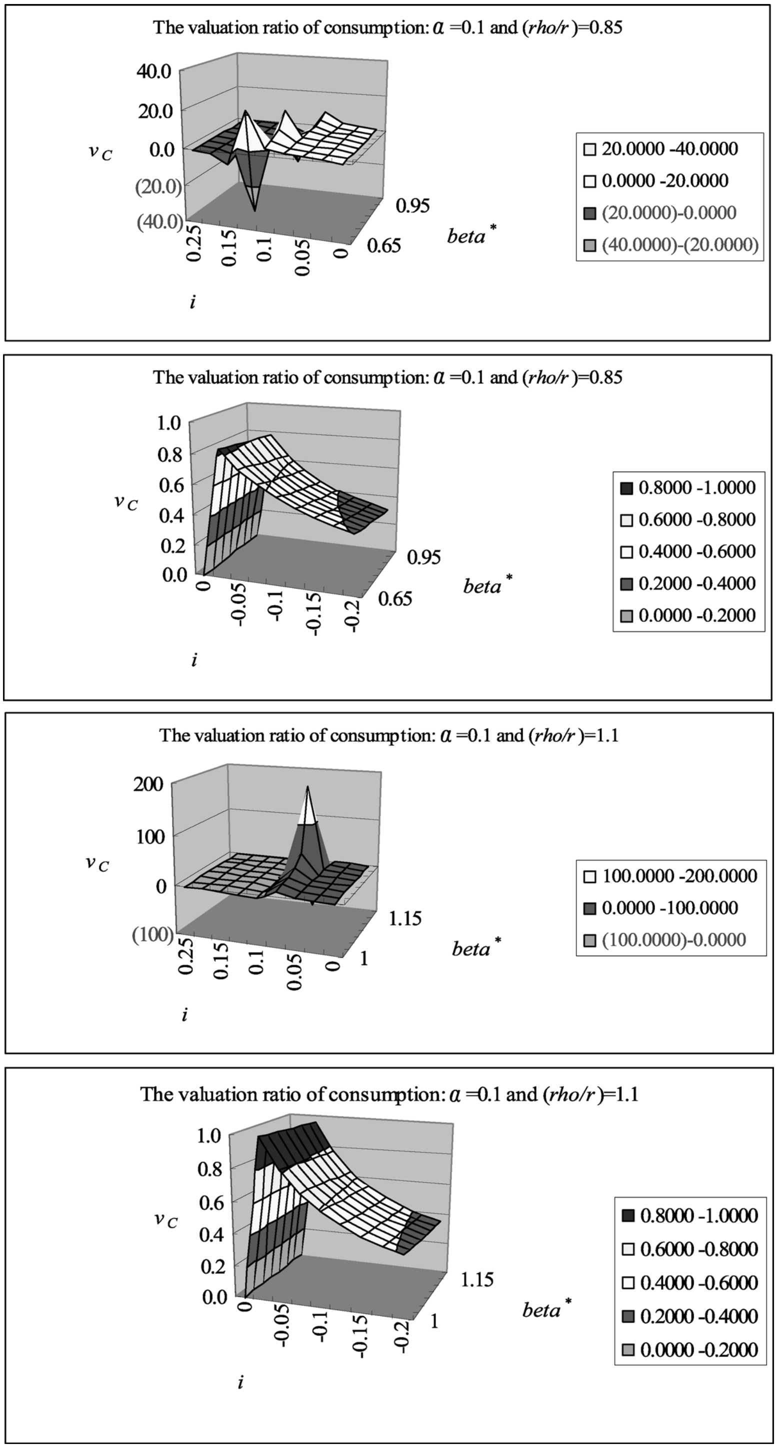

Figure A-2 The valuation ratio of consumption, v_C: with i, $beta^*$, *alpha, and* (rho/r)