Towards the Relationship between Constant Returns to Scale and Diminishing/Increasing Returns to Scale Using Two Production Functions: With Recursive Programming[1)]

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1. Introduction

The purpose of this paper is to clarify the relationship between constant returns to scale (CRS) and the diminishing/increasing returns to scale (DRS/IRS) in the transitional path, basically using my endogenous growth model. My endogenous growth model (as Model A in this paper) has been based on the Cobb-Douglas production, whose convergence point of time is under the constant returns to scale (CRS). I confirm that any model involved in the transitional path will become CRS at the convergence or in the balanced growth-state. Does the Cobb-Douglas production function really express DRS or IRS in the transitional path? This is my first doubt raised in this paper. To get rid of this doubt, I need to establish such a model that is clearly under DRS/IRS at the current situation. I call it Model B in this paper. I need to compare the DRS/IRS-oriented Model B with my Model A.

Now, referring to the relationship between 'to scale' and 'to capital,' the diminishing returns to capital (DRC), the increasing returns to capital (IRC), or the constant returns to capital (CRC) shows each the ratio of rents to capital. I use 'rents' instead of surplus or profit in my models so that I can measure DRC, IRC, or CRC not only for the private sector but also for the government sector. What is the relationship between DRS, IRS, or CRS and DRC, IRC, or CRC? I set DRS, IRS, or CRS such a situation as is related to the scale or the form of the production function. I set DRC, IRC, or CRC such a performance as shows the ratio of rents/surplus to capital. DRC, IRC, or CRC each corresponds with DRS, IRS, or CRS. For example, DRC is shown under DRS, DRC is shown under DRS, and CRC is shown under

1) I am much obliged to Dr. Toshimi Fujimoto. He reviewed my endogenous growth model (here Model A) repeatedly and finalized a short paper that explained how to measure the convergence/speed-coefficient by using effective labor, *L*/*A*. Without his help to the measurement of *delta*, I could not complete my model. He also allowed me to include the above paper of his in Appendix in this paper.

CRS. Thus, for the above discussion, this paper principally uses DRS, IRS, or CRS. Yet, the ratio of rents to capital is tightly related to the growth rate of output at the convergence. And, for the test of discussions in this paper, I need to use such variables as the ratio of rents to capital and the growth rate of output both at the current situation and those at the convergence.

Returning back to my purpose, if I use a certain fixed production function, can I clarify these transitions from DRS or IRS to CRS? Can I justify that Model A holds as a CRS production function? Does Model A express DRS/IRS at the current situation? Model B is a reformed model that shows DRC or IRC at the current situation under DRS or IRS. Does Model B express CRC or CRS at the convergence? If both models are true, then how are these models related to each other? Let me decide that in the path to the convergence I cannot replace a production function by a different production function and that each of the two models converges to CRC under CRS. Can I maintain both models such that show DRS or IRS at the current situation and CRS at the convergence? If the above challenges are possible, what is the logic to show DRC or IRC and CRS in the transitional path?

In this respect, what is the relationship between Models A and B? Is Model B consistent with Model A? Model A is correct since I have proved its justification together with empirical results in my several papers since 2003. If Model B is inconsistent with Model A, Model B is not correct. By using Model A, if I can endogenously and accurately show such variables at the convergence as the rate of technological progress, the growth rates of per capita capital and per capita output, and the ratio of rents to capital, then why do I need Model B? I must justify the existence of Model B. Does Model B clarify essential relationships between variables which Model A cannot clarify? For this test, I need to measure the relationship between the capital-output ratio and the level of technology in the transitional path, by using both models in parallel. I suggest that Model B may be supplemental to Model A, yet guarantees the existence of Model A and presents an equation to measure *delta*, which Model A could not. Throughout these discussions, the relationships between DRS, IRS, or CRS and DRC, IRC, or CRC will be clarified more numerically than in the literature.[2)]

2) I admit that my model as Model A shows a DRS/IRS at the current situation. And, Model B under DRS/IRS will become a CRS Model at the convergence: At the current situation, Model A is shown as $y = A_{RESIDUAL} k^{2\alpha-\delta}$ and Model B is shown as $y = B^{1-\delta} k^{1-\delta+\alpha}$, if I insert both 'the capital-labor ratio in the level of technology' and 'the original k that obeys my endogenous growth model' into an integrated capital-labor ratio. Note that Model A's power of k at the convergence will be equal to *alpha* while Model B's power of k at the convergence will be 1.0 (as an Ak model).

Then, what is the logic to converge from DRS/IRS to CRS in each model? I suggest: the convergence is originally possible in my endogenous growth model since this model (Model A) divides net investment into qualitative and quantitative investment and uses both a technology-oriented or structural reform parameter, *beta*, and a unique parameter neutralizing DRS/IRS, *delta*. And, Model B will succeed in measuring *delta* as shown in this paper. The *delta* is a value at the current situation and *delta* gradually converges to *alpha* at the convergence, where *delta* = *alpha*. The speed of convergence will be measured by using *delta* and *alpha*. Model B finally clarifies the interrelationship between *beta* and *delta* at the convergence since *delta* integrates *beta* and the capital-output ratio. Given *delta*, the essence of the capital-output ratio and accordingly, the rate of technological progress will be clarified endogenously more than in the literature.

Finally, why are the results in Model B be trustworthy? I suggest: (1) by an introduction of my endogenous growth model commonly into Models A and B, the capital-labor ratio is the same over time in the transitional path in Models A and B, (2) Model B is set so that it has the same per capita output over time in the transitional path as Model A has, by introducing an adjustment parameter[3)] and, (3) in both models all the values are the same at the convergence. And, I doubt: why does the literature so strictly distinguish the capital-labor ratio with the level of technology/the rate of technological progress? I suggest that if the capital-labor ratio is completely separated from the level of technology/the rate of technological progress, the results become extremely unstable in the transitional path.

If some part of capital and labor is involved in the level of technology, what is the character of the level of technology? Or, if a model moves from DRS/IRS to CRS under a condition that the level of technology is tightly related to the capital-labor ratio together with *delta* and *alpha*, what is the character of the level of technology? What justifies the combination of the capital-labor ratio and technology involved in the level of technology? If the level of technology excludes the capital-labor ratio, then the level of technology as a residual may be meaningless. I must give a rise to these doubts in the discussions of Models A and B. The literature generally drives the level of technology into a residual by taking capital broadly, but my approach is reversed. I will set the level of technology in various ways, by fixing capital to physical quantitative capital.

Concerning my tests and data, I will test the relationship between Models A and B, by

3) Accordingly, the capital-output ratio of Model A differs that of Model B in the transitional path except for the convergence point of time.

comparing the results shown using equations with those shown in my recursive programming.[4)] As a result, my phase diagram will differ from that in the textbooks. For these tests, I will use my data-sets that start with those original data as IFSY and GFSY, IMF.[5)] My data-sets are consistent with $Y = AK^{\alpha}L^{1-\alpha}$ or $y = Ak^{\alpha}$ whose base is shown as $Y = W + \Pi = C + S$.[6)] My data-sets are arranged by using the functions of consumption and investment as I have presented in 2004 and 2005. As a result, capital and rents by country and by sector are estimated wholly consistently in the data-sets. In this paper, I use these data-sets. I do not describe the details of my endogenous growth model and my data-sets in this paper (see IARIW, Finland, 2006).

2. Framework of Models A and B

This section discusses the characteristics of Models A and B using equations step by step, paying attention to the characters of different types of technology. For symbols, principally I will omit time/year, *t*, in each equation for simplicity.

4) I have formulated various types of recursive programming since 1995. After my PhD in 2003, I have concentrated on one type of recursive programming and improved the contents step by step while I have each time solved some part of a whole set of equations. My recursive program in this paper will be a final one since I succeeded in measuring *delta* in this paper. Any recursive programming must take into consideration the difference between the discrete case and the continuous case.

5) I am thankful to Steve Landefeld, Brian Sliker, and David Wasshausen at BEA, Francois Bourguignon, Alan Gelb, Luis Serven, and Lucie Albert-Drucker at the World Bank, and IMF people, and also Andrew Sharpe, IARIW, Ottawa, Louis Rouillard, Peter koumanakos, Richard Landry, and Roger Jullion at Statistics Canada, and Steve Pugliese and Helen Cutts at Finance Canada, when I made two week trip for discussions in early Oct 2005. In particular, I am much obliged to Carole Brookins and Shigeru Endo.

6) Both Models A and B are based on $Y = W + \Pi = C + S$, where Y shows national disposable income, W is wages/compensation, Π is rents/surplus, C is consumption, and S is saving. This framework is justified by using the functions of consumption and investment (see Kamiryo, 2005). For the estimation of wages, rents, and capital, I have used the following two methods, each externally: how to adjust the relative share of rents by using a coefficient, (*rho*/*r*), and how to adjust the capital-output ratio by using a coefficient, (*r*/*w*). As a supplemental device, I have taken into consideration the difference between disembodied technology that are shown in the level of technology as stock and the (embodied) rate of technological progress as flow (see Kamiryo, IARIW 2006).

2.1 *Framework of Model A*

The basic form is shown as $y = Ak^{\alpha}$. Model A shows my endogenous growth model. Based on $y = Ak^{\alpha}$, both the rate of technological progress and the growth rates of capital and per capita capital are measured, as I showed already in Kamiryo (2004, 2005), by using two unique parameters in my model: (1) the ratio of quantitative investment to total investment, *beta*, and (2) a parameter, *delta*, that neutralizes DRS/IRS necessary for reaching CRS in the transitional path, where the relationship between *delta* and the relative share of rents/surplus, *alpha*, plays an important role.

$$y = Ak^{\alpha}. \tag{1}$$

$g_K = i \cdot \beta \cdot A \cdot k^{-(1-\alpha)}$ and $g_k = i \cdot \beta \cdot A \cdot k^{-(1-\alpha)} - n$,

where n is the growth rate of population/employed persons. (2)

$$g_A = i \cdot (1-\beta) \cdot k^{-(\delta-\alpha)}. \tag{3}$$

Model A expressed by Eq. 1 is the Cobb-Douglas production function. The level of technology, A, is shown as $A = k^{1-\alpha} / \Omega$, where k is the capital-labor ratio and Ω is the capital-output ratio. The level of technology in Model A , A_{MA}, however, is divided into two terms: the level of residual-technology, $A_{MA(RESI:\alpha-\delta)}$, and the capital-labor ratio, $k^{\alpha-\delta}$, where $A_{MA(RESI:\alpha-\delta)}$ refuses the cooperation with quantitative capital.

$$A_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha-\delta}. \tag{4}$$

If I insert Eq. 4 into Eq. 1, $y = Ak^{\alpha}$ will be replaced by $y = A(k) \cdot (k)$. This is a DRS/IRS model. Eq. 4 implies that the level of technology does not hold without the cooperation of the capital-labor ratio. And, if I take out $k^{\alpha-\delta}$ in A_{MA} and insert $k^{\alpha-\delta}$ into k^{α}, the production function does not back to a CRS model:

$$y_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{2\alpha-\delta} = A_{MA(RESI:\alpha-\delta)} \cdot (k^{\alpha-\delta} \cdot k^{\alpha}). \tag{5}$$

In short, Eq. 1 expresses a CRS model when the level of technology hides the existence of the capital-labor ratio endowed in the level of technology. Eq. 5 holds as a DRS/IRS model. However, at the convergence, where *alpha* = *delta*, Eq. 5 reduces to:

$$y_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha}, \tag{6}$$

where $A_{MA} = A_{MA(RESI:\alpha-\delta)}$ and $y = y_{MA}$ (Eq. 1 = Eq. 6).

In the above discussion, I will raise a question and my answer. Why can the level of technology A_{MA} include the term of the capital-labor ratio? How can I justify $A_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha-\delta}$? My answer is: first, I assume that the level of technology, A_{MA}, is not a residual but shows the results derived from the factors' performances of capital, labor, and 'qualitative' investment and capital. I interpret that the literature introduces human capital (as a factor) instead of the level of technology into various models. Second, the level of technology A_{MA} does not hold if I separate 'quantitative' capital from the level of technology like $A_{MA(RESI:\alpha-\delta)}$. For example, $A_{MA(RESI:\alpha-\delta)}$ as a residual will decrease over time when I separate the performance of $k^{2\alpha-\delta}$ from the level of technology (for the test, see **Figures 1** and **2** below). To avoid this unrealistic result, I need to separate the performance of $k^{\alpha-\delta}$ from of k^{α} in Eq. 1. I deny such an integration of the capital-labor ratio as $k^{2\alpha-\delta}$. I approve that the corporation of $A_{MA(RESI:\alpha-\delta)}$ with $k^{\alpha-\delta}$ for the increase in the level of technology. Therefore, $A_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha-\delta}$ and $y_{MA} = (A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha-\delta})k^{\alpha}$ hold in Model A. In other words, $y_{MA} = (A(k) \cdot (k))$ is essential in Model A. Model A is unique in that it shows DRS/IRS in the transitional path, yet it reduces to CRS at the convergence.

Let me discuss the relationship between *delta* ands *alpha*, here using examples (see **Table 1**). The transitional path from DRS/IRS to CRS is determined by the difference between *delta* ands *alpha* as follows:

If $\alpha < \delta$, the production function is under DRS, where variables are explosive.
If $\alpha = \delta$, the production function is under CRS, where variables remain constant.
If $\alpha > \delta$, the production function is under IRS, where variables are not explosive.
The literature treats the case of IRS or IRC. Is there any actual case of IRS in the world? I investigated the case of IRS using my data-sets for thirty countries, 1995 to 2004. I find the case of IRS in several countries for both the government sector and the private sector: a few in Euro countries, Russia in 2003 and 2004 (see **Figure 3**), and the private sector in Japan. It is difficult for an economy to maintain a *delta* close to zero and accordingly, IRS. I will discuss this issue in the measurement of *delta* below.

Before going to Model B, I will just raise a question for formulating the value of *delta*. In my model, I need an equation to formulate *delta* and yet, I have not found a final equation in my papers. I knew that Model A could not obtain this equation. A reason why I formulate Model B in this paper lies in the measurement of *delta*. Why does Model A fail in obtaining the final equation? In Model A, the value of *delta* is offset when I depict *delta*. This is explained using the capital-output ratio as follows:

In any Cobb-Douglas production function, the level of technology is inversely related to the capital-output ratio. This is shown by:

$$k_{BASE}^{1-\alpha} = A \cdot \Omega, \text{ where } k_{BASE}^{1-\alpha} = k^{1-\alpha}. \tag{7}$$

I pay attention to the usual capital-labor ratio, $k_{BASE}^{1-\alpha}$, since it is always used for any model that uses the Cobb-Douglas production function both at the current situation and at the

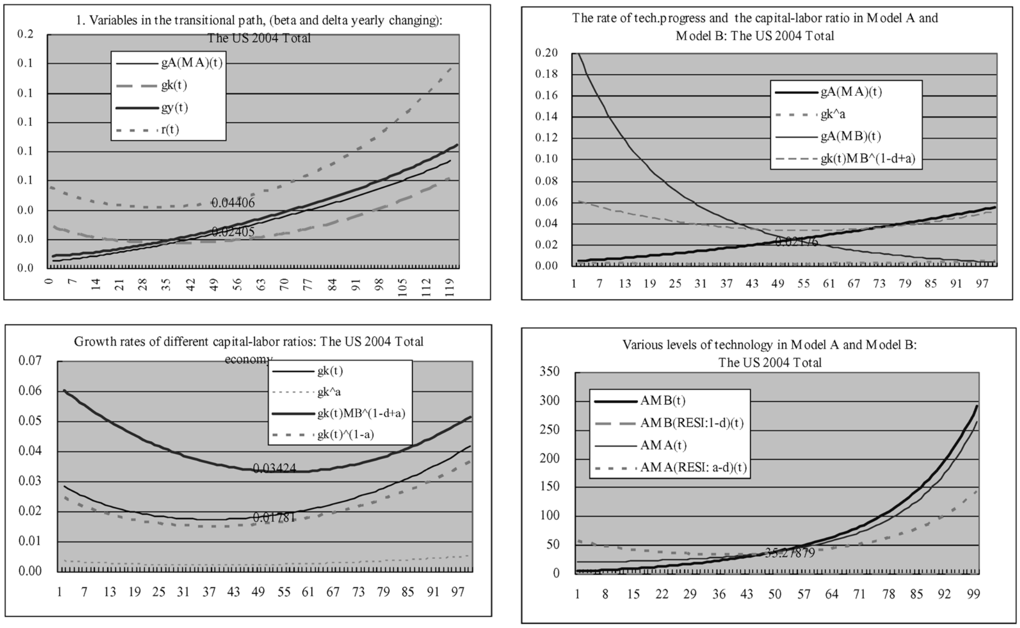
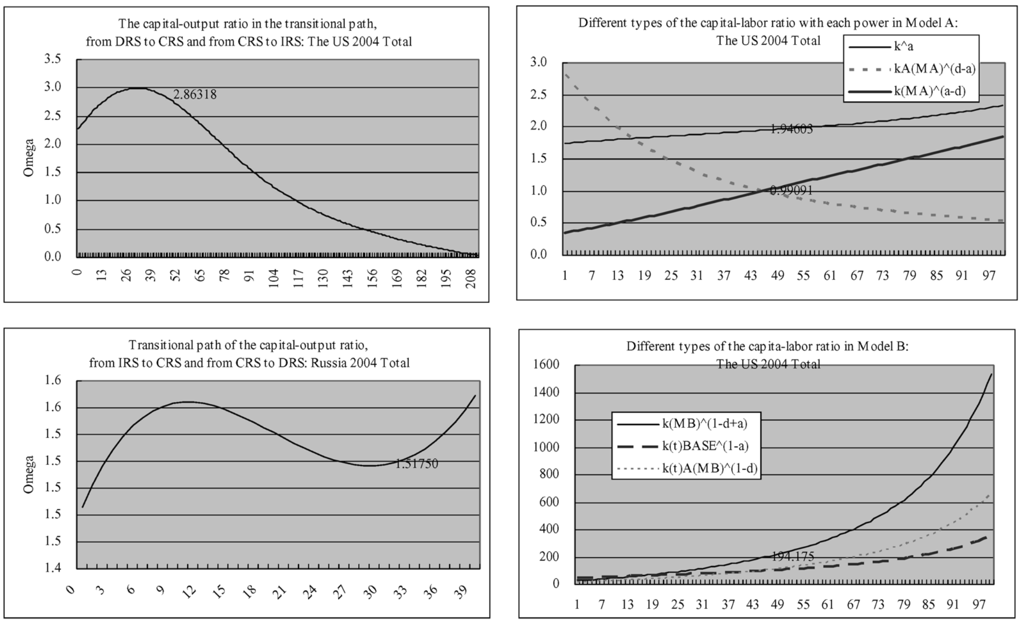


Figure 1　Characteristics of Model A and Model B (1): US 2004 Total economy

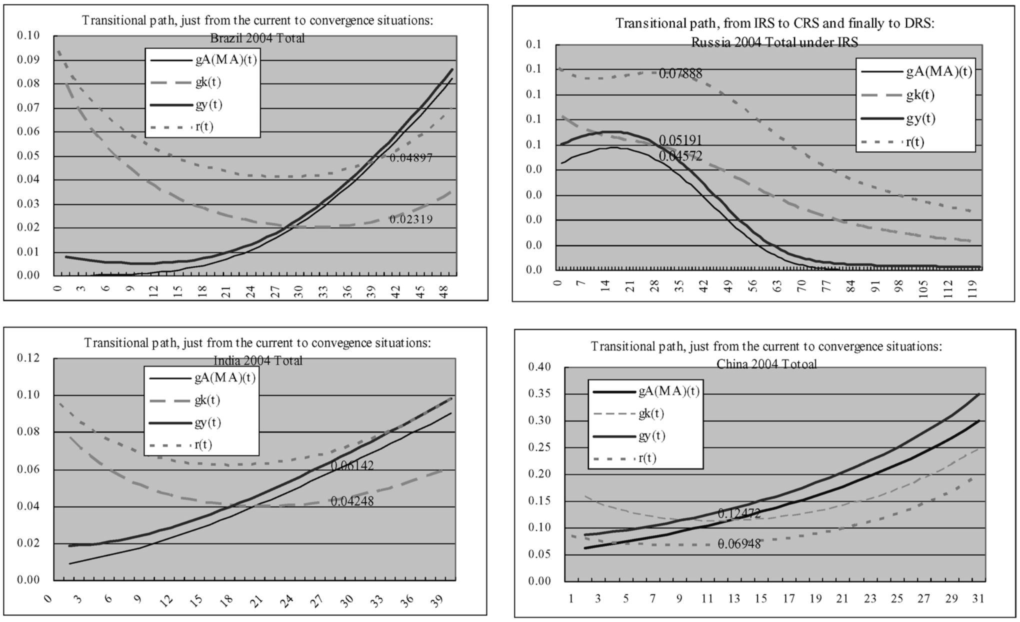


Note: See the difference of the transitional path comparing the capital-output ratio of the US (DRS → CRS → IRS) with that of Russia (IRS → CRS → DRS).

Figure 2　Characteristics of Model A and Model B (2): US 2004 Total economy

Table 1 The discount rate of convergence using the years for convergence

	Current (0)	at conv. (*)	Diff.of LN	r_{CONV}=LN/years	**Variables at * using $k\hat{}(1-\delta)=A^{*}\Omega^{*}$**			
The US 2004 Total			**years for convergence**	**44.58**	The current situation: DRS			
A	20.53	35.72	0.5538	0.0124		$A*\Omega*$	k*	y*
k	81.10	199.47	0.9000	0.0202	$\Omega^{*}_{(\delta\to\alpha)}$	102.270	199.466	69.666
y	35.74	69.67	0.6673	0.0150	$\Omega^{*}_{(\alpha\to\alpha)}$	109.4543	215.581	70.352
k/A	3.9503	*5.5843*	0.3462	0.0078				
y/A	1.7411	*1.2423*	(0.3375)	(0.0076)	g_A^{*}	0.0221	θ=I/S	2.1368
$\Omega^{*}_{(\delta\to\alpha)}$	2.2689	2.8632	0.2326	0.0052	$g_y^{*}=g_k^{*}$	0.0249	$\Omega(0)$	2.2689
delta	0.3607	0.1261	(1.0505)	(0.0236)	delta	0.3607	α	0.1261
beta	0.8586	0.7827	(0.0925)	(0.0021)	$\lambda=(1-\alpha)$r	0.0224	n	0.0095
$\Omega^{*}_{(\alpha\to\alpha)}$	2.2689	3.0643	0.3005	0.0067	$1/(\delta-\alpha)$(n+i	44.5779	i	0.1015
Brazil 2004 Total			**years for convergence**	**39.79**	The current situation: DRS			
A	3223	5444	0.5243	0.0132		$A*\Omega*$	k*	y*
k	9309	34435	1.3081	0.0329	$\Omega^{*}_{(\delta\to\alpha)}$	11590	34435	16176
y	8355	16176	0.6607	0.0166	$\Omega^{*}_{(\alpha\to\alpha)}$	8086	23038	15512
k/A	2.8885	6.3247	0.7837	0.0197				
y/A	2.5924	1.2120	(0.7603)	(0.0191)	g_A^{*}	0.0522	θ=I/S	0.8953
$\Omega^{*}_{(\delta\to\alpha)}$	1.1142	2.1288	0.6474	0.0163	$g_y^{*}=g_k^{*}$	0.0573	$\Omega(0)$	1.1142
delta	0.7551	0.1042	(1.9801)	(0.0498)	delta	0.7551	α	0.1042
beta	0.7927	0.6086	(0.2643)	(0.0066)	$\lambda=(1-\alpha)$r	0.0251	n	0.0138
$\Omega^{*}_{(\alpha\to\alpha)}$	1.1142	1.4851	0.2873	0.0072	$1/(\delta-\alpha)$(n+i	39.7855	i	0.1334
Russia 2004 Total			**years for convergence**	**25.93**	The current situation: **IRS**			
A	56.14	186.70	1.2017	0.0467		$A*\Omega*$	k*	y*
k	149.76	600.39	1.3886	0.0539	$\Omega^{*}_{(\delta\to\alpha)}$	280.821	600.394	399.169
y	101.78	399.17	1.3666	0.0531	$\Omega^{*}_{(\alpha\to\alpha)}$	266.892	566.716	396.441
k/A	2.6676	3.2158	0.1869	0.0073				
y/A	1.8129	1.1488	(0.4562)	(0.0177)	g_A^{*}	0.0454	θ=I/S	0.4413
$\Omega^{*}_{(\delta\to\alpha)}$	1.4714	1.5041	0.0220	0.0009	$g_y^{*}=g_k^{*}$	0.0517	$\Omega(0)$	1.4714
delta	0.0475	0.1188	0.9168	0.0356	delta	0.0475	α	0.1188
beta	0.7347	0.6000	(0.2025)	(0.0079)	$\lambda=(1-\alpha)$r	0.0388	n	(0.0050)
$\Omega^{*}_{(\alpha\to\alpha)}$	1.4714	1.4295	(0.0289)	(0.0011)	$1/(\delta-\alpha)$(n+i	25.7439	i	0.1135
India 2004 Total			**years for convergence**	**25.91**	The current situation: DRS			
A	16.33	34.73	0.7547	0.0296		$A*\Omega*$	k*	y*
k	34.05	116.34	1.2287	0.0482	$\Omega^{*}_{(\delta\to\alpha)}$	64.161	116.338	62.972
y	25.39	62.97	0.9084	0.0356	$\Omega^{*}_{(\alpha\to\alpha)}$	61.276	110.379	62.559
k/A	2.0853	3.3498	0.4740	0.0186				
y/A	1.5548	1.1633	(0.2901)	(0.0114)	g_A^{*}	0.0572	θ=I/S	1.0824
$\Omega^{*}_{(\delta\to\alpha)}$	1.3411	1.8474	0.3203	0.0126	$g_y^{*}=g_k^{*}$	0.0642	$\Omega(0)$	1.3411
delta	0.5477	0.1251	(1.4766)	(0.0579)	delta	0.5477	α	0.1251
beta	0.7434	0.6568	(0.1239)	(0.0049)	$\lambda=(1-\alpha)$r	0.0392	n	0.0152
$\Omega^{*}_{(\alpha\to\alpha)}$	1.3411	1.7644	0.2743	0.0108	$1/(\delta-\alpha)$(n+i	25.5071	i	0.1667
China 2004 Total			**years for convergence**	**10.93**	The current situation: DRS			
A	5.80	14.17	0.8930	0.0817		$A*\Omega*$	k*	y*
k	18.09	66.92	1.3082	0.1197	$\Omega^{*}_{(\delta\to\alpha)}$	33.311	66.920	28.464
y	9.38	28.46	1.1101	0.1016	$\Omega^{*}_{(\alpha\to\alpha)}$	29.696	58.310	27.820
k/A	3.1185	4.7232	0.4151	0.0380				
y/A	1.6169	1.2939	(0.2229)	(0.0204)	g_A^{*}	0.1167	θ=I/S	0.9424
$\Omega^{*}_{(\delta\to\alpha)}$	1.9287	2.3511	0.1980	0.0181	$g_y^{*}=g_k^{*}$	0.1390	$\Omega(0)$	1.9287
delta	0.2597	0.1660	(0.4477)	(0.0410)	delta	0.2597	α	0.1660
beta	0.7962	0.7083	(0.1170)	(0.0107)	$\lambda=(1-\alpha)$r	0.0915	n	0.0062
$\Omega^{*}_{(\alpha\to\alpha)}$	1.9287	2.0959	0.0831	0.0076	$1/(\delta-\alpha)$(n+i	10.9266	i	0.4001



Note: Compare the above set of the growth rates in Russia with the capital-output ratio of Russia in Figure 2: The transitional path in Russia is IRS → CRS → DRS.

Figure 3 Comparison of growth rates; tech. progress, per capita output, per capita capital, and the ratio of rents to capital in the transitional path

convergence.

In the case of Model A,

$$k_{MA}^{1-\alpha} = A_{MA} \cdot \Omega_{MA}, \text{ where } \Omega_{MA} = k_{MA}^{1-\alpha} / A_{MA}. \tag{8}$$

In Eqs. 7 and 8, let me assume that $k^{(1-\alpha)} = 1.0$. Then the relationship between the level of technology and the capital-output ratio will be simply clarified. However, the assumption of $k^{(1-\alpha)} = 1.0$ is unrealistic since it is impossible for *alpha* to be 1.0 (or, no wages). The measurement of *delta* will be discussed using Model B soon below.

2.2 *Framework of Model B*

Model B is formulated using a meaningful combination of two parameters, *beta* and *delta*, and defining the level of residual-technology.

$$A_{MB} = A_{MB(RESI:1-\delta)} \cdot k^{1-\delta} \text{ using } A_{MB} \equiv (B \cdot k)^{1-\delta}, \tag{9}$$

where $B \equiv (1-\beta)/\beta$ and

$$A_{MB(RESI:1-\delta)} \equiv B^{1-\delta} \tag{10}$$

And, $y_{MB} = A_{MB(RESI:1-\delta)} \cdot k^{1-\delta+\alpha}$. (11)

Model B will show a production function under DRS/IRS. However, Model B corresponds with a kind of ak model in the literature. or $y = Bk$ at convergence,

$$y = B^{1-\delta} \cdot k^{1-\delta+\alpha}. \tag{12}$$

Apparently, Eq. 12 may be a Cobb-Douglas production function, if $k^{1-\delta}$ is unknown/ hidden in A_{MB} (I will return to this discussion below when I measure the value of *delta*).

When Model B uses as a base the same capital-labor ratio over years in the transitional path as that in Model A, what is the difference between Models A and B? Model A is supported by a robust base that separates qualitative investment from quantitative investment and accordingly, separates the level of technology from capital stock each as an accumulation of qualitative investment and as an accumulation of quantitative investment. The difference between Models A and B only comes from each power of the capital-labor ratio newly set in the level of technology and the capital-labor ratio. As a result, Model B's level of technology in the transitional path differs from Model A's one. And accordingly, per capita output in Models A and B differ in the transitional path. Nevertheless, at the convergence, both models have all the same values under CRS. Here, to match both models, I need an adjustment-parameter, $\varepsilon_{B(MB)}$.

$$A_{MA} = \varepsilon_{B(MB)} \cdot A_{MB}, \text{ and} \tag{13-1}$$

$$y_{MA} = \varepsilon_{B(MB)} \cdot y_{MB}. \tag{13-2}$$

At the convergence, $\varepsilon_{B(MB)} = 1.0$ holds. What does this adjustment-parameter indicate? Conclusively speaking, the adjustment parameter is composed of the following three terms as shown in Eq. 14.

$$\varepsilon_{B(MB)} = A_{MA(RESI:\alpha-\delta)} \,/\, A_{MB(RESI:1-\delta)} \,/\, k_{BASE}^{1-\alpha}, \tag{14}$$

or, $k_{BASE}^{1-\alpha} = A_{MA(RESI:\alpha-\delta)} \,/\, A_{MB(RESI:1-\delta)} \,/\, \varepsilon_{B(MB)}$

where $k_{BASE}^{1-\alpha} = k^{1-\alpha}$ (see the above Eq. 7).

In Eq. 14, the power of the base-capital-labor ratio, $1-\alpha$, comes from the difference between $\alpha-\delta$ and $1-\delta$: $\alpha-\delta-(1-\delta) = -(1-\alpha)$. By this way, the adjustment parameter, $\varepsilon_{B(MB)}$, connects Model B with Model A under the assumption of $y_{MA} = y_{MB}$ and $k_{MA} = k_{MB}$.

If I tentatively assume that Model B is a Cobb-Douglas production function,

$$k_{MB}^{1-\alpha} = A_{MB} \cdot \Omega_{MB}, \text{ where } \Omega_{MB} = k_{MB}^{1-\alpha} / A_{MB}, \text{ using Eq. 7.} \tag{15}$$

Dividing Eq. 8 by Eq. 15,

$$\frac{A_{MA}}{A_{MB}} = \frac{\Omega_{MB}}{\Omega_{MA}} \quad \text{since } k_{MA}^{1-\alpha} = k_{MB}^{1-\alpha} = k_{BASE}^{1-\alpha} = k^{1-\alpha}. \tag{16}$$

Eq. 16 is tentative (see below), but it implies that how important $k_{BASE}^{1-\alpha} = k^{1-\alpha}$ is as a base lying between the level of technology and the capital-output ratio.

Now, I will turn to the capital-output ratio in the transitional path. For this, I will express the transitional path using time, *t*, as an independent variable. The purpose of the discussion is to obtain the equation necessary for formulating *delta* at the current situation. The measurement of *delta* is only possible only by taking advantage of Model B, as I already indicated above.

In the transitional path from DRC/IRS to CRS, $B(t) = (1-\beta(t)) / \beta(t)$ together with *delta*(*t*) converges to $B^* = (1-\beta^*) / \beta^*$ together with *delta*(*t*) = *alpha*. A part of this process was discussed already in my previous papers, finding a method for measuring *beta*(0) and $beta^*$ but without finding a method for measuring *delta*. I will now justify that the above tentative Eq. 15 is true only at convergence. At convergence, the value of y^* in Model B is equal to the value of y^* in Model A as the Cobb-Douglas production function: $y^*_{MA} = y^*_{MB}$. At convergence, the difference between Model A and Model B comes from each level of technology as a residual, which is adjusted under the same value of k^*: $k^*_{MA} = k^*_{MB}$. By these reasons, Model B is replaced by Model A only at convergence.

Therefore, the above Eq.15 only holds at convergence:

$$k_{MB}^{*(1-\alpha)} = A^*_{MB} \cdot \Omega^*_{MB} \text{ or } \Omega^*_{MB} = k_{MB}^{*(1-\alpha)} / A^*_{MB}, \tag{17}$$

where $A^*_{MB} = A^*_{MB(RESI:1-\delta)} \cdot k_{MB}^{*(1-\delta)}$

$$A^*_{MB(RESI:1-\delta)} \equiv \left(\frac{1-\beta^*}{\beta^*}\right)^{1-\delta} = B^{*(1-\delta)}.$$ Thus,

$$\Omega^*_{MB} = \frac{k_{MB}^{*(\delta-\alpha)}}{B^{*(1-\delta)}_{MB(RESI:1-\delta)}} = \frac{k_{MB}^{*(1-\alpha)}}{B^{*(1-\delta)}_{MB(RESI:1-\delta)} \cdot k_{MB}^{*(1-\delta)}}. \tag{18}$$

To be convenient, at the convergence, $k_{MB}^{*(\delta-\alpha)} = 1.0$ [7] holds since *delta* = *alpha* at the convergence. Thus,

$$\Omega^*_{MB} = \frac{1}{((1-\beta^*)/\beta^*)^{1-\delta}} = \frac{k_{MB}^{*(\delta-\alpha)}}{B^{*(1-\delta)}_{MB(RESIDUAL)}}. \tag{19}$$

Using Eq. 19, *delta* is formulated as,

$$\delta = 1 - \frac{LN(1/\Omega^*_{MB})}{LN((1-\beta^*)/\beta^*)}. \tag{20}$$

7) $k_{MB}^{*(\delta-\alpha)}$ is indispensable to the measurement of *delta*. The actual value of k^* is derived using my conventional endogenous growth model (Model A). I cannot measure the values of k^* and A^* by using my equations. This is because these two values are determined at the same time.

Table 2 Ω^* by *delta* in five countries 2004 Total economy

	delta	(0.4000)	(0.2000)	0	0.1261	0.5	1	1.5
Ω^*	The US	6.0139	4.6542	3.4241	2.9318	1.8504	1	5.8018
	delta	(0.4000)	(0.2000)	0	0.1216	0.5	1	1.5
Ω^*	Brazil	1.8554	1.6986	1.5551	1.4738	1.2470	1	88.9757
	delta	(0.4000)	(0.2000)	0	0.1198	0.5	1	1.5
Ω^*	Russia	1.7905	1.6475	1.5160	1.4423	1.2312	1	13.9267
	delta	(0.4000)	(0.2000)	0	0.1251	0.5	1	1.5
Ω^*	India	2.3896	2.1100	1.8631	1.7235	1.3649	1	6.0762
	delta	(0.4000)	(0.2000)	0	0.1660	0.5	1	1.5
Ω^*	China	3.4630	2.9000	2.4284	2.0959	1.5583	1	3.9944

And, reversely, $\Omega^*_{MB} = \dfrac{1}{B^{*(1-\delta)}}$.[8] (21)

Eqs. 20 and 21 are key equations which I need in the transitional path. The value of *delta* in Eq. 20 is the *delta* at the current situation since Eq. 20 is based on the assumption that $\Omega^* = \Omega(0)$ (see **Table 2**). Eq. 21, however, essentially differs from Eq. 20. The capital-output ratio in Eq. 21 is the current capital-output ratio measured under an assumption that CRS continuously holds from the current to the convergence situation. This will be discussed below in the next section.

3. Measurement of *delta*, the capital-output ratios, and the speed of convergence at the convergence

3.1 *Implication of three capital-output ratios*

This section discusses three kinds of the capital-output ratios. I attach importance to the capital-output ratio more than the capital-labor ratio. The capital-output ratio has its upper limit to sustainable growth and thus, is easier to interpret its character in the transitional path than the capital-labor ratio that is rather difficult to grasp. The capital-output ratio usually falls between 0.5 to 3.0[9] in any country and any sector in the world, as I discussed earlier. Furthermore, the capital-output ratio is inversely connected with the level of technology. This

8) If I use Model A instead of Model B, I cannot extract the value of *delta* as follows: Starting with $\Omega^*_{MA} = \dfrac{k_{MA}^{*(1-\alpha)}}{A^*_{MA(RESI:\alpha-\delta)} \cdot k_{MA}^{*(\alpha-\delta)}}$, $\Omega^*_{MA} = \dfrac{k_{MA}^{*(1-\alpha)}}{A^*_{MA(RESI:\alpha-\delta)}}$ holds. In Model A, the power of $k^{*(1-(2\alpha-\delta))}$ cannot be 1.0 since $1-\alpha = 1-(2\alpha-\delta)$ under convergence. Thus, I cannot obtain the value of *delta* in Model A.

9) The condition necessary for this range is that budget deficit is within a certain level of *GDP* or national disposable income. In the Japanese case, budget deficit is much beyond this condition and the current capital-output ratio is close to 4.0.

relationship is, however, integrated by the base-capital-labor ratio: $k^{1-\alpha} = \Omega \cdot A$.

First let me briefly summarize the three kinds of capital-output ratios and next, let me explain each by each in detail. In the transitional path, I need to distinguish three kinds of the capital-output ratios. I will begin with the review of Eqs. 20 and 21. The capital-output ratio in Model B does not equal to the capital-output ratio in Model A in the transitional path, yet at the convergence, both models converge with the same variables, as I set above.

Given *delta*, the current capital-output ratio is calculated using Eq. 20. Then, at the convergence, I need to distinguish two kinds of the capital-output ratio: One is the capital-output ratio at the convergence whose current situation started under DRS/IRS, $\Omega^*_{\delta\to\alpha}$. The other is the capital-output ratio at the convergence whose current situation started under DRS/ IRS, $\Omega^*_{\alpha\to\alpha}$. Model A cannot measure both of $\Omega^*_{\delta\to\alpha}$ and $\Omega^*_{\alpha\to\alpha}$. Model B measures $\Omega^*_{\alpha\to\alpha}$ by using Eq. 21, but cannot directly measure $\Omega^*_{\delta\to\alpha}$. However, $\Omega^*_{\alpha\to\alpha} > \Omega^*_{\delta\to\alpha}$ holds in a DRS model and, $\Omega^*_{\alpha\to\alpha} < \Omega^*_{\delta\to\alpha}$ holds in a IRS Model. The smaller the difference between *delta* and *alpha*, the smaller the difference between $\Omega^*_{\delta\to\alpha}$ and $\Omega^*_{\alpha\to\alpha}$. Given *delta*, the current capital-output ratio, $\Omega(0)$, is calculated using Eq. 20. However, $\Omega(0)$ is usually much lower than $\Omega^*_{\alpha\to\alpha}$. In short, I need to distinguish three kinds of the capital-output ratios in the transitional path: $\Omega(0), \Omega^*_{\delta\to\alpha}$, and $\Omega^*_{\alpha\to\alpha}$.

Next, let me discuss the above three capital-output ratios in detail each by each. First, I will explain the implication of the current capital-output ratio. When I measure *beta**, I need to use the current capital-output ratio. The *beta** is measured under an assumption that $\Omega^* = \Omega(0)$ as shown in Kamiryo (7(2), 2004; 9(1), 2005), where the current capital-output ratio was a surrogate of the capital-output ratio at the convergence, Ω^*:

$$\beta^*_{\delta=\alpha} = \frac{\Omega^*\left(n(1-\alpha)+i\,(1+n)\right)}{i(1-\alpha)+\Omega^*\cdot i(1+n)}. \quad (22)$$[10]

The above equation enabled me to formulate *delta* in this paper. This equation suggests that *delta* will be obtained by using the current capital-output ratio, the ratio of net investment to output, the relative share of rents, and the growth rate of population. This is proved using the following equations:

$B^* \equiv (1-\beta^*)/\beta^*$ is reformed using the above 22 as,

10) I formulated this equation first in Kamiryo (7 (2), Eq. 22, p. 21, 2004) by setting *delta* = 0 at the convergence. However, in these two papers, I had to assume that $\Omega(0) = \Omega^*$. As long as I measure *delta*, I need the assumption of $\Omega(0) = \Omega^*$, regardless of whether *delta* is a temporal value (as in 7(2)) or a final value as in this paper.

$$B^* = \frac{(1-\alpha)(i-\Omega^* \cdot n)}{\Omega^* \cdot i\,(1+n) + \Omega^* \cdot n(1-\alpha)}. \tag{23}$$

Since $\Omega^* = \dfrac{1}{((1-\beta^*)/\beta^*)^{1-\delta}}$ holds from Eq. 19,

$1 = \Omega^* \left(\dfrac{(1-\alpha)(i-\Omega^* \cdot n)}{\Omega^* \cdot i\,(1+n) + \Omega^* \cdot n(1-\alpha)} \right)^{1-\delta}$ holds under the assumption of $\Omega^* = \Omega(0)$.

The above equation is another expression of Eq. 21. And, Eq. 23 will justify the existence of absolute convergence in the literature.

Then, I will discuss the capital-output ratios at the convergence: $\Omega^*_{\alpha\to\alpha}$ and $\Omega^*_{\delta\to\alpha}$. I will discuss the case of $\Omega^*_{\delta\to\alpha}$ later since it is difficult to measure this $\Omega^*_{\delta\to\alpha}$ without measuring the years for convergence. When I insert *delta* into Eq. 21, the result only shows the current capital-output. When I insert *alpha*, instead of *delta*, into Eq. 21, the capital-output ratio at the convergence, $\Omega^*_{\alpha\to\alpha}$, is shown, and the current capital-output converges to $\Omega^*_{\alpha\to\alpha}$. These cases are expressed as,

$\Omega^*_{\delta\to\alpha} \neq \dfrac{1}{B^{*(1-\delta)}_{\delta\to\alpha}}$, where $\Omega^*_{\delta\to\alpha}$ is not measured. (24-1)

$\Omega^*_{\alpha\to\alpha} = \dfrac{1}{B^{*(1-\alpha)}_{\alpha\to\alpha}}$, where $\Omega^*_{\alpha\to\alpha} > \Omega^*_{\delta\to\alpha}$. (24-2)

Interestingly, $\Omega^*_{\alpha\to\alpha} = \Omega(0)((\delta+\alpha)/\delta)$ holds by adding *alpha* each to both the numerator and denominator of the equation of $\delta/(\delta-\alpha)$. (24-3)

The result of Eq. 24-1 comes from Eq. 23 that assumes $\Omega(0) = \Omega^*$. Eq. 24-1 does not show $\Omega^*_{\delta\to\alpha}$ but shows $\Omega(0) = \frac{1}{B^{*(1-\delta)}_{\delta\to\alpha}}$. Eq. 24-2 shows the balanced growth-state that is under constant returns to capital (CRC) all over years from the current to the convergence situations. I will raise a proposition.

Proposition 1: If the current capital-output, $\Omega(0)$, is significantly lower than the capital-output ratio in the balanced growth-state, $\Omega^*_{\alpha\to\alpha}$, the current situation is under an extreme DRS, where the capital-labor ratio and the level of technology do not cooperate effectively at all.

Why does the difference between $\Omega(0)$ and $\Omega^*_{\alpha\to\alpha}$ occur? This inevitably comes from a fact that *delta* differs from *alpha* at the current situation and that *delta* is formulated based on Eq. 23.

In short, I distinguish three kinds of the capital-output ratios: $\Omega(0) = \Omega^*$, $\Omega^*_{\delta\to\alpha}$, and $\Omega^*_{\alpha\to\alpha}$. The inequality of $\Omega(0) < \Omega^*_{\delta\to\alpha} \approx \Omega^*_{\alpha\to\alpha}$ will be clarified together with *delta* more in the next section.

3.2 *Measurement of the speed of convergence and key variables at the convergence*

In this section, I will present my method and equations for measuring the speed of convergence as a base and then present such key variables as the capital-output ratio, the level of technology, the capital-labor ratio, and per capita output each at the convergence by using the years for convergence. As a result, I will present a version of the phase diagram of my own.

First, the speed of convergence is composed of the convergence-coefficient and the years for convergence. The convergence-coefficient, λ, is shown as,

$$\lambda = (1-\alpha)n + (1-\delta)g_A^*. \tag{25-1}$$

The years for convergence: $Years_{1/\lambda} = 1/\lambda.$ (25-2)

The years for convergence, $1/\lambda$, are used not only for the years for convergence but also for the measurement of variables at the convergence (see below). The λ in Eq. 25-1 that cooperates with an endogenous rate of technology is comparable with the *beta* in Barro and Sala-i-Martin (1995, pp. 40, 53, and 83) that cooperates an exogenous model. Toshimi Fujimoto established Eq. 25-1 in March 2006 as our cooperative work. He used 'effective–labor, '*AL*,' to Model A instead of using labor (for detail, see Appendix) and I tested the results by using my data-sets of thirty countries 1995–2004.

I use the above convergence-coefficient to calculate the years/times required for convergence. The years for convergence, $years_{1/\lambda}$, is $1/\lambda$ as shown in Eq. 25-2. In the transitional path, the values of *delta*(*t*) and *beta*(*t*) will each approach *alpha* and *beta**. For *delta*, the discount rate, $r_{CONVERGE(\delta)}$, is obtained by dividing the logarithmic difference between *delta* (as the initial *delta*) and *alpha* with the years for convergence, $1/\lambda$. For *beta*, similarly to *delta*, the discount rate, $r_{CONVERGE(\beta)}$, is obtained by dividing the logarithmic difference between *beta* (as the initial *beta*) and *beta** with the years for convergence, $years_{1/\lambda} = 1/\lambda$.

$$r_{CONVERGE(\delta)} = POWER(2.7182818,((LN(\alpha) - LN(\delta))/years_{(1/\lambda)})) - 1. \tag{26-1}$$

$$r_{CONVERGE(\beta)} = POWER(2.7182818,((LN(\beta^*) - LN(\beta))/years_{(1/\lambda)})) - 1. \tag{26-2}$$

When I introduced the above equations into my recursive programming, the programming is consistent in every process of the transitional path. Note that $years_{1/\lambda}$ are commonly used but the discount rate differs according to each variable or parameter.

The above method for the speed of convergence differs from Barro and Sala-i-Martin's (1995) that uses 0.69 (instead of 1.0 of mine). In my case, it is not necessary to neglect the latter half/tail of convergence periods, which differs from Barro and Sala-i-Martin.[11] This is

11) If $e^{-x} = 0.5$, x = 0.69 holds as in Barro and Sala-i-Martin (1995).

partly because the years for convergence are much shorter than those of Barro and Sala-i-Martin: it does not take many years to converge in my case. This is also partly because, more importantly, I stopped separating two situations in my recursive programming: convergence now realizes at a point of time when variables move over time in the transitional path.[12)] If the convergence point of time is shown at the end of a hyperbolic curve designed for the balanced growth-state, it takes much more time since the tail of this curve is long. My previous recursive programming distinguished non-hyperbolic curves of variables under DRC/IRC with hyperbolic curves of variables under CRC.[13)] In this case, I needed such an approach as Barro and Sala-i-Martin tried.

The above discount rate is interestingly extended to such case as the capital-labor ratio, $k(t)$, at the convergence. In this case, what is the character of the discount rate? This discount rate is a kind of the growth rate of the capital-labor ratio. The discount rate of $k(t)$, $r_{CONVERGE(k)}$, is a specific value of the corresponding growth rate of the capital-labor ratio. After testing by my recursive programming, I found that $r_{CONVERGE(k)}$ is the middle point[14)] lying between the current capital-labor ratio and the capital-labor ratio at the convergence. Then, this middle point is obtained using an equation or recursive programming and thus, per capita capital at the convergence is measured using $r_{CONVERGE(k)}$ as

$$k^* = k(0)(1 + r_{CONVERGE(k)})^{years_{1/\lambda}}, \tag{26-3}$$

12) In one of my recursive programs in the past, I had to separate the condition of DRC/IRC from the condition of CRC/CRS. This is because I could not get the years for convergence correctly. In a CRC case, it takes much longer years to reach the convergence since the tail approaches flat taking many years.

13) At the same time, I used a provisional equation for *delta*. I will supplement this *delta* in this note. The starting equation was $\delta = \dfrac{n + \alpha(g_A^* - n)}{g_A^*} = \dfrac{n + \alpha(i\,(1 - \beta_{\alpha=\delta}^*) - n)}{i\,(1 - \beta_{\alpha=\delta}^*)}$. By reforming it, $1 - \delta = (1 - \alpha)\left(1 - \dfrac{n}{i\,(1 - \beta^*)}\right)$ is obtained. This form immediately reminds us of the difference between no growth in technology (as in Harrod) and an endogenous rate of technological progress (as in my model). I stated in my previous papers that I had to use this equation until I found a final equation for *delta*. Incidentally, the values of *delta* calculated using the above equation do not much differ from those calculated using Eqs. 25-1 and 25-2, $(1 - \alpha)n + (1 - \delta)\,i\,(1 - \beta^*)$.

14) This curve is a quadratic equation, which differs from the exponential equation of the growth rate of per capita capital. When the transitional path is divided into before convergence and after convergence, a quadratic equation before convergence will be a surrogate of the corresponding exponential equation, but after convergence, the difference between the quadratic and exponential equations becomes wider. This is because after convergence, the situation is reversed: from DRS to IRS and from IRS to DRS. I will discuss these issues in a separate paper.

where the years for convergence of k is the same as $years_{1/\lambda}$.

Here now, I will present how to measure the capital-output ratio, the level of technology, the capital-labor ratio, and output each at the convergence: $\Omega^*_{\alpha\to\alpha}$, A^*, k^*, and y^*. For A^*, k^*, and y^*, I will use the above k^* in Eq. 26-3 as a base. In this case, A^* and y^* are obtained using k^*. In the literature I have seldom found the above variables exactly measured at the convergence. This is partly because equations at the convergence are formulated using derivatives and integrals[15)] while these variables are differently measured by using equations devised in the transitional path and confirmed by recursive programming as shown below (see **Table 1**).

Let me summarize the two capital-output ratios at the convergence: $\Omega^*_{\alpha\to\alpha}$ and $\Omega^*_{\delta\to\alpha}$. First, the case of $\Omega^*_{\alpha\to\alpha}$ assumes that the situation is in the balanced growth-state, from the current situation to the convergence situation. Thus, the value of $\Omega^*_{\alpha\to\alpha}$ is higher than $\Omega^*_{\delta\to\alpha}$ shown in the other case that starts with DRS/IRS or DRC/IRC at the current situation. This comes from the logic that if *delta* is much higher than *alpha*, the capital-output ratio at the current situation must be much lower than $\Omega^*_{\alpha\to\alpha}$. $\Omega^*_{\alpha\to\alpha}$ is a benchmark for the capital-output ratio at the current situation and reliable more than the case of $\Omega^*_{\delta\to\alpha}$. $\Omega^*_{\alpha\to\alpha}$ is obtained using $\Omega^*_{\alpha\to\alpha} = \frac{1}{B^{*(1-\alpha)}_{\alpha\to\alpha}}$ as Eq. 24-2. Then, the level of technology at the convergence, A^*, will be obtained using the following equations in Models A and B:

$$A^* = k^{*(1-\alpha)} / \Omega^*_{\alpha\to\alpha} \qquad \text{(from Eq. 8).}$$

$$A^* = B^{*(1-\alpha)}_{\alpha\to\alpha} \cdot k^{*(1-\alpha)} \qquad \text{(from Eq. 9).}$$

Note that these equations reduce to $B^{*(1-\alpha)}_{\alpha\to\alpha} = 1/\Omega^*_{\alpha\to\alpha}$ and cannot extract the capital-labor ratio due to offset. This implies that I must measure the value of k^* differently.[16)] Once k^* is measured, the value of y^* is obtained using this k^* and also A^*in Eq. 9.

Let me close this section by showing the case of the capital-output ratio, $\Omega^*_{\delta\to\alpha}$, at the

15) See Appendix that was generously given by Dr. Toshimi Fujimoto. His approach is compared with mine by using my recursive programming. Our common consensus is that we must control the difficulties lying between the continuous case and the discrete case.

16) The capital-labor ratio at the convergence, k^*, is measured using Eqs. 26-2 and 26-3. The value of k^* will be confirmed in recursive programming. A problem remains unsolved: how to obtain an equation to directly calculate the value of k^* without using the years for convergence. In my model, the product of the level of technology and the capital-output ratio is equal to $k^{1-\alpha}$. A problem for measuring the capital-labor ratio under convergence is that $k^{\delta-\alpha} = 1$ holds at *delta* = *alpha*. The value of k^* may be determined by the relationship between *delta* and *alpha*, if I devise the power of k^* so as to be more than zero, instead of using $\delta - \alpha$. I will review the capital-output ratio and *delta* by using an optimal case of $\beta^*(\delta(\Omega^*))$.

convergence. The value of $\Omega^{*}_{\delta\to\alpha}$ is not obtained by using Eq. 24-2. However, once the years for convergence, $years_{(1/\lambda)}$, are measured by Eq. 25-2, $\Omega^{*}_{\delta\to\alpha}$ is obtained using the following equations (see Table 1).

$r_{CONVERGE(\Omega(\delta\to\alpha))} = (LN(\Omega_{\delta\to\alpha}) - LN(\Omega(0)) / years_{(1/\lambda)}$ or,

$r_{CONVERGE(\Omega(\delta\to\alpha))} = POWER(2.7182818,((LN(\Omega_{\delta\to\alpha}) - LN(\Omega(0))) / years_{(1/\lambda)})) - 1$

and then, $\Omega_{\delta\to\alpha} = \Omega(0)(1 + r_{CONVERGE(\Omega(\delta\to\alpha))})^{years_{(1/\lambda)}}$. (26-4)

4. Empirical results in Models A and B and in the relationship between *delta* and the capital-output ratio at the convergence

This section first empirically clarifies the differences between Models A and B with each characteristics and second, shows the relationships among *delta*, *beta**, the capital-output ratio in the transitional path. I arrange the data-sets by country 1995-2004, starting with the original data of IFSY and GFSY, IMF. It is essential for my data-sets to first estimate capital and rents by sector. The method for estimating capital and rents will be discussed in Kamiryo (PRSCE, 47(1), 2005), together with examples of my data-sets.

For confirmation of all the above related values, I will use the spread sheets that are fitted for my recursive programming, as will be shown in **Figures A1-1** to **A1-3** in Appendix. I do not discuss the differences between countries in this paper, yet I will compare some results by country to help to understand the whole picture in the transitional path.

4.1 *Empirical results and characteristics of Models A and B*

First, let me summarize the characteristics of Models A and B. Model A can be a CRS model, but it is shifted to a DRS/IRS model if its level of technology discloses the capital-labor ratio: $A_{MA} = A_{MA(RESI:\alpha-\delta)} \cdot k^{\alpha-\delta}$. Model B holds under DRS/IRS, but it is shifted to a CRS model at the convergence regardless of whether or not its level of technology is limited to the level of residual-technology: $y_{MB} = A_{MB(RESI:1-\delta)} \cdot k^{1-\delta+\alpha}$. Let me explain both models in detail (see **Figures 1** and **2**):

In both models, I stress that the level of technology cooperates with capital-labor ratio. When the level of technology excludes the influence of quantitative capital, it turns to the level of residual-technology. However, this residual-technology does not show real results of technology. The level of residual-technology only shows reduced something: this something is unknown in Model A, while in Model B this something indicates $(1-((1-\beta^{*})/\beta^{*})^{1-\delta}$.

First, let me compare each value of several capital-labor ratios specified by different powers. I distinguish a group of the capital-labor ratios whose values are not exponential and a group of the capital-labor ratios whose values are exponential. The purpose of setting these two groups is to clarify the relationships lying between the capital-labor ratio and the residual-technology.

Non-exponential group: ${k_{MA}}^{\alpha}$, ${k_{MA}}^{\alpha-\delta}$, and ${k_{MA}}^{\delta-\alpha}$.

Exponential group: $k_{BASE}^{1-\alpha}$, ${k_{MB}}^{1-\delta+\alpha}$, and ${k_{A(MB)}}^{1-\delta}$.

Incidentally, the capital-labor ratios used in Model A all belong to the non-exponential group, while the capital-labor ratios used in Model B all belong to the exponential group. The capital-labor ratio in Models A and B differ over time until the situation reaches each convergence. I summarize the character of each group or model: Model A is stable in that the cooperation between the capital-labor ratio, the levels of technology, and residual-technology are moderate and reliable. Its capital-output ratio increases very slowly towards the convergence or the rate of technological progress increases slowly and steadily. Model B is supplemental and makes it possible to measure *delta* and the cooperation among the capital-labor ratio, the levels of technology, and the residual-technology are rather extreme. Its capital-output ratio starts at a high level and decreases as a hyperbolic curve towards the convergence or the rate of technological progress decreases sharply. In short, Model A is more steadily technology-oriented over time while Model B is more vividly capital-oriented.

Nevertheless, such variables as the growth rate of technological progress and the level of technology each show the same values at a point of time or at the convergence. And, y_{MA} and y_{MB} after adjustment are maintained as the same values over time, by introducing the adjustment parameter, $\varepsilon_{B(MB)}$, into y_{MB} so that both values are the same over time.[17] At the convergence, the adjustment parameter becomes 1.0. $\varepsilon_{B(MB)} = 1.0$ and $A_{MA} = A_{MB} = \varepsilon_B \cdot A_{MB}$ at the convergence holds since in my model I distinguish the qualitative investment/the level of technology with the quantitative investment/capital in the transitional path. These imply that I am able to justify the method for formulating the value of *delta* using Model B, as shown in Eqs. 20 and 21. As results, the level of residual-technology in Model A is much stable compared with that in Model B. It is inevitably suggested that technology is qualitative capital-oriented and qualitative investment-oriented but must cooperate with quantitative capital and investment. Technology cannot stand itself without the help of quantitative capital

17) $A_{MA} = \varepsilon_{B(MB)} \cdot A_{MB}$, $y_{MA} = \varepsilon_{B(MB)} \cdot y_{MB}$, and $\varepsilon_{B(MB)} = A_{MA(RESI:\alpha-\delta)} / A_{MB(RESI:1-\delta)} / k_{BASE}^{1-\alpha}$ hold at the same time (as shown in Eqs. 12, 13-1, 13-2, and 14).

and investment.[18)]

Then, how is the growth rate of capital related to the growth rate of output? In the transitional path, the growth rate of capital is higher than the growth rate of output before the convergence and the growth rate of capital is lower than the growth rate of output after the convergence. This is acceptable but, a true picture in the phase diagram differs a little bit. Solow (1956) presents equations in the balanced growth-state: the growth rate of per capita capital, g_k^*, is equal to the growth rate of per capita output, g_y^*, at the convergence. And, this g_y^* is equal to the rate of technological progress divided by the relative share of labor, $g_y^* = g_A^* / (1-\alpha)$. This is true in the case of an exogenous growth model. In the cases of such endogenous growth models as Models A and B, the convergence-timing of $g_k^* = g_y^*$ is a little bit earlier than that of $g_y^* = g_A^* / (1-\alpha)$.[19)] I confirmed this fact by my recursive programming as shown in **Figures 1** and **2**. Also, I confirmed, comparing Model A with Model B, that $g_{A(MA)}^* = g_{A(MB)}^*$ exists at the convergence of $g_y^* = g_A^* / (1-\alpha)$. Furthermore, the phase diagram in Model A is asymmetric before and after the convergence point of time under an assumption that all the parameters at the current situation remain unchanged. This diagram also differs from that of the textbooks. The version of this diagram will be clarified by comparing the three kinds of the capital-output ratios (see below).

In short, by using both Models A and B, the essence of the balanced growth-state will be clarified differently of the literature and this is more discussed in the next section.

4.2 *Empirical results in delta, the capital-output ratios, and the speed of convergence*

This section tests the relationships among *delta*, the capital-output ratios, the convergence-coefficient, and the years for convergence applying my data-sets to the related equations. The above asymmetric convergence is more clarified when I use the three kinds of capital-output ratios. This is because *delta* integrates *beta** and the capital-output ratio in an economy. I will test the relationships, by showing **Tables A1-1** and **A1-2** in Appendix that include the three dimensional graphs and then, by referring to Table 1 with Figures 1 and 2.

The value of *delta* using the capital-output ratio, $\boldsymbol{\Omega}$(0), and *beta:**

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18) This will be applicable to the introduction of human capital, although I do not take human capital into my model.

19) I suspect that the difference of timing between $g_k^* = g_y^*$ and $g_y^* = g_A^* / (1-\alpha)$ will come from the difference of results between the discrete case and the continuous case. I need to review this issue in the future since the above difference of timing is a little bit higher than the difference of results caused by the discrete and continuous cases.

1. The higher the $\Omega(0)$, the lower the *delta*.
2. The higher the $beta^*$, the lower the *delta*, where $beta^* < 1.0$.
3. If $\Omega(0)$ is high and $beta^*$ is low the value of *delta* becomes negative/minus.
4. If $\Omega(0)=1.0$, $delta=1.0$ holds regardless of the value of $beta^*$.
5. Most interestingly, I find a fact that the value of *delta* is the same by country if I use the same $\Omega(0)$ and $beta^*$.

What does this fact imply? The value of $beta^*$ is formulated using such current values as the ratio of net investment to output, the relative share of rents, the growth rate of population, and the capital-output ratio (see Eq. 23). Therefore, this fact implies that the value of *delta* is the same if the above current values are the same.

The convergence-coefficient using the capital-output ratio, $\Omega(0)$, and $beta^*$:

This differs from the above finding in the relationship among *delta*, $\Omega(0)$, and $beta^*$ by country (see **Table A1-1** in Appendix). The values of the convergence-coefficient, λ, differ by country. With the same *delta*, the convergence-coefficient differs due to the differences of the capital-output ratio and $beta^*$. The characteristics of the convergence-coefficient are summarized:

1. The higher the $\Omega(0)$, the higher the λ.
2. The higher the $beta^*$, the lower the λ.
3. If $\Omega(0)$ is low and $beta^*$ is low, the value of λ becomes negative/minus.
4. If $\Omega(0)=1.0$, λ shows 1.0,[20] regardless of the value of $beta^*$. The fixed/constant value of λ does not differ by country.

The years for convergence using the capital-output ratio, $\Omega(0)$, and $beta^*$:

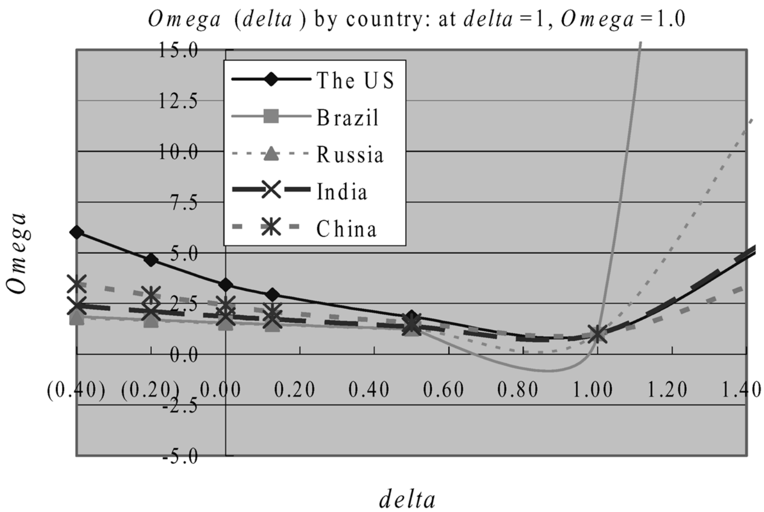
The years for convergence is just measured by using $1 / \lambda$, as the reciprocal number of λ. However, the years for convergence explain the characteristics of convergence in terms of its sign more clearly than λ (see **Tables A1-2** in Appendix).

1. The higher the $\Omega(0)$, the shorter the $years_{1/\lambda}$.
2. The higher the $beta^*$, the longer the $years_{1/\lambda}$.
3. If $\Omega(0)$ is low and $beta^*$ is low, the value of $years_{1/\lambda}$ becomes negative/minus.
4. If $\Omega(0) = 1.0$, $years_{1/\lambda}$ is positively or negatively fixed regardless of the value of $beta^*$. This fixed/constant $years_{1/\lambda}$ is much longer than other plus/minus $years_{1/\lambda}$ and differs by country. Using the above characteristics of the years for convergence, the problem of

20) This is proved using Eq. 20: $\delta = 1 = 1 - \frac{LN(1/1)}{LN((1-\beta^*)/\beta^*)}$, where $\Omega(0) = \Omega^*$.

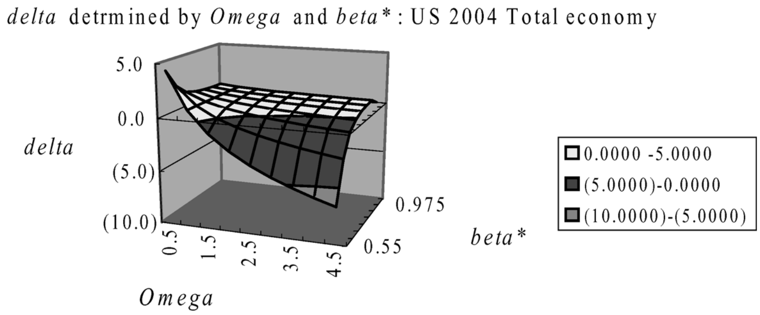
absolute versus conditional convergence in the literature will be solved by country.

Finally, I will show the relationship between *delta* and the capital-output ratio by country (see **Figures 4-1** and **4-2**). When the current capital-output ratio (measured at the convergence) is 1.0, *delta* is always 1.0, regardless of the value of $beta^*$, except for $beta^*$=1.0 (i.e., no qualitative investment), where *delta* has no number since $\Omega^*(beta^*)$ has a vertical asymptote at $beta^*$=1.0. Thus, an economy must, as much as possible, maintain *delta* less than 1.0. Regardless of whether the current situation is under DRS or IRS, *delta* increases rapidly after this point or when *delta*>1.0. When *delta* lies between 0 and 1.0, the corresponding capital-output ratio is comparatively stable. The capital-output ratio at *alpha*, $\Omega^*_{\alpha\to\alpha}$, lies in 0.08 to 0.16 and



Note: The capital-output ratio increases positively when *delta* increases negatively.

Figure 4-1 Ω^* by *delta* in five countries 2004 Total economy



Note: there is no difference among countries if the three values are each the same.

Figure 4-2 Three dimensional graph of Ω^*, *beta and delta in the US 2004 Total economy**

is slightly higher than that at *delta*=1.0. These characteristics slightly differ by country. Yet, if I pay attention to three dimensional graphs (that use the capital-output ratio, *beta**, and *delta* by country), there is no difference by country. This is because *beta** offsets the above differences in $\Omega(0)(\delta)$.

The capital-output ratio continuously under CRS, $\Omega^*_{\alpha\to\alpha}$, is close to the highest value of the capital-output ratio in the transitional path. If *delta* is lower than the *delta* at $\Omega^*_{\alpha\to\alpha}$, the current situation is under IRS. This IRS disappears at the convergence, but after the convergence under CRS the situation turns to DRS. If *delta* is much higher than *alpha*, the current situation is under DRS. This DRS disappears at the convergence, but after the convergence under CRS the situation turns to IRS. Why do these reverses happen after convergence? If *delta* at the current situation remains unchanged before and after convergence, the discount rates of convergence, $r_{CONVERGE(\delta)}$ and $r_{CONVERGE(\beta)}$, remain unchanged. Then after convergence, the situation turns reversed as above. And, different from the textbooks, convergence does not form a symmetric phase diagram. This is because the point of $\Omega^*_{\alpha\to\alpha}$ usually exists before convergence and, this makes the phase diagram before convergence different from the phase diagram after convergence that is far from the point of $\Omega^*_{\alpha\to\alpha}$. I call the phase diagram before and after convergence 'asymmetric' as I touched in the previous section. The IRS situation not often happens as seen in Russia in 2003 and 2004. I will investigate the conditions for IRS using my data-sets in the near future. I assert that the asymmetric convergence occurs without influenced by any policy as long as the current situation reaches the convergence according to the above discount rates.

Proposition 2: If *delta* determined at the current situation remains unchanged after convergence and the current situation is under DRS, the situation after convergence will be under IRS. Reversely, if *delta* determined at the current situation remains unchanged after convergence and the current situation is under IRS, the situation after convergence will be under DRS. And, in each case, the phase diagram is not symmetric but asymmetric which differs from the text-books as shown in Barro-Sala-i-Martin (1995) and Jones, Charles, I. (1998).

4. Conclusions

Let me first compare Model A with Model B, second summarize the relationship between *beta* and *delta* together with the capital-output ratio and the speed of convergence, and lastly interpret empirical results, suggesting a version of the phase diagram.

First, Models A and B have each a common base of the same capital-labor ratio over time in the transitional path. This comes from the fundamental characteristic of my endogenous growth model that divides investment into qualitative and quantitative investments whose accumulation is shown each as the level of technology and capital stock. In the transitional path, both Models A and B turn to CRS at the convergence, after DRS/IRS. The differences between Models A and B are derived from each difference of the power of the capital-labor ratio included in the level of technology. The above common capital-labor ratio is divided into two: the level of technology and a specified capital-labor ratio. This is a key for clarifying various relationships between DRS/IRS at the current situation and CRS at the convergence. The level of technology should not be a residual but a combination of factors, which makes it possible to produce IRS/IRS. My endogenous growth model starting with the Cobb-Douglas production function, $y = Ak^{\alpha}$, must be shown as $y = (A(k)\cdot(k))$ in each Models A and B, where $A = A(k)$. This shows that it is essential for the level of technology to integrate capital (some part of the capital-labor ratio) and technology in a narrow sense (called the level of residual-technology). Even in the Cobb-Douglas production function, the capital-labor ratio and the level of technology tightly cooperate together. I admit that this function is able to express the transitional path from DRS/IRS to CRS by introducing such a device as shown in this paper. A production function at the convergence is a final form, yet we need to express the transitional path differently expressing the same production function. This paper challenged for the reformation to show DRS/IRS in $y = Ak^{\alpha}$. Note that Model A after absorbing *k* of *A*(*k*) into *k* as a whole reduces back to $y = Ak^{\alpha}$.

Second, the most important finding in Model B is that the value of *delta* is determined by *beta* and the current capital-output ratio $\Omega(0)$ (not at the convergence), $\delta = 1-((LN(1/\Omega(0))/$ $(LN((1-\beta^{*})/\beta^{*}))$ (for numerical results, see Tables A1-1 and A1-2 in Appendix). This *delta* integrates *beta*[*] and the capita-output ratio. As a result, the speed of convergence (using the convergence-coefficient $\lambda = (1-\alpha)n + (1-\delta)g_A^{*}$ and the years for convergence, $1/\lambda$) were clarified and measured differently from Barro and Sala-i-Martin (1995). And also, at the convergence, a unique capital-output ratio function of *delta*, $\Omega^{*}_{\alpha\to\alpha}(\delta)$, is derived as $\Omega^{*}_{\alpha\to\alpha} = 1/((1-\beta^{*})/\beta^{*})^{*(1-\alpha)}_{\alpha\to\alpha}$, where *delta*=*alpha* continuously over time holds in the transitional path. Here I distinguish three capital-output ratios: $\Omega(0)$ as the current capital-output ratio, $\Omega^{*}_{\delta\to\alpha}$ to show *delta* ≠ *alpha* at the current situation, and $\Omega^{*}_{\alpha\to\alpha}$ as a benchmark.

Findings are: The stronger the DRS (is the higher the *delta* than *alpha*) the lower the current capital-output ratio than the capital-output ratio at the convergence. As a benchmark, I use the capital-output ratio continuously under CRS, $\Omega^*_{\alpha\to\alpha}$ in the transitional path (or the capital-output ratio 'in the balanced growth-state' set continuously from the current to convergence situations). This $\Omega^*_{\alpha\to\alpha}$ is usually much higher than the current capital-output ratio under DRS or IRS. In the phase diagram, if the current situation shows DRS, the capital-output will finally fall into zero after convergence. This implies that DRS before convergence turns to IRS after convergence. If the current situation shows IRS, the capital-output will reversely be explosive after convergence. I raised two propositions: (1) If the current capital-output is significantly lower than $\Omega^*_{\alpha\to\alpha}$, the current situation is under an extreme DRS, where the capital-labor ratio and the level of technology does not cooperate effectively. (2) If *delta* determined at the current situation remains unchanged after convergence and if the current situation is under DRS, the situation after convergence will be under IRS. Reversely, if *delta* determined at the current situation remains unchanged after convergence and the current situation is under IRS, the situation after convergence will be under DRS. In each case, the phase diagram that uses the capital-output ratio is not symmetric but asymmetric. And, the capital-output ratio will be replaced by the capital-labor ratio in the transitional path and, its phase diagram will differ from the symmetric diagram in the text-books. My approach differs from Jorgenson's (1966, 1967) and Hall's (1968). Comparing with the embodiment hypothesis, I will discuss the differences in a separate paper, hopefully with my phase diagram.

Finally, empirical results by country prove that *delta* will be the same if *beta** and the current capital-output ratio are the same. In other words, if the ratio of investment to output, the growth rate of population/employed persons, the relative share of rents, and the current capital-output ratio are the same among countries, *delta* is the same by country. Or, if the speed of convergence is the same among countries, the above parameters are the same. This will contribute to the discussions for the conventional 'convergence' that compares the annual average growth rate with per capita output at the beginning by country. This will be discussed in the future.

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Appendix: An equation to the speed of convergence[21)]

Only skeletons of the Kamiryo model of endogenous economic growth as discussed above in the text are presented here.

Main features of the model

It is obvious that the model depends upon the Cobb-Douglas type production function $Y = BK^{\alpha}L^{1-\alpha}$, where Y, B, K and L are output, technology level, capital input and labor input, respectively. In order to treat B as of labor-augmenting type and to base the model on the efficient labor basis throughout this appendix for convenience of analysis, we redefine

$$B = A^{1-\alpha},$$

$$y = Y / AL,$$

$$k = K / AL,$$

$$g_A = \frac{dA/dt}{A},$$

$$g(k) = \frac{dk/dt}{k}$$

so that the model can be represented compactly as

(1) $\quad y = k^{\alpha},$

(2) $\quad \dfrac{dk}{dt} = i_K k^{\alpha} - g_A k - nk,$

(3) $\quad g_A = i_A k^{-(\delta-\alpha)},$

where, as already defined in the text

$$n = \frac{dL}{dt} \Big/ L,$$

$$i_K = \frac{\text{saving appropriate to increasing } K}{Y},$$

$$i_A = \frac{\text{saving appropriate to increasing } A}{Y}$$

Thus, the system of nonlinear differential equations (2) (3) determines the dynamics of the model.

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21) I am thankful to the kindness of Dr. Toshimi Fujimoto who allowed me to raise his study using my model in this Appendix.

Clearly, from (2)

(4) $\quad g(k) = i_K k^{-(1-\alpha)} - (g_A + n)$

is obtained and from the definition $k = K / AL$, it follows that

$$g(k) = g(K) - (g_A + n)$$

so that, comparing this with (4), it is evident that

(5) $\quad g(K) \equiv i_K k^{-(1-\alpha)}$.

Now, let us analyze the structure of the model. To begin with, substituting (3) into (2),

(6) $\quad \dfrac{dk}{dt} = i_k k^{\alpha} - i_A k^{1-(\delta-\alpha)} - nk,$

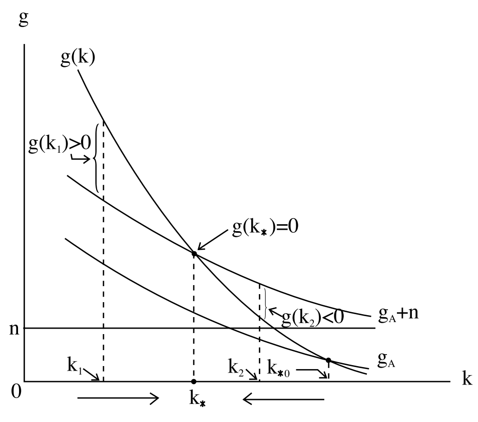
(7) $\quad g(k) = i_k k^{-(1-\alpha)} - i_A k^{-(\delta-\alpha)} - n,$

are obtained. The so-called steady-state value of the pivotal variable k of the model is, as is well-known, nothing but the value, k_*, which makes $\dfrac{dk}{dt} = 0$ in (6). I.e., k_* is the solution of

(8) $\quad i_K k_*^{-(1-\alpha)} - i_A k_*^{-(\delta-\alpha)} - n = 0.$

The moving process of k toward k_* can be sketched in the following diagram in case of $\delta > \alpha$. It seems sufficiently apparent at a glance for the process to be stable that the gradient

Diagram



of $g(K)$ be steeper than that of g_A. In this connection, when $\delta < \alpha$, the stability necessarily holds, because g_A then comes under the increasing function of k, and $g(K)$ as a decreasing function of k always cuts g_A from above.

Before proceeding to solve (8), let us mention how to determine the steady-state values of the other endogenous variables, g_A and y. It is almost self-evident that by inserting k_* in (3) and (1), g_{A*} and y_* are determined respectively as follow,

(9) $g_{A*} = i_A k_*^{-(\delta-\alpha)}$,

(10) $y_* = k_*^{\alpha}$.

How to determine k_*

The solution method adopted here is of a kind of linear approximation by way of comparative statics. That is, first of all, put $n = 0$ in (8) to obtain

$$(11) \quad k_{*0} = \left(\frac{i_K}{i_A}\right)^{\frac{1}{1-\delta}}.$$

In short, k_{*0} is the value of k_* in the condition of a constant L.

Secondly, totally differentiate (8) with respect to only $(k_*,\ n)$ to deduce

$$\left[-(1-\alpha)i_K k_*^{-(1-\alpha)-1} + (\delta-\alpha)i_A k_*^{-(\delta-\alpha)-1}\right]dk_* = dn$$

and evaluate it at ($n = 0$, $k_* = k_{*0}$), then after rearranging,

$$(12) \quad k_* = k_{*0}\left[1 + \frac{nk_{*0}^{1-\alpha}}{(\delta-\alpha)i_A k_{*0}^{1-\delta} - (1-\alpha)i_K}\right]$$

follows, which is found to give what we want to obtain, i.e., k_*. Here, note that

(13) $dk_* = k_* - k_{*0}$, $dn = n - 0 = n$

are assumed as a matter of course.

Lastly, substitute (11) in (12) to lead to the final or reduced form of the endogenous variable k_* in the sense of expressing endogenous k_* exclusively in terms of parameters and exogenous variables such as $\alpha, \delta, i_K, i_A, n$. However, the reduced form thus obtained is found too much complicated to deduce any additional meaningful outcomes from it, but substituting (11) in only the denominator of (12) seems to make much contribution to simplify (12) as follows,

$$(14) \quad k_* = k_{*0}\left[1 - \frac{nk_{*0}^{1-\alpha}}{(1-\delta)i_K}\right].$$

Convergence analysis

First, from the Taylor expansion of (6) at $k = k_*$, a linear approximation

$$(15) \quad \frac{dk}{dt} \simeq \frac{\partial}{\partial k}\left(\frac{dk}{dt}\right)\bigg|_{k=k_*}(k - k_*)$$

is obtained. Second, taking (8) (9) into consideration

(16) $\quad \dfrac{\partial}{\partial k}\left(\dfrac{dk}{dt}\right)\Bigg|_{k=k_*} = -\left[(1-\alpha)n + (1-\delta)g_{A^*}\right]$

is found. Now, define for convenience

(17) $\quad \lambda = \left[(1-\alpha)n + (1-\delta)g_{A^*}\right]$

(18) $\quad x = (k - k_*)$

to lead to a differential equation of the simplest type, in place of (15),

(19) $\quad \dfrac{dx}{dt} = -\lambda x$

so that its solution is given as follows, expressing here each time concerned, *t*,

$$x(t) - x(0)e^{-\lambda t},$$

or more concretely,

(20) $\quad k(t) - k_* = e^{-[(1-\alpha)n+(1-\delta)g_{A^*}]t}(k(0) - k_*).$

This is the instrument appropriate for convergence analysis.

(by Toshimi Fujimoto)

Table A1-1 The speed coefficient, λ, determined by delta, *beta, and $\Omega^* = \Omega(0)$: four countries04**

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
US2004 Total	**0.12180**	**0.10201**	**0.00953**	**2.19061**	**0.77396**	**0.36287**	0.02306	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(0.0713)	(0.0310)	(0.0105)	(0.0032)	0.0011	0.0029	0.0040	#NUM!
1	0.0084	0.0084	0.0084	0.0084	0.0084	0.0084	0.0084	#NUM!
1.5	0.0550	0.0314	0.0194	0.0151	0.0126	0.0115	0.0109	#NUM!
2	0.0880	0.0478	0.0272	**0.0199**	0.0156	0.0138	0.0127	#NUM!
2.5	0.1137	0.0605	0.0333	0.0236	0.0180	0.0155	0.0141	#NUM!
3	0.1346	0.0708	0.0383	0.0266	0.0199	0.0170	0.0153	#NUM!
3.5	0.1523	0.0796	0.0425	0.0292	0.0215	0.0182	0.0163	#NUM!
4	0.1677	0.0872	0.0461	0.0314	0.0229	0.0192	0.0171	#NUM!
4.5	0.1812	0.0939	0.0493	0.0334	0.0242	0.0202	0.0178	#NUM!

The Z axis: *lambda*, the X axis: Ω, and the Y axis: *beta**. $\delta=1-((LN(1/\Omega)/LN((1-\beta*)/\beta*))$

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
Russia2004 T	**0.11982**	**0.11347**	**(0.00498)**	**1.48613**	**0.60254**	**0.04779**	0.04510	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(0.1602)	(0.0815)	(0.0413)	(0.0269)	(0.0186)	(0.0150)	(0.0129)	#NUM!
1	(0.0044)	(0.0044)	(0.0044)	(0.0044)	(0.0044)	(0.0044)	(0.0044)	#NUM!
1.5	0.0867	**0.0407**	0.0172	0.0088	0.0039	0.0018	0.0006	#NUM!
2	0.1514	0.0727	0.0325	0.0182	0.0098	0.0062	0.0042	#NUM!
2.5	0.2015	0.0975	0.0444	0.0254	0.0144	0.0097	0.0069	#NUM!
3	0.2425	0.1178	0.0541	0.0314	0.0182	0.0124	0.0091	#NUM!
3.5	0.2772	0.1350	0.0623	0.0364	0.0213	0.0148	0.0110	#NUM!
4	0.3072	0.1498	0.0694	0.0407	0.0241	0.0169	0.0127	#NUM!
4.5	0.3336	0.1629	0.0757	0.0445	0.0265	0.0187	0.0141	#NUM!

$\beta*=(\Omega*(n\,(1-\alpha)+i\;(1+n)))/(\iota\;(1-\alpha)+\Omega*i\,(1+n))$ $g_A{}^*=i_A=i\,(1\text{-}beta^*)$

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
India2004 Tot	**0.12511**	**0.16667**	**0.01524**	**1.30995**	**0.65072**	**0.56609**	0.05821	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(0.1877)	(0.0862)	(0.0343)	(0.0158)	(0.0050)	(0.0004)	0.0023	#NUM!
1	0.0133	0.0133	0.0133	0.0133	0.0133	0.0133	0.0133	#NUM!
1.5	0.1310	0.0715	**0.0412**	0.0304	0.0241	0.0214	0.0198	#NUM!
2	0.2144	0.1128	0.0610	0.0424	0.0317	0.0270	0.0243	#NUM!
2.5	0.2791	0.1449	0.0763	0.0518	0.0376	0.0314	0.0279	#NUM!
3	0.3320	0.1711	0.0888	0.0595	0.0424	0.0351	0.0308	#NUM!
3.5	0.3767	0.1932	0.0994	0.0659	0.0465	0.0381	0.0332	#NUM!
4	0.4155	0.2124	0.1086	0.0715	0.0501	0.0407	0.0354	#NUM!
4.5	0.4497	0.2293	0.1167	0.0765	0.0532	0.0431	0.0372	#NUM!

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
China2004To	**0.16596**	**0.40006**	**0.00615**	**1.92871**	**0.70832**	**0.25968**	0.11669	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(0.3979)	(0.1943)	(0.0903)	(0.0532)	(0.0317)	(0.0223)	(0.0169)	#NUM!
1	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	0.0051	#NUM!
1.5	0.2409	0.1218	0.0610	0.0393	0.0267	0.0212	0.0180	#NUM!
2	0.4082	0.2046	**0.1006**	0.0635	0.0419	0.0326	0.0272	#NUM!
2.5	0.5379	0.2688	0.1313	0.0823	0.0538	0.0414	0.0343	#NUM!
3	0.6440	0.3213	0.1564	0.0976	0.0635	0.0487	0.0401	#NUM!
3.5	0.7336	0.3657	0.1777	0.1106	0.0717	0.0548	0.0450	#NUM!
4	0.8113	0.4041	0.1961	0.1218	0.0788	0.0601	0.0493	#NUM!
4.5	0.8797	0.4380	0.2123	0.1317	0.0850	0.0647	0.0530	#NUM!

Table A1-2 Years of convergence, $1/\lambda$, determined by *delta*, *beta**, and $\Omega^* = \Omega(0)$: four countries04

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
US2004 Total	**0.12180**	**0.10201**	**0.00953**	**2.19061**	**0.77396**	**0.36287**	0.02306	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(14.0)	(32.2)	(95.3)	(316.9)	909.4	339.5	249.3	#NUM!
1	119.4	119.4	119.4	119.4	119.4	119.4	119.4	#NUM!
1.5	18.2	31.8	51.5	66.2	79.2	86.6	91.5	#NUM!
2	11.4	20.9	36.7	**50.2**	63.9	72.5	78.5	#NUM!
2.5	8.8	16.5	30.0	42.3	55.6	64.3	70.7	#NUM!
3	7.4	14.1	26.1	37.5	50.2	58.9	65.4	#NUM!
3.5	6.6	12.6	23.5	34.2	46.5	55.0	61.5	#NUM!
4	6.0	11.5	21.7	31.8	43.6	52.0	58.5	#NUM!
4.5	5.5	10.6	20.3	29.9	41.4	49.6	56.1	#NUM!

The Z axis: *lambda*, the X axis: Ω, and the Y axis: *beta**. $\delta=1-((\mathrm{LN}(1/\Omega)/\mathrm{LN}((1-\beta*)/\beta*))$

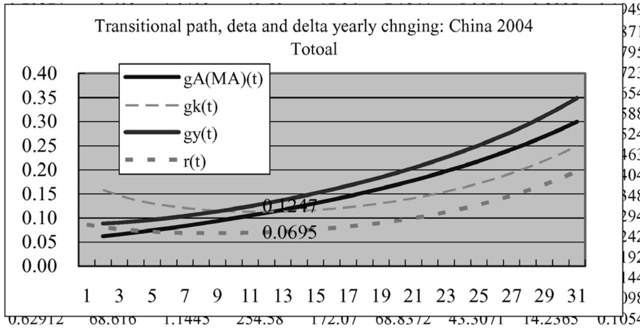
	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
Russia2004 T	**0.11982**	**0.11347**	**(0.00498)**	**1.48613**	**0.60254**	**0.04779**	0.04510	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(6.2)	(12.3)	(24.2)	(37.1)	(53.7)	(66.7)	(77.4)	#NUM!
1	(228.2)	(228.2)	(228.2)	(228.2)	(228.2)	(228.2)	(228.2)	#NUM!
1.5	11.5	**24.6**	58.1	113.5	253.8	546.9	1641.2	#NUM!
2	6.6	13.8	30.8	55.0	101.6	160.4	240.9	#NUM!
2.5	5.0	10.3	22.5	39.3	69.3	103.6	145.0	#NUM!
3	4.1	8.5	18.5	31.9	55.0	80.4	109.4	#NUM!
3.5	3.6	7.4	16.1	27.5	46.9	67.5	90.6	#NUM!
4	3.3	6.7	14.4	24.6	41.5	59.3	78.8	#NUM!
4.5	3.0	6.1	13.2	22.4	37.8	53.6	70.8	#NUM!

$\beta*=(\Omega*(n(1-\alpha)+i(1+n)))/(\iota(1-\alpha)+\Omega*i(1+n))$ $g_A{}^*=i_A=i(1\text{-}beta^*)$

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
India2004 To	**0.12511**	**0.16667**	**0.01524**	**1.30995**	**0.65072**	**0.56609**	0.05821	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(5.3)	(11.6)	(29.2)	(63.4)	(198.8)	(2705.1)	431.0	#NUM!
1	75.0	75.0	75.0	75.0	75.0	75.0	75.0	#NUM!
1.5	7.6	14.0	**24.3**	32.9	41.5	46.8	50.6	#NUM!
2	4.7	8.9	16.4	23.6	31.5	37.0	41.1	#NUM!
2.5	3.6	6.9	13.1	19.3	26.6	31.8	35.9	#NUM!
3	3.0	5.8	11.3	16.8	23.6	28.5	32.5	#NUM!
3.5	2.7	5.2	10.1	15.2	21.5	26.2	30.1	#NUM!
4	2.4	4.7	9.2	14.0	20.0	24.5	28.3	#NUM!
4.5	2.2	4.4	8.6	13.1	18.8	23.2	26.9	#NUM!

	alpha	*i*	*n*	$\Omega(0)$	*beta**	*delta*	$g_A{}^*=i_A$	
China2004To	**0.16596**	**0.40006**	**0.00615**	**1.92871**	**0.70832**	**0.25968**	0.11669	*beta**
	0.55	0.6	0.7	0.8	0.9	0.95	0.975	1
0.5	(2.5)	(5.1)	(11.1)	(18.8)	(31.6)	(44.8)	(59.0)	#NUM!
1	194.8	194.8	194.8	194.8	194.8	194.8	194.8	#NUM!
1.5	4.2	8.2	16.4	25.5	37.5	47.2	55.4	#NUM!
2	2.4	4.9	**9.9**	15.8	23.8	30.7	36.8	#NUM!
2.5	1.9	3.7	7.6	12.2	18.6	24.1	29.1	#NUM!
3	1.6	3.1	6.4	10.2	15.8	20.5	24.9	#NUM!
3.5	1.4	2.7	5.6	9.0	14.0	18.3	22.2	#NUM!
4	1.2	2.5	5.1	8.2	12.7	16.6	20.3	#NUM!
4.5	1.1	2.3	4.7	7.6	11.8	15.4	18.9	#NUM!

1. China 2004		**New Recursive Progamming stating with Y=S+C, (rho/r) being consistent with Y=Π+W**												r/w=(α/(1-α)/k(0)	*s*	s_{Π}	$s_{S\Pi/Y}$	s_H	$s_{SH/Y}$
beta	0.79624	LN(beta)	(0.2279)	LN(delta)	(1.3483)	β from k:	rho	0.0594	The initial data:			w=W/L	7.8225	0.01100	**0.42451**	0.70000	0.11617	0.41320	0.4052
beta*	0.70832	LN(beta*)	(0.3449)	LN(deleta	(1.7960)	0.70832	r	0.0860	I	S	C	Y=S+C	W	Π	L	K	$S_{\Pi}(0)$	D(0)	$S_H(0)$
delta	0.25968	difference	(0.1170)	difference	(0.4477)	β from Ω:	rho/r	**0.6900**	**4907.9**	*5207.9*	**7060.0**	**12267.9**	**10231.9**	*2036.0*	**1308.00**	**23661**	*236.5*	1799.5	4971.4
λ=(1−α)n	0.09152	1/λ→	10.93	the same y	10.93	0.70832	$\mathbf{r_{CB}}$	0.0333	Dividing by Y:			I-S							
1/(δ−α)(n	10.93	0.25968	(0.0107)		(0.0410)	m=C/K	θ_{OPEN}=I/S	0.9424	**i**	**s**	**c**	-300.0	1-α=c/(ρ/r	**α**	**n**	**k**	**Ω=K/Y**	**r**	i=*s* θ_{OPRN}
change in β & δ per	(0.01065)	0.9893	(0.04014)	0.9599		0.2984	(I-S)/Y	(0.0245)	*0.4001*	*0.4245*	*0.57549*		0.83404	*0.1660*	**0.00615**	*18.090*	*1.9287*	*0.0860*	0.4001
β*=(Ω*(n(1-α)+i (1+n)))/(i (1-α)+Ω*i(1+n)), β=(Ω*(n(1-α)k^(-α)+i (1+n)))/(i (1-α)k^(-α)+Ω*i(1+n)), and **δ=1-(LN**											Nomore δ	**Given Ω for δ(0), 1. enter 1/λ, 2. change conv.time as t+11, and watch :**							t+11
t	t+11	**14.1683**	**1.0698**	**66.9195**	**28.4636**	**11.3871**	**8.0594**	**1.6599**	**0.1655**	**0.1109**	**0.1130**	**0.1308**	**0.1198**	**0.1378**	**2.3511**	**0.0706**	**1.9237**	**14.1683**	**2.3511**
	β→β*	$A_{MA}(t)$	L(t)	k(t)	$y_{MA}(t)$	*i(t)*	$i_{K(t)}$	$i_{A(t)}$	**δ→δ***	$\mathbf{g_{A(MA)}(t)}$	$g_k(t)$	$g_y(t)$	$g_K(t)$	$g_Y(t)$	$\Omega_{MA}(t)$	r(t)	$i_A \to i_A^*$	0.1167	**0.1167**
0	0.79624	5.801	1.0000	18.09	9.38	3.7522	2.9876	0.3605	0.25968						1.9287	0.0860	$i_K \to i_K^*$	0.2834	0.2834
1	0.78776	6.161	1.0062	20.95	10.21	4.0836	3.2169	0.4060	0.24925	0.0621	0.1580	0.0883	0.1652	0.0950	2.0522	0.0809	$g_A(1) \to g_A$	0.0382	0.1167
2	0.77937	6.567	1.0123	24.02	11.13	4.4526	3.4702	0.4592	0.23925	0.0659	0.1465	0.0904	0.1536	0.0971	2.1579	0.0769	$g_k(1) \to g_k^*$	0.0389	0.1390
3	0.77107	7.026	1.0186	27.32	12.17	4.8669	3.7527	0.5213	0.22964	0.0699	0.1375	0.0930	0.1445	0.0998	2.2457	0.0739	$g_y(1) \to g_y^*$	0.0444	0.1390
4	0.76285	7.548	1.0248	30.88	13.34	5.3355	4.0702	0.5940	0.22042	0.0742	0.1304	0.0963	0.1374	0.1030	2.3156	0.0717	$g_K(1) \to g_K$	0.0454	0.1461
5	0.75473	8.142	1.0312	34.74	14.67	5.8689	4.4294	0.6795	0.21157	0.0787	0.1249	0.1000	0.1318	0.1067	2.3680	0.0701	$g_Y(1) \to g_Y$	0.0509	0.1461
6	0.74669	8.821	1.0375	38.93	16.20	6.4800	4.8386	0.7803	0.20308	0.0835	0.1206	0.1041	0.1275	0.1109	2.4033	0.0691			
7									0493	0.0									$\Omega_{MA}(t)$
8									3710	0.0									2.4255
9									7959	0.0									2.4140
10									7238	0.1									2.3888
11									5546	0.1									2.3511
12									5882	0.1									2.3021
13									5245	0.1									2.2431
14									4633	0.1									2.1755
15									4045	0.1									2.1007
16									3481	0.1									2.0199
17									2940	0.1									1.9346
18									2421	0.1									1.8458
19									1922	0.1									1.7550
20									1443	0.1									1.6630
21									0984	0.1									1.5709
22	0.62912	68.616	1.1445	234.38	172.07	68.8372	43.3071	14.2365	0.10543	0.1972	0.1548	0.2261	0.1619	0.2337	1.4796	0.1122	1.3394	68.62	1.4796
23	0.62242	82.852	1.1515	296.07	213.04	85.2284	53.0481	18.0920	0.10120	0.2075	0.1630	0.2381	0.1701	0.2457	1.3897	0.1194	1.3218	82.85	1.3897
24	0.61579	100.944	1.1586	346.98	266.49	#######	65.6499	23.2064	0.09714	0.2184	0.1720	0.2509	0.1792	0.2586	1.3021	0.1275	1.2859	100.94	1.3021
25	0.60924	124.151	1.1658	410.11	336.97	#######	82.1294	30.0615	0.09324	0.2299	0.1819	0.2645	0.1892	0.2723	1.2170	0.1364	1.2515	124.15	1.2170



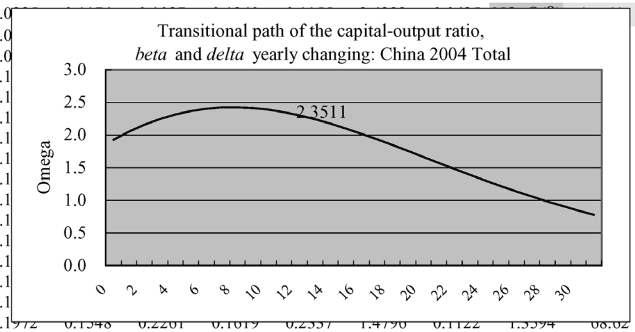
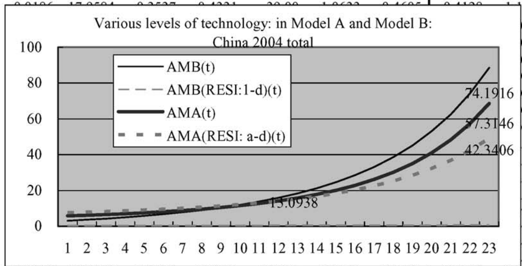


Figure A1-1 Recursive programming for China 2004 Total economy (1)

2. China 2004			MODEL A vs. MODEL B		At convergence unde $1/\Omega^*=B^*$^1-δ								At the current year			A(t)$_{MA}$ equals A(t)$_{MB}$ if ε(t)=1 at the convergence.		
AΩ=k^1-α			**delta**	**Ω***	Assume Ω(0)=Ω*, y	Ω(0)	**set Ω**	1/Ω*	beta*	1-beta*	϶=(1-β*)/β	**delta**	i=I/Y	i=i$_A$+i$_K$	0.4001	Current	At convergence	
Ω(t)=k^(δ–α)/B^1–δ			(0.2000)	2.9000	Ω* differs from Ω(0	1.9287	1.7639	0.5669	0.7083	0.2917	0.4118	0.36035	i$_A$=i·(1-beta*)		0.1167	**5.8007**	**14.1683**	$A_{MA}(t)=\varepsilon_B(t)\cdot A(t)_{MB}$
A(t)=k^(1–α)/Ω(t)			0.0000	2.4284	Before Goal Seek	**Case 1**	2.1797	0.4588	0.7083	0.2917	0.4118	0.1218	i$_K$=i·beta*		0.2834	**3.1095**	**15.9550**	$A_{MB}(t)$=(B(t)k(t))^(1-δ(t))
ε$_B$=A(t)$_{MA}$/A(t)$_{MB}$			0.1660	2.0959	After Goal Seek	**Case 2**	1.0001	0.9999	0.7083	0.2917	0.4118	0.9999	n/g$_A$*		0.0527	**5.8007**	**15.9550**	
ε$_B$=y$_{MA}$(t) /(B^(1-δ)·k^(1-δ+α)			0.5000	1.5583		**Case 3**	1.7600	0.5682	0.7083	0.2917	0.4118	0.3629	**If λ is accurate, discrete case**			**0.6801**	**0.4780**	$A_{MB(RESI:1-\delta)}(t)=A_{MB}(t)$/k(t)^(1–δ(t))
Ω*/Ω(0)	Ω*MA=1/	delta*ΩMA	1.0000	1.0000		**Case 4**	2.7164	0.3681	0.7083	0.2917	0.4118	(0.1263)	**is consistent with equations.**			**5.0278**	**32.0529**	$y_{MB}(t)=A_{MB(RESI:\ 1-\delta)}(t)$·k^(1-δ)·k^α
1.2190	2.0959	0.1660	1.5000	3.9944		**Case 5**	3.1902	0.3135	0.7083	0.2917	0.4118	(0.3075)	λ=(1–α)n+(1–δ)g$_A$*					$y_{MA}(t)=\varepsilon_B(t)\cdot y_{MB}(t)$

2.0090	**0.0179**	**28.4636**	**0.4129**	**0.4780**	**67.0598**	**0.8880**	**0.4245**	**0.4253**	**0.9979**	**2.0878**	**33.3105**	**15.9550**	**15.9550**	**0.8880**	**0.0833**	**28.4636**	**0.1450**	**0.0934**	**33.3803**	**0.4780**	**32.0529**	**15.9550**	**14.1683**
k^α	g$_{k^\alpha}$	y$_{MA}$=Ak^	B(t)	B(t)^(1-δ)	k$_{(MB)}$^(1-δ	**ε$_B$**	ε$_B$·B(t)^(1	1/Ω(t)	k$_{A(MA)}$^(δ-	Ω(t)$_{MB}$=k^	k(t)$_{BASE}$^(	**A$_{MB}$(t)**	(Bk)^(1–δ	A(t)/A$_{MB}$(	**g$_{A(MB)}$(t)**	**y$_{MA}$=e$_B$·y**	g$_{k(t)MB}$^(1-	g$_{k(t)^(1-\alpha)}$	k(t)$_{A(MB)}$^(	A$_{MB(RESI:1}$	**y$_{MB}$(t)**	**A$_{MB}$(t)**	A$_{MA}$(t)=ε$_E$
1.6169		9.3791	0.2559	0.3646	13.79	1.8655	0.6801	0.5185	1.3117	3.5979	11.19	3.11	3.11	1.8655		9.38			8.53	0.3646	5.03	3.11	5.8007
1.6568	0.0246	10.2075	0.2694	0.3736	16.26	1.6804	0.6278	0.4873	1.2884	3.4486	12.64	3.67	3.67	1.6804	0.3792	10.21	0.1790	0.1302	9.81	0.3736	6.07	3.67	6.1611
1.6948	0.0229	11.1300	0.2831	0.3829	19.03	1.5279	0.5850	0.4634	1.2623	3.2970	14.17	4.30	4.30	1.5279	0.3197	11.13	0.1702	0.1208	11.23	0.3829	7.28	4.30	6.5672
1.7314	0.0216	12.1655	0.2969	0.3924	22.13	1.4009	0.5497	0.4453	1.2345	3.1459	15.78	5.02	5.02	1.4009	0.2715	12.17	0.1632	0.1134	12.78	0.3924	8.68	5.02	7.0264
1.7670	0.0206	13.3367	0.3109	0.4022	25.62	1.2943	0.5206	0.4319	1.2054	2.9972	17.48	5.83	5.83	1.2943	0.2318	13.34	0.1577	0.1076	14.50	0.4022	10.30	5.83	7.5477
1.8018	0.0197	14.6701	0.3250	0.4122	29.55	1.2044	0.4965	0.4223	1.1757	2.8520	19.28	6.76	6.76	1.2044	0.1988	14.67	0.1533	0.1031	16.40	0.4122	12.18	6.76	8.1417
1.8362	0.0191	16.1977	0.3392	0.4225	33.98	1.1281	0.4767	0.4161	1.1456	2.7113	21.20	7.82	7.82	1.1281	0.1711	16.20	0.1500	0.0996	18.51	0.4225	14.36	7.82	8.8213
1.8704	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.5756	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20.85	0.4331	16.89	9.03	9.6016
1.9045									[illegible]	2.4454									[illegible]	0.4439	19.84	10.42	10.5012
1.9388									[illegible]	2.3206									[illegible]	0.4550	23.28	12.01	11.5428
1.9736									[illegible]	2.2014									[illegible]	0.4664	27.31	13.84	12.7538
2.0090									[illegible]	2.0878									[illegible]	0.4780	32.05	15.95	14.1683
2.0452									[illegible]	1.9795									[illegible]	0.4899	37.65	18.41	15.8282
2.0824									[illegible]	1.8765									[illegible]	0.5020	44.27	21.26	17.7858
2.1209									[illegible]	1.7785									[illegible]	0.5144	52.16	24.59	20.1064
2.1609									[illegible]	1.6853									[illegible]	0.5271	61.61	28.51	22.8725
2.2026									[illegible]	1.5967									[illegible]	0.5400	72.97	33.13	26.1884
2.2464									[illegible]	1.5123									[illegible]	0.5532	86.75	38.62	30.1876
2.2925									[illegible]	1.4321									[illegible]	0.5667	103.54	45.16	35.0413
2.3414									[illegible]	1.3557									[illegible]	0.5805	124.18	53.04	40.9718
2.3932									[illegible]	1.2829									[illegible]	0.5945	149.74	62.57	48.2690
2.4485									[illegible]	1.2135									[illegible]	0.6087	181.66	74.19	57.3146
2.5077	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.1473	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.6233	221.89	88.48	68.6157
2.5713	0.0254	#######	0.6066	0.6381	428.00	0.7801	0.4978	0.7196	0.6917	1.0841	115.14	106.21	106.21	0.7801	0.0136	213.04	0.2022	0.1342	166.45	0.6381	273.11	106.21	82.8522
2.6399	0.0267	#######	0.6239	0.6532	518.97	0.7861	0.5135	0.7680	0.6686	1.0236	131.44	128.40	128.40	0.7861	0.0114	266.49	0.2125	0.1415	196.58	0.6532	338.98	128.40	#######
2.7142	0.0281	#######	0.6414	0.6685	635.21	0.7935	0.5305	0.8217	0.6456	0.9658	151.10	156.46	156.46	0.7935	0.0095	336.97	0.2240	0.1496	234.03	0.6685	424.65	156.46	#######



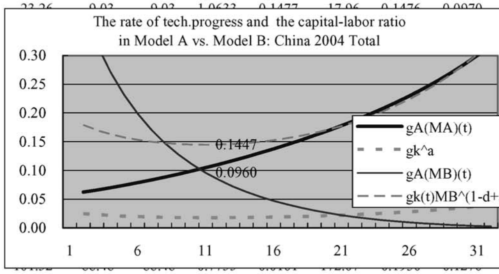


Figure A1-2 Recursive programming for China 2004 Total economy (2)

3. China 2004 $i(t)=(I/L)(t)=i\cdot y(t)$ $g_A(t)=i\cdot(1-\beta(t-1))\cdot k(t-1)$^$(\alpha-\delta(t-1))$ $w(t)=W(t)/L(t)=y(t)(1-\alpha)$

$i_A(t)=i\cdot(1-\beta(t-1))\cdot A(t-1).k(t-1)$^$(\alpha-\delta(t-1))$ $g_K(t)=i\cdot\beta(t-1)\cdot A(t-1)k(t-1)$^$\alpha$

$i_K(t)=i\cdot\beta(t-1)\cdot A(t-1)k(t-1)$^$\alpha$ $g_k(t)=((i\cdot\beta(t-1)\cdot A(t-1)k(t-1)$^$\alpha)-n)/(1+n)$ Test for Ricardo's

$y_{MA}=Ak$^α: at the convergence, $y_{MA}=y_{MB}$, where $\varepsilon_B=1$. $A_{MB}=(Bk)$^$(1-\delta)$ since α is fixed, gw-gy=0..

$y_{MB}=\varepsilon_B\cdot B$^$(1-\delta)\cdot k(1-\delta+\alpha)$ $\Omega_{MB}=k$^$(\delta-\alpha)/B$^$(1-\delta)$

$g_{A(STOCK)}(t)=(A(t)-A(t-1))/A(t-1)$ $A_{MA}=\varepsilon_B\cdot A_{MB}$

$g_{A(FLOW)}(t)=gy(t)-\alpha\cdot gk(t)$ $gk(t)=(gK(t)-n)/(1+n$ as k^$(1-\alpha)/\Omega$.

$g_{A(STOCK)}(t)=g_{A(FLOW)}(t)$ $g_K(t)=gk(t)+n+gk(t)\cdot n$

Review of Model A:

When Model A Separates k^$(\alpha-\delta)$ from A(t).

$A_{MA(PE\Sigma I:\ \alpha-\delta)}(t)=A_{MA}(t)/k$^$(\alpha-\delta)$

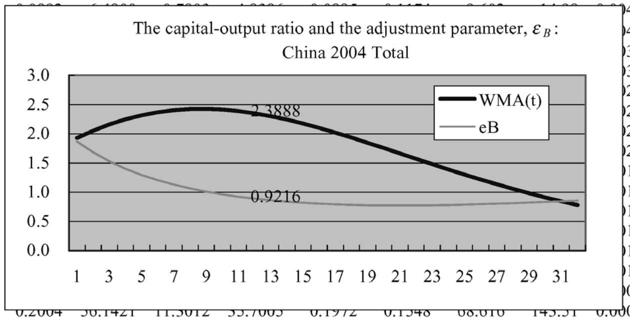
$y_{MA(REV)}=A_{MA(t)}.k(t)$^$(2\alpha-\delta)$ | $RESI_{A(MA)/A(MB)}=A_{MA(RESI)}/A_{MB(RESI)}$

$\Omega(t)_{(REV)}=k$^$(1-\alpha)/A_{MA(RESI)}$ | $k(t)_{BASE}$^$(1-\alpha)=A_{MA(RESI)}/A_{MB(RESI)}/\varepsilon_B$

Model A cannot produce the value of *delta* ,

since $\Omega(t)=k$^$(1-\delta)/A_{MA(RESI)}$ is only reduced to k^$(1-\alpha)/A(t)$.

$g_k(t)$	$g_{AFLOW}(t)$	$i\ (t)$	$i_A(t)$	$i_{K(t)}$	$g_A(t)=i_A(t)$	$g_k(t)$	$A_{MA}(t)$	w(t)	r(t)/w(t)	$g_w(t)$	g_w-g_y	$\delta(t)-\alpha$	$k_{(MA)}$^$(\alpha-\delta$	k_{MA}^$(2\alpha-$	$A_{MA(RESI:}$	$y_{MA(REV)}=$	$\Omega(t)_{(REV)}=$	$A_{MA(RESI)}/$	$k(t)_{BASE}$^(
0.1130	**0.1121**	**10.0699**	**1.4145**	**7.2038**	**0.1109**	**0.1130**	**14.1683**	**23.7397**	**0.0030**	**0.1308**	**0.0000**	**(0.0005)**	**1.0021**	**2.0132**	**14.1387**	**28.4636**	**2.3560**	**29.5804**	**33.3105**
							5.801	7.82	0.01100			0.0937	0.7624	1.2327	7.61	9.38	1.4704	20.87	11.19
0.1580	0.0621	3.7522	0.3605	2.9876	0.0621	0.1580	6.161	8.51	0.00950	0.08833	0.00000	0.0833	0.7762	1.2859	7.94	10.21	1.5929	21.25	12.64
0.1465	0.0661	4.0836	0.4060	3.2169	0.0659	0.1465	6.567	9.28	0.00829	0.09037	0.00000	0.0733	0.7922	1.3426	8.29	11.13	1.7095	21.65	14.17
0.1375	0.0702	4.4526	0.4592	3.4702	0.0699	0.1375	7.026	10.15	0.00728	0.09304	0.00000	0.0637	0.8101	1.4026	8.67	12.17	1.8191	22.10	15.78
0.1304	0.0746	4.8669	0.5213	3.7527	0.0742	0.1304	7.548	11.12	0.00644	0.09627	0.00000	0.0545	0.8296	1.4659	9.10	13.34	1.9210	22.62	17.48
0.1249	0.0793	5.3355	0.5940	4.0702	0.0787	0.1249	8.142	12.24	0.00573	0.09998	0.00000	0.0456	0.8506	1.5326	9.57	14.67	2.0142	23.22	19.28
0.1206	0.0841	5.8689	0.6795	4.4294	0.0835	0.1206	8.821	13.51	0.00511	0.10413	0.00000	0.0371	0.8729	1.6028	10.11	16.20	2.0979	23.92	21.20
0.1174										0.10870									23.26
0.1151										0.11366									25.47
0.1137										0.11901									27.86
0.1130										0.12472									30.47
0.1130										0.13081									33.31
0.1136										0.13728									36.44
0.1148										0.14413									39.90
0.1167										0.15137									43.74
0.1192										0.15902									48.05
0.1222										0.16710									52.90
0.1259										0.17562									58.40
0.1303										0.18462									64.68
0.1353										0.19413									71.90
0.1411										0.20419									80.27
0.1476										0.21484									90.03
0.1548										0.22612									101.52
0.1630	0.2111	68.8372	14.2365	43.3071	0.2075	0.1630	82.852	177.68	0.00067	0.23812	0.00000	(0.0648)	1.4456	3.7171	57.31	213.04	2.0090	89.82	115.14
0.1720	0.2223	85.2284	18.0920	53.0481	0.2184	0.1720	100.944	222.26	0.00057	0.25088	0.00000	(0.0688)	1.4957	3.9485	67.49	266.49	1.9475	103.33	131.44
0.1819	0.2343	#######	23.2064	65.6499	0.2299	0.1819	124.151	281.04	0.00049	0.26449	0.00000	(0.0727)	1.5489	4.2040	80.15	336.97	1.8851	119.90	151.10



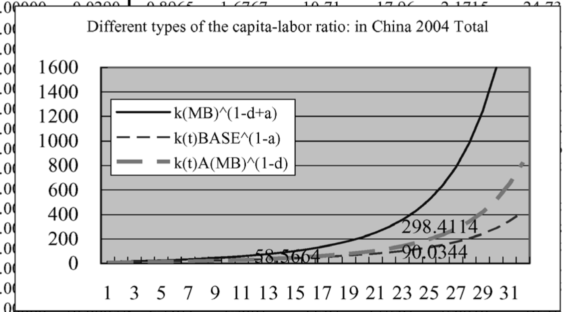


Figure A1-3 Recursive programming for China 2004 Total economy (3)