On Long-Run Effects of the Hypothetical Fertility-Related Social Security System

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1. Introduction

The effects of social security have been analyzed in a substantial literature. But in almost every literature, for example, in Feldstein (1974), Barro (1974), Becker and Barro (1988), Laintner (1988), Barro and Becker (1989), Lapan and Enders (1990) and Seater (1993), their framework is an exogenous growth setting.

In recently, the effects of social security begin to be analyzed by the framework of endogenous growth setting, for example, in Zhang and Zhang (1995), Zhang (1995), Zhang and Zhang (1998) and Wigger (1999). In these studies, the interrelation between the fertility rate, the per capita income growth rate and the social security tax rate on the balanced growth path (BGP) is analyzed.

There are two types of assumption about individual utility function in these studies: non-altruistic and altruistic. Furthermore there are two types of altruism: the altruism of children for parents and of parents for children. It is important aspect that whether there are voluntary intergenerational transfers or not, because the effects of social security is influenced by it crucially. If we take non-altruistic approach, there is no incentive to make intergenerational transfer. In this case, to make the transfer operative, it is necessary to make an assumption about some kind of social norm[1]. But there is a problem in such approach that the transfer is arbitrary and exogenously determined, as mentioned in Zhang and Nishimura (1993).

In this paper, We will take an altruistic approach of Veall (1986). As mentioned in Veall (1986), the consumption by the parents has externality experienced by their children. So we explicitly model the altruism of children for parents. According to Veall's approach, Nishimura

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1) For example, Samuelson (1958) calls it Kant's Categorical Imperative ('enjoining like people to follow the common pattern that makes each best off').

and Zhang (1992), Zhang and Nishimura (1993), and, Nishimura and Zhang (1995) analyze the effects of pay-as-you-go public pension system. But their framework is not an endogenous growth setting.

Zhang and Zhang (1995), Zhang (1995), Zhang and Zhang (1998), and, Wigger (1999) analyze the effects of pay-as-you-go social security system by employing an endogenous growth setting. Zhang (1995), Zhang and Zhang (1998), and, Wigger (1999) studies the interrelation between the fertility rate, the per capita income growth rate, and, the social security tax rate on the BGP when social security tax rate changes. But, in their approach, individual utility function is non-altruistic or altruistic one, and the altruism of children for parents or of parents to children. Zhang (1995) takes non-altruistic approach, Zhang and Zhang (1998) takes altruistic approach and their altruism is from parents to children, and Wigger (1999) takes the altruism of both side[2].

In these studies, there are two social security systems: the conventional one and the hypothetical fertility-related one. The fertility-related social security system relates its benefits to individual fertility. So social security benefits would be proportional to an individual's production of future tax payers[3]. From the point of view that whether the hypothetical social security system that relates its benefits to individual fertility would stimulate fertility or not, the analysis of this hypothetical social security system is important because the conventioanl social security system would not stimulate fertility[4].

In this paper, we will build a general equilibrium model with both endogenous fertility and growth. Following the recent literature on endogenous growth, a simple engine of growth — the Romer (1986) type of capital externality — is chosen. And we will analyze the effects of the fertility-related social security system on the fertility rate and the per capita income growth rate on the BGP.

The main results are as follows: If the social security tax rate is low, there is no effect of the social security system on the fertility rate and the per capita growth rate because the existence of private intergenerational transfer cancels out the effect of the social security system. If the social security tax rate exceeds the regime-switching tax rate, there are

2) To introduce the altruism of parents for children, Wigger (1999) introduces the consumption aspect of children to his model.

3) In Bental (1989), the idea of conditioning social security benefit to individual fertility is discussed.

4) See, for example, Zhang (1995) and Zhang and Zhang (1998). In this paper, we examine this point in the Appendix A. 2.

negative and positive interrelations between the fertility rate and the per capita income growth rate with no private intergenerational transfer. These results depend on the fertility effect and the saving effect of the social security system.

The rest of the paper is organized as follows: In section 2, the model with endogenous fertility and Romer's type of endogenous growth is built. In section 3, the effects of social security on the BGP is examined. Finally, the paper concludes in section 4.

2. The model

2.1 *Individual*

As in related studies, a two-period overlapping-generations model is used. Each individual lives for two periods. In the first period of life (young), each individual works and earns income in the labor market, gives birth to children, makes gift to parents and saves for old age. In the second period of life (old), each individual lives in retirement, i.e, consumes savings, gift and pension from the social security system. The motive of young individual to make gift to parents is an altruistic one.

Each working individual chooses own consumption in the first period, C_t^t, saving, S_t, gift, δ_t, and the number of children, $1 + n_t$. Children are raised with costs. The cost function of raising children is assumed to be a function of the number of children, and takes the form $h(1 + n_t) = a\,(1 + n_t)^b$, where a and b are constant parameters, as in Nishimura and Zhang (1992). It is assumed that $b \geq 1$. This assumption ensures that the maximum problem of the young is strictly concave and that the respective second order conditions are satisfied[5].

We assume that the utility of each individual depends on his lifetime consumption and that of his parents during retirement. As mentioned in Veall (1986), the utility from the consumption of individual's parents represents an externality experienced by their children. The caring for parents provides the incentive for individuals to have children[6].

We assume a logarithmic utility function.

$$U(C_t^t,\ C_{t+1}^t,\ C_t^{t-1}) = \ln\ C_t^t + \rho \ln\ C_{t+1}^t + \theta \ln\ C_t^{t-1}, \tag{1}$$

where superscripts denote the time of birth and subscripts the time of consumption. Thus, C_t^t and C_{t+1}^t are the first- and second-period consumption of the younger generation t, and C_t^{t-1} is

5) See the Appendix A. 1.

6) For discussion and application of this type of effect, see Hochman and Rogers (1969), Pauly (1973).

the second-period consumption of the older generation $t-1$. ρ is an intertemporal discount factor and θ is an intergenerational discount factor.

The budget constrains are given by:

$$C_t^t = \{1-\delta_t - \tau - h(1+n_t)\}W_t - S_t, \tag{2}$$

$$C_{t+1}^t = (1+r_{t+1})S_t + (1+n_t)\delta_{t+1}W_{t+1} + (1+n_t)\tau W_{t+1}, \tag{3}$$

$$C_t^{t-1} = (1+r_t)S_{t-1} + (1+n_{t-1})\delta_t W_t + (1+n_{t-1})\tau W_t, \tag{4}$$

where W_t is the wage rate and r_t is the interest rate in period t. τ is social security tax rate. Each young individual at time t consumes C_t^t units of a homogeneous output, gives birth to $1 + n_t$ children, transfers a fraction τ of income to the public pension system, a fraction δ_t as a gift to parents, and a fraction $h(1 + n_t)$ for rearing children. At time t + 1, old individual consumes C_{t+1}^t units using the proceeds of saving, the gift from children and a public pension. We assume the fertility-related social security system, so it is assumed that parents realize that the social security benefit depends on their production of future tax payers, and takes these into account while deciding the number of children.

2.2 *Public pension system*

The public pension system is assumed to be a pay-as-you-go financed one. It taxes individual income at a constant rate, τ, and uses its revenues to transfer it to the old. As in (3) and (4), the transfer are related to individual fertility. Each individual with fertility $1 + n_t$ excepts $(1 + n_t)\tau$ out of the wage from the next generation.

2.3 *Firm*

Firms are assumed to behave competitively. They employ the available labor force which equals the size of the young working generation, and the aggregate capital stock to produce the homogeneous output.

A firm i rents K_t^i unit of capital from the old generation and employs L_t^i young workers and generates net output of

$$Y_t^i = F(K_t^i,\ A_t L_t^i), \tag{5}$$

where the production function F is assumed to exhibit constant return to scale. A_t represents labor productivity with a Romer type externality that depends on the size of the aggregate capital stock. This is the engine of growth in this model. The role of the externality is to eliminate diminishing returns on capital so as to ensure the existence of endogenous growth.

The homogeneity of degree one of F implies the following relationships at the aggregate level.

$$r_t = F_1(K_t, A_t L_t) = f'(k_t), \tag{6}$$

$$W_t = [F(K_t, A_t L_t) - K_t F_1(K_t, A_t L_t)] / L_t = A_t [f(k_t) - k_t f'(k_t)], \tag{7}$$

where $k_t \equiv K_t / (A_t L_t)$ (capital per unit of effective labor), $f(k_t) \equiv F(K_t / (A_t L_t), 1)$ (output per unit of effective labor).

Product market equilibrium obtains when aggregate investment equals aggregate savings:

$$K_{t+1} = S_t L_t. \tag{8}$$

In order to ensure the existence of a balanced growth path, as in Grossman and Yanagawa (1993), it is assumued that A_t takes on a particular form as follows:[7)]

$$A_t = \overline{K}_t / v, \tag{9}$$

where $\overline{K}_t \equiv K_t / L_t$.

In equilibrium, it follows that $k_t = v$ for all t. Denote $w_t = W_t / A_t$ (the wage rate per effective unit of labor), then r_t and w_t become constant for all t given by (6) and the above definition of w_t.

We must mention the definition of the balanced growth path before proceed the analysis. A balanced growth path is a competitive equilibrium path in which gift, fertility and per capita income growth rate (δ_t, n_t, g_t) are all constant, and per capita output, capital and consumption all grows at the same rate, g.

Using equation (8), per capita capital stock along a balanced growth path grows at the rate

$$1 + g_t = \frac{S_t w}{v(1+n_t) W_t}. \tag{10}$$

It is suggested that social security could affect the growth rate through its effects on savings and fertility from (10).

2.4 *The competitive equilibrium*

The decision problem faced by individual is as follows: individual seeks maximize (1) by choosing savings, S_t, the gift ratio, δ_t, and the number of children, $1 + n_t$, subject to the budget constraints (2) – (4). In solving this problem, it is assumed that each individual takes decision of future generations as given. For an interior solution, utility maximization yields the following first-order conditions:

7) The general form of A is $A = (\overline{K})^{\psi} / v$ as in Romer (1986). If $\psi > 1$, the growth rate is increasing; if $0 < \psi < 1$, it is decreasing. A BGP exists only if $\psi = 1$. Hence, this model reduces to a version of the AK model in which there are no transitional dynamics.

$$-U_1 + (1 + r_{t+1})U_2 = 0, \tag{11}$$

$$-U_1 + (1 + n_{t-1})U_3 = 0, \tag{12}$$

$$-h'(1 + n_t)U_1 + (\delta_{t+1} + \tau)(W_{t+1} / W_t)U_2 = 0, \tag{13}$$

where U_i (i = 1, 2, 3) denotes the partial derivative of U with respect to the ith argument.

Equation (11) – (13) correspond to the first order conditions for saving, the gift ratio and the number of children, respectively. Equation (11) states the marginal return of a unit of saving must equal the marginal cost in terms of losses in utility. Equation (12) states the interrelation between the marginal return of a unit of the gift ratio and the marginal cost in terms of losses in utility as in (11). Equation (13) states the interrelation between the marginal return of having a child and the marginal child rearing cost in terms of losses in utility. The first term of the left hand side of equation (13) states the marginal cost of having a child and the second term states the marginal return in terms of ulility. In the second term, there are the marginal return from the gift and the social security system. We must notice that the per capita income grows at the rate, g[8].

To proceed the analysis, we use the functional form of utility function, (1), to get the following relation between consumptions.

$$\rho(1 + r_{t+1})C_t^t = C_{t+1}^t, \tag{14}$$

$$\theta(1 + n_{t-1})C_t^t = C_t^{t-1}, \tag{15}$$

$$\rho(W_{t+1} / W_t)(\delta_{t+1} + \tau)C_t^t = h'(1 + n_t)C_{t+1}^t. \tag{16}$$

3. Equilibrium analysis

3.1 *Operative gift*

We focus on the balanced growth equilibrium. The balanced growth is a competitive equilibrium in which gift, fertility and growth rate (δ_t, n_t, g_t) are all constant, and per capita output, capital and consumption all grow at the same rate, g.

We have the interrelation between the fertility rate, n, and the per capita income growth rate, g, as follows from (14) and (15).

$$(1 + g)(1 + n) = \frac{\rho(1 + r)}{\theta}. \tag{17}$$

Equation (17) implies a negative interrelation between n and g, if gift are operative. This

8) From the definition of w_t, A_t ((9)) and the constancy of w_t, we have $W_{t+1} / W_t = \bar{K}_{t+1} / \bar{K}_t = 1 + g$ on the BGP.

is because the interest rate, r, is constant.

Using (2), (3), (14) and (17), we can write the first-period consumption on the BGP, $\hat{C}_t^l$, as follows.

$$\hat{C}_t^l = \frac{1}{1+\rho} W_t \left[\{1-\delta-\tau-h(1+n)\} + \frac{\rho(\delta+\tau)}{\theta} \right] \tag{18}$$

Substituting (18) into the budget constraint (2), we find the saving function on the BGP, $\hat{S}_t$.

$$\hat{S}_t = \frac{1}{1+\rho}\{1-\delta-\tau-h(1+n)\}W_t - \frac{\rho(\delta+\tau)}{\theta(1+\rho)} W_t \tag{19}$$

We substitute (17) and (19) into (10) to derive

$$h(1+n) = \frac{w\rho\theta - \rho v(1+r)(1+\rho)}{w\theta\{b(1+\theta)+\rho\}}. \tag{20}$$

From (14) , (16) and (17), we have

$$\delta+\tau = \frac{b\theta}{\rho} h(1+n). \tag{21}$$

It depends on the public pension system that whether individuals make gift to their parents or not. The following lemma provides a necessary and sufficient condition for gift being operative in the balanced growth equilibrium[9].

Lemma 1 *The gift ratio, δ, is strictly positive if and only if the social security tax rate satisfies:*

$$\tau < \bar{\tau} \equiv \frac{b\{\theta w - v(1+r)(1+\rho)\}}{w\{b(1+\theta)+\rho\}}. \tag{22}$$

Proof. From (20) and (21), we have

$$\delta = \frac{b\{w\theta - v(1+r)(1+\rho)\}}{w\{b(1+\theta)+\rho\}} - \tau. \tag{23}$$

Let $\delta > 0$. Then we have $\tau < \bar{\tau}$. Next let $\delta \leq 0$. Then we have $\tau \geq \bar{\tau}$. This is a contraposition. □

$\bar{\tau}$ is smaller than 1. On the other hand, whether $\bar{\tau}$ is positive or not depends on the parameters. A necessary and sufficient condition for $\bar{\tau}$ to be possitive is that:

9) The analysis in section 3.1 owes to Wigger (1999).

$$\theta > \frac{v(1+r)(1+\rho)}{w}. \tag{24}$$

It is assumed that θ meets condition (24). Equation (24) means that the altruism of children for parents has to be strong enough to render the gift operative.

Equation (23) states the interrelation between the gift ratio, δ, and the social security tax rate, τ, and there is a negative interrelation. The following lemma indicates this relation.

Lemma 2 *If $\tau < \bar{\tau}$, a rise in τ reduce the gift ratio, δ.*

Proof. δ is given as in (23) from the proof of *Lemma* 1. Differentiation of (23) with respest to τ leads to

$$\frac{d\delta}{d\tau} = -1 < 0. \ \square \tag{25}$$

A rise in τ reduces the first period consumption and raises the second period consumption. Hence, marginal utility of the first period consumption increases and of the second period one decreases. So young individual intends to reduce the gift to parents.

Equation (17) implies a negative interrelation between the fertility rate, n, and the per capita income growth rate, g. A decrease of the fertility rate is accompanied by an increase of the per capita growth rate and vice versa. The following proposition indicates the effect of the change of τ on n and g.

Proposition 1 *If $\tau < \bar{\tau}$, a rise in τ has no effect on the fertility rate, n, the per capita income growth rate, g, and the saving rate, $s_t (\equiv \hat{S}_t / W_t)$.*

Proof. Using the functional form of $h(1 + n)$, (17) and (20), we can write the fertility rate, n, and the per capita income growth rate, g, on the BGP as follows:

$$g = (1+r)(\rho/\theta)\left[\frac{\rho\theta w - v(1+r)\rho(1+\rho)}{a\theta w\{b(1+\theta)+\rho\}}\right]^{-1/b} - 1, \tag{26}$$

$$n = \left[\frac{\rho\theta w - v(1+r)\rho(1+\rho)}{a\theta w\{b(1+\theta)+\rho\}}\right]^{1/b} - 1. \tag{27}$$

Differentiation of (26) and (27) with respect to τ leads to

$$\frac{dg}{d\tau} = 0, \tag{28}$$

$$\frac{dn}{d\tau} = 0. \tag{29}$$

Next we can write the saving rate, $s_t (\equiv \hat{S}_t / W_t)$, as follows from (19).

$$s_t = \left(\frac{\rho}{1+\rho}\right)\{1-\delta-\tau-h(1+n)\} - \left(\frac{1}{1+\rho}\right)(\rho/\theta)(\delta+\tau) \tag{30}$$

We substitute (21) into (30) to drive

$$s_t = \left(\frac{\rho}{1+\rho}\right) - \left(\frac{\rho+(1+\theta)b}{1+\rho}\right)h(1+n). \tag{31}$$

Differentiation of (31) with respect to τ leads to

$$\frac{ds_t}{d\tau} = -\frac{\rho+(1+\theta)b}{1+\rho}\frac{dh(1+n)}{d\tau}. \tag{32}$$

From (20), $dh(1+n)/d\tau = 0$. Therefore we have following relation.

$$\frac{ds_t}{d\tau} = 0. \ \square \tag{33}$$

If $\tau < \bar{\tau}$, there is no effect of the social security system on the fertility rate, the per capita income growth rate, and, the saving rate. This is because the existence of private inter-generational transfer cancels out the effect of the social security system. *Proposition* 1 would be the neutrality-theorem of social security system[10].

3.2 *No private intergenerational transfer*

From *lemma* 1 and *lemma* 2, the gift ratio, δ, is monotonically decreasing with the social security tax rate, τ. So there would exist a regime-switching tax rate, $\bar{\tau}$, such that no gift will be made when $\tau \geq \bar{\tau}$. This requires that the gift ratio, δ, be set to zero. In this case, from (2), (3) and (4), the budget constraints of individual will be replaced by

$$C_t^t = \{1-\tau-h(1+n_t)\}W_t - S_t, \tag{34}$$

$$C_{t+1}^t = (1+r_{t+1})S_t + (1+n_t)\tau W_{t+1}, \tag{35}$$

$$C_t^{t-1} = (1+r_t)S_{t-1} + (1+n_{t-1})\tau W_t. \tag{36}$$

Utility maximization yields the following first-order condition for an interior solution.

$$-U_1 + (1+r_{t+1})U_2 = 0, \tag{37}$$

$$-h'(1+n_t)U_1 + \tau(W_{t+1}/W_t)U_2 = 0. \tag{38}$$

Equation (37) and (38) correspond to the first order conditions for saving and the number

10) We can not find the neutrality-theorem of the conventioanl social security system. See the Appendix A. 2.

of children, respectively. Equation (37) states the marginal return of a unit of saving must equal the marginal cost in terms of losses in utility. Equation (38) states the interrelation between the marginal return of having a child and the marginal child rearing cost in terms of losses in utility.

To proceed the analysis, we use the functional form of utility function, (1), to get the following interrelation between consumptions.

$$\rho(1+r_{t+1})C_t^t = C_{t+1}^t, \tag{39}$$

$$\rho(W_{t+1}/W_t)\tau C_t^t = h'(1+n_t)C_{t+1}^t. \tag{40}$$

As in section 3.1, we focus on the BGP again. Fertility and growth rate(n_t, g_t) are all constant on the BGP. From (39) and (40), we have

$$(1+g)(1+n) = b(1+r)h(1+n)/\tau. \tag{41}$$

Equation (41) implies a positive or negative interrelation between the fertility rate, n, and the per capita income growth rate, g, since n and g would change with the social security tax rate, τ.

Using (34), (35), (39) and (41), we can write the first-period consumption on the BGP, $\tilde{C}_t^t$, as follows.

$$\tilde{C}_t^t = \frac{1}{1+\rho} W_t\{1-\tau-(1-b)h(1+n)\}. \tag{42}$$

Substituting (42) into the budget constraint (34), we find the saving function on the BGP, $\tilde{S}_t$.

$$\tilde{S}_t = \frac{\rho}{1+\rho}\{1-\tau-h(1+n)\}W_t - \frac{b}{1+\rho}h(1+n)W_t. \tag{43}$$

We substitute (41) and (43) into (10) to derive

$$h(1+n) = \frac{w\rho(1-\tau)\tau}{w(b+\rho)\tau + vb(1+r)(1+\rho)}. \tag{44}$$

From (44) and the functional form of $h(1 + n)$, we have

$$n = \left[\frac{w\rho(1-\tau)\tau}{aw(b+\rho)\tau + abv(1+r)(1+\rho)}\right]^{1/b} - 1. \tag{45}$$

n is not monotonic to τ, but instead a humped curve from (45). We analize this point in detail. Differentiation of (45) with respest to τ leads to

$$\frac{dn}{d\tau} = -\frac{1}{b}\left[\frac{w\rho(1-\tau)\tau}{aw(b+\rho)\tau + abv(1+r)(1+\rho)}\right]^{(1-b)/b}$$
$$\times\left[\frac{aw\rho\{w(b+\rho)\tau^2 + 2bv(1+r)(1+\rho)\tau - bv(1+r)(1+\rho)\}}{[aw(b+\rho)\tau + abv(1+r)(1+\rho)]^2}\right]. \tag{46}$$

From (46), we difine $\hat{\tau}$ as follows.

$$\hat{\tau} \equiv \frac{-2bv(1+r)(1+\rho)+\sqrt{4b^2v^2(1+r)^2(1+\rho)^2+4bwv(1+r)(1+\rho)(b+\rho)}}{2w(b+\rho)}. \tag{47}$$

The sign of $dn/d\tau$ depends on $\hat{\tau}$: whether τ is greater than $\hat{\tau}$ or not. The following lemma indicates this point.

Lemma 3 *If $\tau<\hat{\tau}$, a rise in τ increases the fertility rate, n, and if $\hat{\tau}<\tau$, a rise in τ decreases the fertulity rate, n.*

Proof. Let $\tau<\hat{\tau}$. From (46), $dn/d\tau>0$. Next let $\hat{\tau}<\tau$. From (46), $dn/d\tau<0$. □

To proceed our analysis, it is important to examine that whether $\hat{\tau}$ is greater than $\bar{\tau}$ or not. The following lemma indicates this point.

Lemma 4 *If $\theta<2v(1+r)(1+\rho)/w$, then $\bar{\tau}<\hat{\tau}$.*

Proof. Let $\bar{\tau}\geq\hat{\tau}$. We substitute (22) and (47) into $\bar{\tau}\geq\hat{\tau}$, and reduce the inequality, we find

$$\begin{aligned}&4b^2w^2(b+\rho)[w\theta(b+\rho)\{w\theta-2v(1+r)(1+\rho)\}\\&-wv(1+r)(1+\rho)(1-\theta)\{b(1+\theta)+\rho\}\\&-v^2(1+r)^2(1+\rho)^2\{b(1+2\theta)+\rho\}]-4bw^3\rho v(1+r)(1+\rho)(b+\rho)\{b(1+\theta)+\rho\}\\&\geq 0.\end{aligned} \tag{48}$$

If $w\theta-2v(1+r)(1+\rho)<0$, the inequality of (48) is a contradiction. □

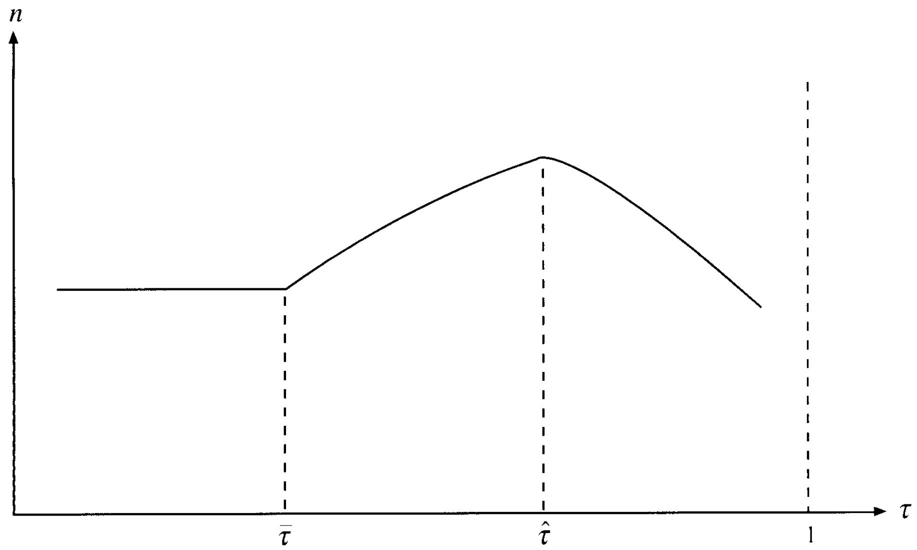

Figure 1 The interrelation between the fertility rate, *n*, and the change of the social security tax rate, *t*.

$\hat{\tau}$ is smaller than 1. From *Lemma* 3 and *Lemma* 4, the interrelation between the fertility rate, n, and the change of social security tax rate, τ, can be illustrated diagramartically in Figure 1. If $\tau < \bar{\tau}$, there is no interrelation between n and τ as in *Proposition* 1. If $\bar{\tau} < \tau < \hat{\tau}$, there is a positive interrelation between n and τ, and if $\hat{\tau} < \tau$, there is a negative interrelation.

The intuition behind this result is as follows: If $\bar{\tau} < \tau < \hat{\tau}$, there is no gift and public pension is the only transfer to the old. So the parents would try to produce more tax payer to get more transfer. In this regime, the transfer increasing effect would dominates the child rearing cost. If $\hat{\tau} < \tau$, the child rearing cost would dominates the transfer increasing effect. So the parents produce less tax payer.

Next we proceed to the analysis of the per capita income growth rate, g. Using (41), (44) and (45), we can write the per capita income growth rate, g, as follows.

$$g = a^{1/b} b(1+r)\tau^{-1}\left[\frac{w\rho\tau(1-\tau)}{w(b+\rho)\tau + vb(1+r)(1+\rho)}\right]^{(1-1/b)} - 1 \tag{49}$$

Differentiation of (49) with respect to τ leads to:

$$\frac{dg}{d\tau} = a^{1/b}(1+r)w\rho\tau^{-1}\left[\frac{w\rho\tau(1-\tau)}{aw(b+\rho)\tau + abv(1+r)(1+\rho)}\right]^{-1/b}$$
$$\times\frac{[w(b+\rho)\tau^2 - \{bw(b+\rho) + bv(b-2)(1+r)(1+\rho)\}\tau - bv(1+r)(1+\rho)]}{[w(b+\rho)\tau + bv(1+r)(1+\rho)]^2}. \tag{50}$$

We define $\tilde{\tau}$ as follows from (50).

$$\tilde{\tau} \equiv -\frac{bw(b+\rho) + bv(b-2)(1+r)(1+\rho)}{2w(b+\rho)}$$
$$+\frac{\sqrt{\{bw(b+\rho) + bv(b-2)(1+r)(1+\rho)\}^2 + 4bwv(1+r)(b+\rho)(1+\rho)}}{2w(b+\rho)} \tag{51}$$

The sign of $dg/d\tau$ depends on $\tilde{\tau}$: whether τ is greater than $\tilde{\tau}$ or not. The following lemma indicates this point.

Lemma 5 *If $\tau < \tilde{\tau}$, a rise in τ decrease the growth rate, g, and if $\tilde{\tau} < \tau$, a rise in τ increase the growth rate, g.*

Proof. Let $\tau < \tilde{\tau}$. From (50), $dg/d\tau < 0$. Next let $\tilde{\tau} < \tau$. From (50), $dg/d\tau > 0$. □

To proceed our analysis, it is important to examine that whether $\tilde{\tau}$ is smaller than 1 or not and whether $\tilde{\tau}$ is greater than $\hat{\tau}$ or not. The following two lemmas indicate these points.

Lemma 6 *If $b = 2$, then $\tilde{\tau} < 1$.*

Proof. Let $\tilde{\tau} \geq 1$. We substitute (51) into $\tilde{\tau} \geq 1$, and reduce the inequality, we find

$$\begin{aligned} & w^2(b-2)^2(b+\rho)^2 + b^3v^2(b-4)(1+r)^2(1+\rho)^2 \\ & +2bwv\{b(b-4)+2\}(1+r)(1+\rho)(b+\rho) \\ & \geq 0. \end{aligned} \tag{52}$$

If $b = 2$, the inequality of (52) is a contradiction. □

Lemma 7 *If $b \geq 2$, then $\bar{\tau} < \tilde{\tau}$.*

Proof. Let $\bar{\tau} \geq \tilde{\tau}$. We substitute (22) and (51) into $\bar{\tau} \geq \tilde{\tau}$, and reduce the inequality, we find

$$\begin{aligned} & 4b^2w^2(b+\rho)^2\{w\theta - v(1+r)(1+\rho)\}\{w\theta(1-b) - v(1+r)(1+\rho) - w(b+\rho)\} \\ & -4b^2w^2v(b-2)(1+r)(1+\rho)(b+\rho)\{b(1+\theta)+\rho\}\{w\theta - v(1+r)(1+\rho)\} \\ & -4bw^3v(1+r)(1+\rho)(b+\rho)\{b(1+\theta)+\rho\}^2 \\ & \geq 0. \end{aligned} \tag{53}$$

If $b \geq 2$, the inequality of (53) is a contradiction. □

From *Lemma* 5, *Lemma* 6, and *Lemma* 7, the interrelation between the per capita income growth rate, g, and the change of social security tax rate, τ, can be illustrated diagramartically in Figure 2.

If $\bar{\tau} < \tau < \tilde{\tau}$, there is a negative interrelation between g and τ. It would be indicated that the fertility effect of the social security system dominates the saving effect because the fertility

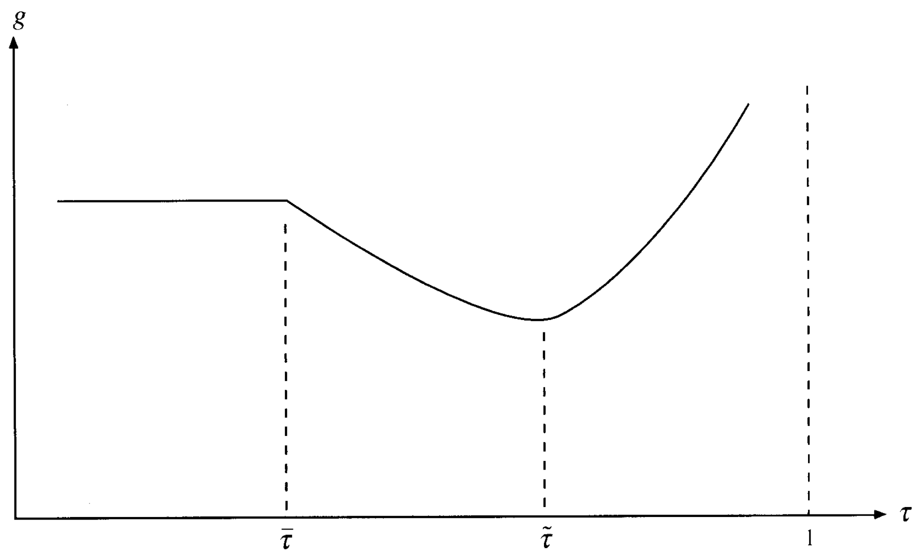

Figure 2 The interrelation between the per capita growth rate, g, and the change of the social security tax rate, τ

increases in this range of τ from Figure 1. If $\tilde{\tau} < \tau$, there is a positive interrelation between the per capita income growth rate and the social security tax rate. It would be indicated that the saving effect dominates the fertility effect.

Next we examine the effect of social security on the saving rate. From (43), we can write the saving rate, $s_t'(\equiv \tilde{S}_t / W_t)$, as follows.

$$s_t' = \frac{\rho}{1+\rho} - \frac{\rho}{1+\rho}\tau - \frac{b+\rho}{1+\rho}h(1+n) \tag{54}$$

Differentiation of (54) with respect to τ leads to

$$\frac{ds_t'}{d\tau} = -\frac{1}{1+\rho}\left\{\rho + (b+\rho)\frac{dh(1+n)}{d\tau}\right\}. \tag{55}$$

Differentiation of (44) with respect to τ leads to

$$\frac{dh(1+n)}{d\tau} = \frac{-\rho(b+\rho)w^2\tau^2 - 2bw\rho v(1+r)(1+\rho)\tau + bw\rho v(1+r)(1+\rho)}{\{w(b+\rho)\tau + bv(1+r)(1+\rho)\}^2}. \tag{56}$$

Substitution (56) into (55) leads to

$$\frac{ds_t'}{d\tau} = -\frac{b\rho v(1+r)\{bv(1+r)(1+\rho) + w(b+\rho)\}}{\{w(b+\rho)\tau + bv(1+r)(1+\rho)\}^2} < 0. \tag{57}$$

Therefore the saving rate is decreasing function of τ.

The interrelation between the saving rate, s, and the change of social security tax rate, τ, can be illustrated diagramatically in Figure 3 from *Proposition* 1 and (57).

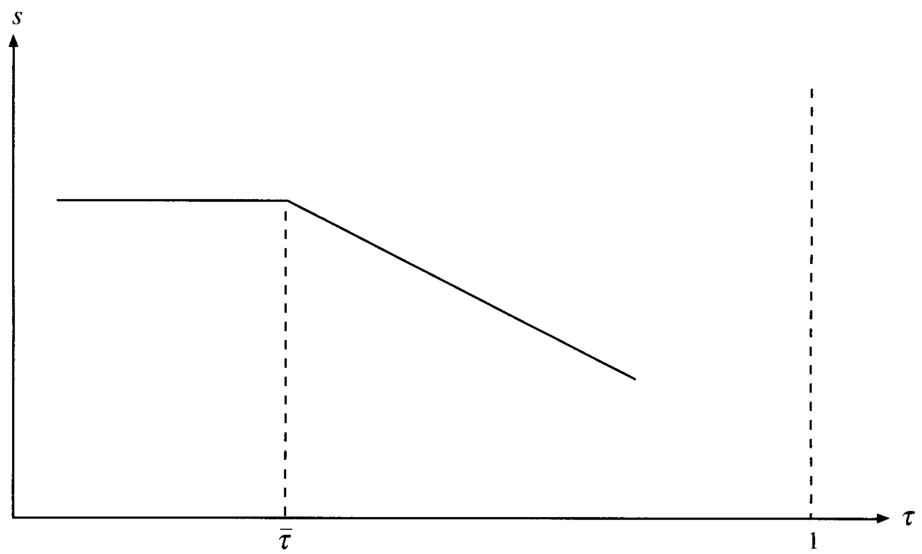

Figure 3 The interrelation between the saving rate, s, and the change of the social security tax rate, τ

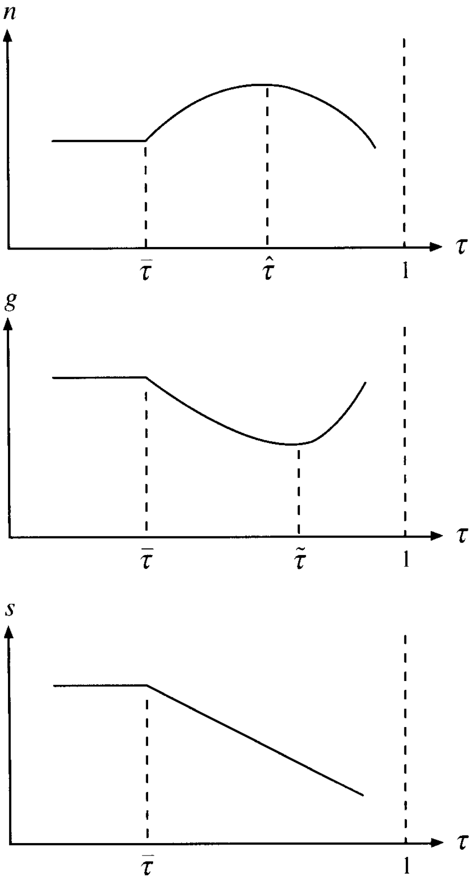

Figure 4 The interrelation between the fertility rate, n, the per capita growth rate, g, the saving rate, s, and the change of the social security tax rate, τ

The interelation between the fertility rate, n, the per capita growth rate, g, the saving rate, s, and, the change of the social security tax rate, τ can be illustrated diagramartically in Figure 4 from Figure 1, 2, and, 3.[11)]

If $\bar{\tau} < \tau < \hat{\tau}$, there is a negative interrelation between the fertility rate, n, and the per capita income growth rate, g. If $\hat{\tau} < \tau < \tilde{\tau}$, there is a positive interrelation between the fertility rate and the per capita income growth rate. This is a interesting case. The saving effect dominates the fertility effect. Higher social security tax rate reduces the saving and the per capita income growth rate. If $\tilde{\tau} < \tau < 1$, there a negative interrelation again. In this range, the fertility effect dominates the saving effect.

11) Comparing $\hat{\tau}$ ((43)) and $\tilde{\tau}$ ((47)), $\hat{\tau} < \tilde{\tau}$ obviously.

3.3 *The optimal solution of the social planner and the neutrality of social security system*

We examine the neutrality of social security system in the central planning economy. We would have the neutrality-theorem of social security system as in *Lemma* 8.

We suppose that a government would maximize the steady growing value of (1), $U(C_1, C_2, C_3)$, where $C_3 = C_2/(1+g)$. The planner chooses the optimal values of saving, fertility rate, and, gift rate plus social security tax rate.

The budget constrains are given by

$$C_1 = \{1-(\delta+\tau)-h(1+n)\}W_t - S_t, \tag{58}$$

$$C_2 = (1+r)S_t + (1+n)(\delta+\tau)W_{t+1}, \tag{59}$$

$$C_3 = (1+r)S_{t-1} + (1+n)(\delta+\tau)W_t. \tag{60}$$

The first-order conditions are given as follows.

$$S_t:\quad -U_1 + (1+r)U_2 = 0 \tag{61}$$

$$\delta+\tau:\quad -U_1 + (1+n)(1+g)U_2 + (1+n)U_3 = 0 \tag{62}$$

$$n:\quad -h'U_1 + (1+g)(\delta+\tau)U_2 + (\delta+\tau)U_3 = 0 \tag{63}$$

We use the functional form of utility function (1) and $C_3 = C_2/(1+g)$ to have

$$-C_2 + (1+r)\rho C_1 = 0, \tag{64}$$

$$-C_2 + (1+n)(1+g)(\rho+\theta)C_1 = 0, \tag{65}$$

$$-h'C_2 + (1+g)(\delta+\tau)(\rho+\theta)C_1 = 0. \tag{66}$$

From (64) and (65), we have

$$(1+g)(1+n) = \left(\frac{\rho}{\rho+\theta}\right)(1+r). \tag{67}$$

Using (58), (59), (64), and, (67), we can write the first-period consumption as follows.

$$C_1 = \frac{1}{1+\rho} W_t \left[\{1-(\delta+\tau)-h(1+n)\} + \left(\frac{\rho}{\rho+\theta}\right)(\delta+\tau)\right] \tag{68}$$

We use (64), (66), and, (67) to have

$$\delta+\tau = bh(1+n). \tag{69}$$

From (58) and (68), we have

$$S_t = \frac{\rho}{1+\rho}\{1-(\delta+\tau)-h(1+n)\}W_t - \frac{\rho(\delta+\tau)}{(1+\rho)(\rho+\theta)}W_t. \tag{70}$$

On the other hand, from (10) and (67), we have

$$S_t = \frac{\rho v(1+r)}{w(\rho+\theta)} W_t \tag{71}$$

Using (69), (70), and, (71), we can write the cost function of raising children as follows.

$$h(1+n)=\frac{w\rho(\rho+\theta)-\rho v(1+r)(1+\rho)}{w\rho\{(1+b)(\rho+\theta)+b\}}. \tag{72}$$

From (69) and (72), we have

$$\delta+\tau=\frac{b\{w\rho(\rho+\theta)-\rho v(1+r)(1+\rho)\}}{w\rho\{b+(1+b)(\rho+\theta)\}} \tag{73}$$

Lemma 8 *If $\rho > \theta$ and the government chooses δ^* as follows, social security system is neutral.*

$$\frac{bw\rho\{bw\rho+v(1+r)(1+\rho)(b\rho+\theta)+w(\rho+\theta)(\rho-\theta)\}}{w\rho\{b+(1+b)(\rho+\theta)\}\{b(1+\theta)+\rho\}}<\delta^*<\frac{b\{w\rho(\rho+\theta)-\rho v(1+r)(1+\rho)\}}{w\rho\{b+(1+b)(\rho+\theta)\}}$$

Proof. Define τ^* as follows from (73).

$$\tau^*\equiv\frac{b\{w\rho(\rho+\theta)-\rho v(1+r)(1+\rho)\}}{w\rho\{b+(1+b)(\rho+\theta)\}}-\delta$$

Let $\tau^*<\bar{\tau}$, we have a following inequality.

$$\frac{bw\rho\{bw\rho+v(1+r)(1+\rho)(b\rho+\theta)+w(\rho+\theta)(\rho-\theta)\}}{w\rho\{b+(1+b)(\rho+\theta)\}\{b(1+\theta)+\rho\}}<\delta \quad \square$$

4. Conclusion

We have analized the effects of the hypothetical fertility-related social security system by employing an overlapping generations endogenous growth model with the altruism of children for parents.

If the social security tax rate is low, there is operative private intergenerational transfer and there is no effect of the social security system on the fertility rate and the per capita income growth rate with operative private intergenerational transfer. This is because the existence of private intergenerational transfer cancels out the effect of the social security system. This would be the neutrality-theorem of social security system.

There is the regime-switching tax rate and if tax rate exceeds it, there would be no private intergenerational transfer. If the social security tax rate exceeds the regime-switching tax rate, there are negative and positive interrelations between the fertility rate and the per capita income growth rate with no private intergenerational transfer.

The interrelation between the fertility rate and the per capita income growth rate depends on the fertility effect and the saving effect of the social security system. If the social security tax rate exceeds the regime-switching tax rate and there is no private intergenerational transfer, the fertility effect is positive. This is because if there are more tax payers, there are more public pension though there is no gift from children. So the per capita income growth rate reduces. If the social security tax rate is too high, the fertility rate changes from positive to negative. The per capita income growth rate reduces because the fertility effect dominates the saving effect.

There would be some extensions of our model. Introducing another types of altruism is important problem. If we introduce several types of altruism, there may be another results[12]. There are another elements of extensions, for example, introducing the elements of human capital investment is important problem.

Appendix

A.1 *The second order conditions*

In this appendix, we examine the second order conditions of the maximum problem. The second order conditions for a maximum at an interior solutions are that the Hessian matrix of (11), (12), and (13) at the solution be negative definite. The Hessian matrix, H, is given as follows.

$$H = \begin{bmatrix} U_{11} + (1+r_{t+1})^2 U_{22} & W_t U_{11} & h' W_t U_{11} + (1+r_{t+1})(\delta_{t+1} + \tau) W_{t+1} U_{22} \\ U_{11} & W_t U_{11} + (1+n_{t-1})^2 W_t U_{33} & h' W_t U_{11} \\ h' U_{11} + \dfrac{(\delta_{t+1} + \tau) W_{t+1} (1+r_{t+1}) U_{22}}{W_t} & h' W_t U_{11} & h'^2 W_t U_{11} - 3h'' U_1 + \dfrac{(\delta_{t+1} + \tau)^2 W_{t+1}^2 U_{22}}{W_t} \end{bmatrix} \tag{A1}$$

We can show the sign of each principal minors from the Hessian matrix, $(A.1)$, as follows.

$$|H_1| = U_{11} + (1+r_{t+1})^2 U_{22} < 0 \tag{A2}$$

$$\begin{aligned} |H_2| &= \begin{vmatrix} U_{11} + (1+r_{t+1})^2 U_{22} & W_t U_{11} \\ U_{11} & W_t U_{11} + (1+n_{t-1})^2 W_t U_{33} \end{vmatrix} \\ &= (1+n_{t-1})^2 W_t U_{11} U_{33} + (1+r_{t+1})^2 W_t U_{11} U_{22} \\ &\quad + (1+r_{t+1})^2 (1+n_{t-1})^2 W_t U_{22} U_{33} > 0 \end{aligned} \tag{A3}$$

12) Zhang and Zhang (1998) examines several types of altruism.

$$|H_3| = |H| =$$

$$\begin{vmatrix} U_{11} + (1+r_{t+1})^2 U_{22} & W_t U_{11} & h' W_t U_{11} + (1+r_{t+1})(\delta_{t+1} + \tau) W_{t+1} U_{22} \\ U_{11} & W_t U_{11} + (1+n_{t-1})^2 W_t U_{33} & h' W_t U_{11} \\ h' U_{11} + \dfrac{(\delta_{t+1} + \tau) W_{t+1} (1+r_{t+1}) U_{22}}{W_t} & h' W_t U_{11} & h'^2 W_t U_{11} - 3h'' U_1 + \dfrac{(\delta_{t+1} + \tau)^2 W_{t+1}^2 U_{22}}{W_t} \end{vmatrix}$$

$$= \{h'(1+r_{t+1})W_t - (\delta_{t+1} + \tau) W_{t+1}\}^2 (1+n_{t-1})^2 U_{11} U_{22} U_{33}$$

$$-3h'' U_1 W_t \{(1+n_{t-1})^2 U_{11} U_{33} + (1+r_{t+1})^2 U_{11} U_{22} + (1+r_{t+1})^2 (1+n_{t-1})^2 U_{22} U_{33}\} \tag{A4}$$

From ($A4$), if $b \geq 1$, then $h'' \geq 0$ from the functional form of $h(1+n)$. So if $b \geq 1$, then $|H| < 0$. Therefore the second order conditions are satisfied.

A.2 *The conventional social security system*

In this appendix, we examine the effects of the conventional social security system on the fertility rate and the per capita income growth rate. A public pension, T_t, is given for the single individual in the conventional social security system.

The budget constrains are given by

$$C_t^t = \{1 - \delta_t - \tau - h(1+n_t)\} W_t - S_t, \tag{A5}$$

$$C_{t+1}^t = (1+r_{t+1}) S_t + (1+n_t) \delta_{t+1} W_{t+1} + T_{t+1}, \tag{A6}$$

$$C_t^{t-1} = (1+r_t) S_{t-1} + (1+n_{t-1}) \delta_t W_t + T_t. \tag{A7}$$

For an interior solution, utility maximization yields the following first-order conditions.

$$-U_1 + (1+r_{t+1}) U_2 = 0 \tag{A8}$$

$$-U_1 + (1+n_{t-1}) U_3 = 0 \tag{A9}$$

$$-h'(1+n_t) U_1 + \delta_{t+1} (W_{t+1} / W_t) U_2 = 0 \tag{A10}$$

We use the functional form of utility function, (1), to get the following relation between consumptions.

$$\rho(1+r_{t+1}) C_t^t = C_{t+1}^t \tag{A11}$$

$$\theta(1+n_{t-1}) C_t^t = C_t^{t-1} \tag{A12}$$

$$\rho\delta(W_{t+1} / W_t) C_t^t = h'(1+n_t) C_{t+1}^t \tag{A13}$$

From ($A11$) and ($A12$), we have

$$(1+n)(1+g) = (\rho / \theta)(1+r). \tag{A14}$$

Equation ($A14$) implies a negative interrelation between the fertility rate, n, and the per capita income growth rate, g, if gift are operative. This is because the interest rate, r, is constant.

Using ($A5$), ($A6$), ($A11$) and ($A14$), we can write the first-period consumption on the BGP, $\hat{C}_t^t$, as follows.

$$\hat{C}_t^t = \left(\frac{1}{1+\rho}\right) W_t [\{1-(\delta+\tau)-h(1+n)\}+\delta(\rho/\theta)]+(1+r)^{-1}(1+\rho)^{-1}T_{t+1} \tag{A15}$$

Substituting $(A15)$ into the budget constraint $(A5)$, we find the saving function on the BGP, $\hat{S}_t$.

$$\hat{S}_t = \left(\frac{\rho}{1+\rho}\right)\{1-(\delta+\tau)-h(1+n)\}W_t - \left(\frac{1}{1+\rho}\right)(\rho/\theta)\delta W_t - (1+r)^{-1}(1+\rho)^{-1}T_{t+1} \tag{A16}$$

From $(A11)$, $(A13)$ and $(A14)$, we have

$$\delta = b(\theta/\rho)h(1+n). \tag{A17}$$

We substitute $(A14)$, $(A16)$, and $(A17)$ into (10) to derive

$$h(1+n) = \frac{\rho\theta - \rho(1+\theta)\tau - w^{-1}\rho v(1+r)(1+\rho)}{\theta\{b(1+\theta)+\rho\}} \tag{A18}$$

It depends on the public pension system that whether individuals make gift to their parents or not. The following lemma provides a necessary and sufficient condition for gift being operative in the balanced growth equilibrium.

Lemma A1 *The gift ratio, δ, is strictly positive if and only if the social security tax rate satisfies:*

$$\tau < \bar{\tau}' \equiv \frac{\theta w - v(1+r)(1+\rho)}{w(1+\theta)}. \tag{A19}$$

Proof. From $(A17)$ and $(A18)$, we have

$$\delta = \frac{b\{\theta - (1+\theta)\tau - w^{-1}v(1+r)(1+\rho)\}}{b(1+\theta)+\rho}. \tag{A20}$$

Let $\delta > 0$. Then we have $\tau < \bar{\tau}'$. Next let $\delta \leq 0$. Then we have $\tau \geq \bar{\tau}'$. This is a contraposition. □

$\bar{\tau}'$ is smaller than 1. On the other hand, whether $\bar{\tau}'$ is positive or not depends on the parameters. A necessary and sufficient condition for $\bar{\tau}'$ to be possitive is that:

$$\theta > \frac{v(1+r)(1+\rho)}{w}. \tag{A21}$$

It is assumed that θ meets condition $(A21)$. Equation $(A21)$ means that the altruism of children for parents has to be strong enough to render the gift operative.

Equation $(A20)$ states the interrelation between the gift ratio, δ, and the social security tax rate, τ, and there is a negative interrelation. The following lemma indicates this point.

Lemma A2 *If $\tau < \bar{\tau}'$, a rise in τ reduce the gift ratio, δ.*

Proof. From the proof of *Lemma A1*, δ is given as in (A20). Differentiation of (A20) with respest to τ leads to

$$\frac{d\delta}{d\tau} = -\frac{b(1+\theta)}{b(1+\theta)+\rho} < 0. \square \tag{A22}$$

A rise in τ reduces the first period consumption and raises the second period consumption. Hence, marginal utility of the first period consumption increases and of the second period one decreases. So young individual intends to reduce the gift to parents.

Equation (A14) implies a negative interrelation between the fertility rate, n, and the per capita income growth rate, g. A decrease of the fertility rate is accompanied by an increase of the per capita growth rate. The following proposition indicates the effect of the change of τ on n, g, and, the saving rate, s_t.

Proposition A1 If $\tau < \bar{\tau}'$, *a rise in* τ *reduces the fertility rate, n, and increases the per capita income growth rate, g, and has no effect on the saving rate,* $s_t (\equiv \hat{S}_t / W_t)$.

Proof. Using the functional form of $h(1 + n)$, (A14) and (A18), we can write the fertility rate, n, and the per capita income growth rate, g, on the BGP as follows.

$$g = \frac{\rho(1+r)}{\theta}\left[\frac{w\rho\theta - w\rho(1+\theta)\tau - \rho v(1+r)(1+\rho)}{aw\theta\{b(1+\theta)+\rho\}}\right]^{-1/b} - 1 \tag{A23}$$

$$n = \left[\frac{w\rho\theta - w\rho(1+\theta)\tau - \rho v(1+r)(1+\rho)}{aw\theta\{b(1+\theta)+\rho\}}\right]^{1/b} - 1 \tag{A24}$$

Differentiation of (A23) and (A24) with respect to τ leads to

$$\frac{dg}{d\tau} = \frac{w\rho^2(1+r)(1+\theta)[aw\theta\{b(1+\theta)+\rho\}]^{1/b}}{b\theta\{w\rho\theta - w\rho(1+\theta)\tau - \rho v(1+r)(1+\rho)\}^{1/b+1}} > 0, \tag{A25}$$

$$\frac{dn}{d\tau} = -\frac{1}{b}\left[\frac{w\rho\theta - w\rho(1+\theta)\tau - \rho v(1+r)(1+\rho)}{aw\theta\{b(1+\theta)+\rho\}}\right]^{1/b-1}\frac{w\rho(1+\theta)}{aw\theta\{b(1+\theta)+\rho\}} < 0. \tag{A26}$$

From (A16) and the consistency condition, $T_{t+1} = (1+n_t)W_{t+1}\tau$, we can write the saving rate, $s_t (\equiv \hat{S}_t / W_t)$, on the BGP as follows.

$$s_t = \frac{\rho}{1+\rho}\{1-\delta-\tau-h(1+n)\} - \frac{\rho\delta}{\theta(1+\rho)} - \frac{(1+g)(1+n)}{(1+r)(1+\rho)}\tau \tag{A27}$$

We substitute (A14), (A17), and, (A18) into (A27) to drive

$$s_t = \frac{\rho}{1+\rho} - \frac{\rho(1+\theta)}{\theta(1+\rho)}\tau - \frac{w\rho\theta - w\rho(1+\theta)\tau - \rho v(1+r)(1+\rho)}{w\theta(1+\rho)}. \tag{A28}$$

Differentiation of (A28) with respect to τ leads to

$$\frac{ds_t}{d\tau} = 0. \ \square \tag{A29}$$

We examine the interrelation between n and τ in detail. There is a maximum tax rate, $\tau_{\max}$, such that the fertility rate would be zero. The following lemma indicates this point.

Lemma A3 *There is a maximum tax rate,* $\tau_{\max}(<\bar{\tau}')$, *as follows such that the fertility rate would be zero.*

$$\tau_{\max} = \frac{w\theta - v(1+r)(1+\rho) - aw\theta\{1+(1+\theta)/\rho\}}{w(1+\theta)}$$

Proof. We substitute n = 0 into (A24) to drive

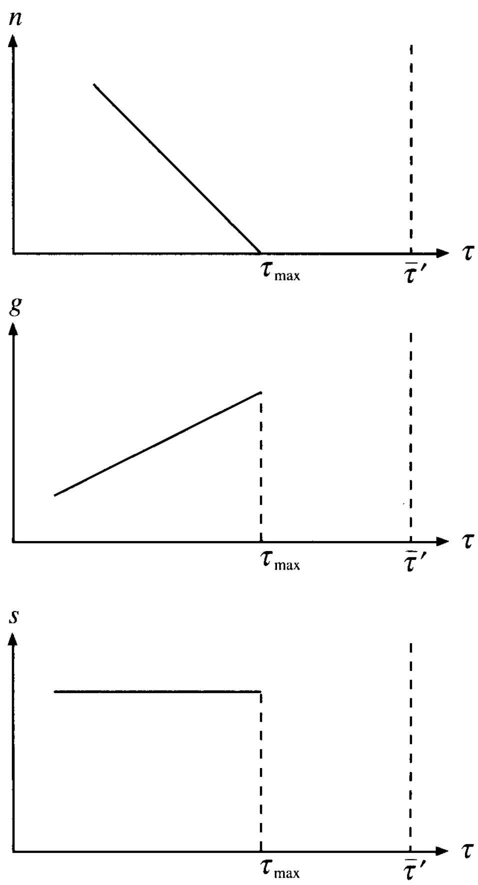

Figure 5 The interrelation between the fertility rate, *n*, the per capita growth rate, *g*, the saving rate, *s*, and, the change of the social security tax rate, *τ*

$$\tau = \frac{w\theta - v(1+r)(1+\theta) - aw\theta\{1+(1+\theta)/\rho)\}}{w(1+\theta)}. \quad \square$$

From *Lemma A*3, the interrelation between the fertility rate, n, the per capita growth rate, g, the saving rate, s, and, the change of the social security tax rate, τ, can be illustrated diagramartically in Figure 5.

If $\tau < \tau_{\max}$, there is a negative interrelation between the fertility rate, n, and the per capita income growth rate, g. There is no neutrality of social security system as in the case of the fertility-related social security system.

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