Basics of an Endogenous Growth Model: the Optimum CRC Situation and Conditional Convergence

Hideyuki Kamiryo

(Received on October 1, 2003)

1. Introduction

This paper intends to develop the characteristics in the endogenous growth model of Kamiryo [2002] that introduced "*beta*" and "*delta*," where "*beta*" is an integrated/weighted value of the financial parameters, θ_1, θ_2, and γ in Kamiryo [2003] but "*delta*" is a decision-making parameter instead of a control parameter in Kamiryo [2003]. This paper and Kamiryo [2002] both treat *delta* as a decision-making parameter. Nevertheless, an important idea in this paper comes from Kamiryo [2003] that started in 1995. The important idea is that the function of financial and corporate institutions divides saving into corporate and household saving and also investment into investment in quality and investment in quantity.

My intuition is that if my model could shift *delta* from a control parameter to a decision-making parameter, we may specify the relationship between parameters under CRC. Kamiryo [2002] could not realize this intuition and this paper intends to finalize this intuition. In other words, CRC can be expressed as a CRC* under an optimum situation to maximize the rate of profit. And, transitional paths from the current situation, DRC, to CRC* can be clarified if the relationship between parameters is clarified under CRC*. Thus, I was stimulated so as to clarify the relationship between *beta** and *delta** (each as a decision-making parameter) under CRC*.

This paper, after formulating final equations in my endogenous growth model, first completes a set of specific equations held under CRC* and second clarifies the relationships between "*beta* and *delta*" under DRC and also between "*beta** and *delta**" under CRC*. The results will show the characteristics of my endogenous growth model. I can use the set of specific equations in the future to connect these equations with such issues as the Penrose effect [Uzawa, 1969], the role of monetary policy [Friedman, 1968], and the valuation ratio [Gorden, 1994].

Let me briefly describe the outline of my model in this paper. My models [2002, 2003] formulate an endogenous growth model by using the Cobb-Douglas production function, dividing total investment into investment in quality and quantity, and introducing *beta* (integrating the three financial parameters) for resource redistribution, structural reform, and deregulation, and *delta* for the improvement in qualitative investment over time (accelerated by R&D and education costs). These parameters reflect the results of the function of financial and corporate institutions/sectors and make it possible to change the Solow model to an endogenous growth model. Both the current and CRC* situations are expressed by using these two decision-making parameters, *beta* and *delta*.

How can my model express transitional paths from the current DRC to the optimum CRC* situation? My models attain CRC* without an assumption of DRC and/or CRC as in the literature. The current DRC situation is determined by *beta* and *delta*, yet the convergence to CRC* over time depends on the work of *delta*. When *delta* is a decision-making parameter, I can clarify the path from DRC to CRC* since *delta*' productivity enhancement over time only neutralizes DRC to CRC*. Note that under constant returns to scale I must manage to find two kinds of DRC, weak and conventional DRC, for the current situation.

In this respect, I indicate here a defect in Kamiryo [2002], which I use as a base of my model in this paper. Kamiryo [2002] calibrates, using recursive programming, *beta* and *delta* independently under DRC and also calibrates *beta** under CRC, by applying the root mean square error (RMSE) method as a method of ordinary least squares (OLS) and using the actual/initial growth rate of per capita output, the initial rate of profit, and the initial rate of technological progress. However, a problem remains unsolved in recursive programming: a specific *delta** under the optimum CRC* situation cannot be calibrated since, for CRC, I have to set *delta* is equal to the relative share of profit (*delta* = *alpha*) in recursive programming. And thus, Kamiryo [2002] cannot specify a *delta** that accurately corresponds with a calibrated *beta**. My intuition is: if I could formulate the relationship between parameters under CRC* by using both *beta** and *delta**, I can calculate *delta** using a calibrated *beta**.

There are numerous CRC situations and I denote the optimum CRC* situation as a situation where the initial capital-output ratio, $\Omega(0)$, becomes horizontal over time ($\Omega^* = \Omega(0)$) under a certain value of *beta**. In this case, along with the decrease in the capital-output ratio starting with a high capital-output ratio, the rate of profit increases and at the point of $\Omega^* = \Omega(0)$ profit is maximized. It is true that if $\Omega^* < \Omega(0)$ the rate of profit will further increase but *beta** and *delta** must further improve by breaking the framework of the initial data. I distinguish CRC under $\Omega^* < \Omega(0)$ with CRC* under $\Omega^* = \Omega(0)$ and I lead this paper from CRC to CRC*.

2. Parameters and equations in the generalized form

This section modifies and corrects the equations formulated in my model [2002] that first uses *beta*. I call these final equations as the basic equations. I pay attention to (1) the discrete time equations that are consistent with the Cobb-Douglas production function and (2) the generalized form commonly used under DRC and CRC.

Before starting, I stress that the level of technology in the Cobb-Douglas production function, $A(t)$, is endogenously obtained under two conditions that (1) the initial level of technology, $A(0)$, is implicitly expressed using the capital-labour ratio, $k(0)$, and the relative share of profit, *alpha*, and the initial capital-output ratio, $\Omega(0)$,[1] and (2) the rate of technological progress is measured endogenously. My model satisfies these two conditions and as a result, I can generally formulate the basic equations, using $Y(t) = A(t) \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha}$.

2.1 *Basic equations from quantity to quality*

1. Profit: $\Pi(t) = \alpha \cdot Y(t)$, where α is the relative share of profit. (1)
2. Compensation: $W(t) = (1-\alpha)Y(t)$. (2)
3. Corporate saving: $S_{\Pi}(t) = s_{\Pi} \cdot \Pi(t)$, where s_{Π} is the retention ratio, and the ratio of corporate saving to output: $s_{S\Pi/Y} = S_{\Pi}(t) / Y(t) = \alpha \cdot s_{\Pi}$. (3)
4. Dividends: $D(t) = (1 - s_{\Pi})\Pi(t)$. (4)
5. Household saving: $S_H(t) = s_H(1 - s_{\Pi} \cdot \alpha)Y(t)$, where s_H is the ratio of household saving to the sum of compensation and dividends (or the rate of household saving), and also the ratio of household saving to output: $s_{SH/Y} = S_H(t) / Y(t) = s_H \cdot (1 - \alpha \cdot s_{\Pi})$. (5)
6. Saving: $S(t) = S_{\Pi}(t) + S_H(t)$, and the rate of saving to output: $s \equiv S / Y = s_{SH/Y} + s_{S\Pi/Y}$ (6)
7. For investment I use net (after depreciation/capital consumption) investment.[2] Three reasons are (1) saving in statistics is separated from depreciation, (2) gross-investment growth is limited to positive values, and (3) I use the function of financial and corporate sectors for my new parameters, resulting in investment (which is equal to saving), by adjusting, in a open economy, the increase in inventories, the balance of payment, and capital transfers. Net

1) $A(t) = Y(t) / K(t)^{\alpha} \cdot L(t)^{1-\alpha} = k(t)^{1-\alpha} / (K(t) / Y(t)) = k(t)^{1-\alpha} / (k(t) / y(t)) = k(t)^{1-\alpha} / \Omega(t)$

2) In more detail, gross investment can be used for both saving and investment as in Kamiryo [2003], where I can distinguish the increase in capital, ΔK, from gross investment, $I = \Delta K +$ depreciation.

investment (hereafter investment), I, and the rate of investment to output, i, are shown using saving:

$I(t)=\theta_1 \cdot S_H(t)+S_\Pi(t)$ and $i=\theta_1 \cdot s_{SH/Y}+s_{S\Pi/Y}$,[3)]

where $I(t)=i\cdot Y(t)$.

8. The per capita rate of investment: $I(t)/L(t)=i\cdot y(t)=(\theta_1 \cdot s_{SH/Y}+s_{S\Pi/Y})y(t)$,

   where if $\theta_l = 1$ (with no banking costs), $i = s = S(t)/Y(t)$, and $y(t) = A(t)k(t)^{\alpha}$. (7)

   The concept of the per capita rate of investment, $i\cdot y(t)$, is essentially required for qualitative investment (since qualitative investment is done in corporation with capital and effective labour: see below soon).

9. Quantitative investment: $I_K(t)=beta\cdot I(t)=\gamma\cdot\theta_1\cdot S_H(t)+\theta_2\cdot S_\Pi(t)$ and

$$\Delta K(t)=I_K(t)=i_K\cdot Y(t)=i_K\cdot A(t)\cdot K(t)^{\alpha}\cdot L(t)^{1-\alpha}, \tag{8}$$

   where $i_K \equiv \beta\cdot i=\gamma\cdot\theta_1\cdot s_{SH/Y}+\theta_2\cdot s_{S\Pi/Y}$ and $\beta=\dfrac{i_K\cdot y(t)}{i\cdot y(t)}=\dfrac{i_K}{i}$.[4)]

10. Per capita quantitative investment:

    Per capita capital investment, $\Delta k(t)$, is obtained using $k(t) \equiv K(t)/L(t)$:

$$k(t+1)=\frac{K(t+1)}{L(t+1)}=\frac{K(t)+\Delta K(t)}{(1+n)\cdot L(t)}=\frac{K(t)+i_K\cdot Y(t)}{(1+n)\cdot L(t)}=\frac{k(t)+i\cdot y(t)}{1+n}.$$

Thusly, $$\Delta k(t)=\frac{i_K\cdot y(t)-n\cdot k(t)}{1+n}. \tag{9}$$

In recursive programming, I first calculate $k(t+1)=(k(t)+i_K\cdot y(t))/(1+n)$ using $y(t)$ obtained in the previous time.

11. Per capita qualitative investment:

    In discrete time, $A(t+1)\equiv A(t)+\Delta A(t))$[5)] and,

    $\Delta A(t)=i_A\cdot Y(t)=i_A\cdot A(t)\cdot K(t)^{\alpha}\cdot L(t)^{1-\alpha}$.

---

3) If the rate of investment, i, is set as the sum of corporate and government investment, $i_{F/Y}$ and $i_{G/Y}$, to output, *beta* is replaced by $\beta=\theta_F\dfrac{i_{F/Y}}{i}+\theta_G\dfrac{i_{G/Y}}{i}$, instead of $\beta=\theta_2\dfrac{s_{S\Pi/Y}}{i}+\gamma\dfrac{\theta_1\cdot s_{SH/Y}}{i}$. In this case, government ineffectiveness is directly expressed by the parameter, θ_G. For this device, I need data of total investment in the corporate sector, which I will discuss in the future as a two-sector model using assumptions less than Uzawa's [1961, 1963] assumptions.

4) I am thankful to Dr. Hiroshi Noma for his suggestion to connect θ_l, θ_2, and γ directly with *beta*. $\beta=\theta_2\left(\dfrac{s_{S\Pi/Y}}{s}\right)+\gamma\left(\dfrac{s_{SH/Y}}{s}\right)$ or $\beta=\theta_2\dfrac{s_{S\Pi/Y}}{i}+\gamma\dfrac{\theta_1\cdot s_{SH/Y}}{i}$. Note that this equation is independent of *delta*, under DRC.

5) Eq. 10 does not divide $A(t)$ by $(1 + n)$ to avoid a double count since Eq. 9 was already divided by $(1 + n)$. Also, note that $A(t)$ is calculated after calculating $k(t)$.

This equation holds only when qualitative investment does not improve over time, where another parameter *delta* is not introduced yet.

The value of *delta* is defined as the elasticity of the improvement in qualitative investment over time with respect to the capital-labour ratio.

12. Per capita qualitative investment introducing *delta* into my model:

$\Delta A(t) = i_A \cdot y(t) = i_A \cdot A(t) \cdot k(t)^{\alpha}$, where *delta* = 0.

$\Delta A(t) = \dfrac{i_A \cdot y(t)}{k(t)^{\delta}} = A(t) \cdot i_A \cdot k(t)^{\alpha-\delta}$, where *delta* $\neq 0$ and

if *delta* = 0, $i_A \,/\, k(t)^{\delta}$ reduces to i_A (which depends on *beta*). (10)

$i_A \equiv (1-\beta)i = (1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2)s_{ST/Y}$ and $1-\beta = \dfrac{i_A \cdot y(t)}{i \cdot y(t)} = \dfrac{i_A}{i}$.

For the introduction of *delta*, it is essential for us to use per capita qualitative investment. Another parameter, *delta*, improves qualitative investment over time, where it is essential for capital and labour to cooperate each other: This implies that qualitative investment must be expressed by per capita qualitative investment. Note that in the case of quantitative investment, it is not necessary for us to use per capita quantitative investment. It is just convenient for quantitative investment to follow the case of per capita qualitative investment: $i \cdot y(t) = i_A \cdot y(t) + i_K \cdot y(t)$.

I finalize this section by indicating the work of *delta*. When *delta* is not introduced into my model, the convergence to constant returns to capital (CRC) does not occur. Or, using *beta* only, my model does not converge. The CRC situation only holds under *alpha* = *delta*, where $k(t)^{\alpha-\delta} = 1$ and $\Delta A(t) = A(t) \cdot i_A$. The work of *delta* makes it possible for an open economy, where *S* = *I*, to converge to CRC from DRC.

2.2 *Growth rates in the generalized form*

In this section, I finalize the rate of technological progress, the growth rates of per capita capital and capital, and the growth rate of per capita output under any situation. The relationships between *beta* and *delta* will distinguish DRC and CRC situations. Also, in these equations, I do not specify the optimum CRC situation, which will be discussed in Section 3.

Before starting, I will explain the method of recursive programming where I calibrate *beta* and *delta* using the root mean square error (RMSE) method. In recursive programming, a horizontal capital-output ratio under CRC is confirmed by setting the number of repeating times to 300,[6] but for repeating times, I usually use at least t = 1000. Recursive programming calibrates (1) both *beta* and *delta* under DRC and (2) *beta* under CRC (since *delta* under CRC is calculated using Eqs. 19 and 20 in Section 3).[7] The value of *beta* is calibrated by assuming that the actual growth rate of output equals a model growth rate of output and that the actual rate of return equals a model rate of return. The value of *delta* is separately calibrated by assuming that the actual growth rate of per capita output equals a model growth rate of per capita output and the actual rate of technological progress, $g_{A(actual)}$, equals a model rate of technological progress, where $g_{A(actual)} = g_{y(actual)} - \alpha \cdot g_{k(actual)}$.

The growth rates commonly used for DRC and CRC situations are summarized as follows:

(1) The rate of technological progress, $g_A(t)$:

Starting with Eq. 10,

$$g_A(t) \equiv \frac{A(t+1)-A(t)}{A(t)} = \frac{i_A \cdot y(t)}{A(t) \cdot k(t)^{\delta}} = i_A \cdot k(t)^{\alpha-\delta}, \tag{11}$$

or, using a calibrated *beta*, $g_A(t) = (1-\beta) \cdot i \cdot k(t)^{\alpha-\delta}$, where $i \equiv s_{S\Pi/Y} + \theta_1 \cdot s_{SH/Y}$ and i_A = $(1-\beta)i = (1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2) \cdot s_{S\Pi/Y}$. Eq. 11 always holds under any situation (DRC and CRC). Note that if a *beta* (or a Ω) changes, $g_A(t)$ also changes.

Under CRC, where *alpha* = *delta*, $g_A(t) = g_A^* = i_A$.

(2) The growth rates of per capita capital and capital, $g_k(t)$ and $g_K(t)$:

Since $\Delta k(t) = \dfrac{i_K \cdot y(t) - n \cdot k(t)}{1+n}$ (see Eq. 9),

$$g_k(t) = \frac{i_K \cdot y(t) - n \cdot k(t)}{(1+n)k(t)} = \frac{1}{1+n}\left(i_K \cdot A(t) \cdot k(t)^{\alpha-1} - n\right). \tag{12}$$

Now let me confirm the consistency between the growth rates of per capita capital, and capital, $g_k(t)$ and $g_K(t)$:

Using Eq. 8, $\Delta K(t) = i_K \cdot Y(t) = i_K \cdot A(t) \cdot K(t)^{\alpha} \cdot L(t)^{1-\alpha}$,

$$g_K(t) \equiv \frac{\Delta K(t)}{K(t)} = i_K \cdot A(t) \cdot K(t)^{\alpha-1} \cdot L(t)^{1-\alpha} = i_K \cdot A(t) \cdot k(t)^{\alpha-1}.\text{[8]} \tag{13}$$

Now let me confirm the relationship between $g_K(t)$ and $g_k(t)$:

6) The capital-output ratio under CRC usually converges to a constant value within t = 300, but in extreme cases it converges at t = 2000 to t = 4000.

7) For *beta*, $RMSE_{MIN} \equiv \sqrt{((g_{Y(model)}(300) - g_{Y(actual)})/g_{Y(actual)})^2 + ((r_{(model)}(300) - r_{(actual)})/r_{(actual)})^2}$.

For *delta*, $RMSE_{MIN} \equiv \sqrt{((g_{Y(model)}(300) - g_{Y(actual)})/g_{Y(actual)})^2 + ((g_{A(model)}(300) - g_{A(actual)})/g_{A(actual)})^2}$.

8) The relationship between $g_k(t)$ and $g_K(t)$ is that $g_K(t) = g_k(t)(1+n) + n$ in discrete time as shown: ↗

$1+g_K(t)=(1+g_k(t))(1+n)$ and $g_K(t)=g_k(t)+n+g_k(t)\cdot n$ holds in discrete time.

Using $g_K(t)=i_K\cdot A(t)\cdot k(t)^{\alpha-1}$,

$g_k(t)=\frac{1}{1+n}\left(i_K\cdot A(t)\cdot k(t)^{\alpha-1}-n\right)$ is obtained as shown in Eq. 12.

Eq. 12 reduces to Eq. 13 when n = 0. Note that for recursive programming we cannot initially use $g_K(t)$: only after calculating $\Delta k(t)$ in the previous period.

(3) The growth rate of output:

The values of the growth rate of per capita output over time are shown in recursive programming. However, the following equation can be used for convenience:

$g_y(t)=g_A(t)+\alpha\cdot g_k(t)$ under any situation using $y(t)=A(t)k(t)^{\alpha}$, where $y=y(A(\alpha,\delta),k(\alpha))$ (see Eq.11 and 12).

The related equations under CRC are separately discussed in the next section.

3. Specific equations under the optimum CRC situation

3.1 The CRC situation versus the optimum CRC situation*

The CRC situation, where the capital-output ratio becomes horizontal over time, is divided into (1) numerous CRC situations and (2) the optimum CRC* situation. The relationship between parameters is only specified under the optimum CRC* situation. What is the difference between CRC and CRC*? I define CRC as such condition that the capital-output ratio higher than the initial/current capital-output ratio becomes horizontal over time. I define CRC* as such condition that the capital-output ratio equal to the initial/current capital-output ratio becomes horizontal over time, resulting in the highest rate of profit.

Numerous CRC situations are realized under the condition that the relative share of profit, *alpha*, is equal to *delta*: *alpha* = *delta*. These situations cannot take into consideration the relationship between *beta**, *beta*, *delta**, and *delta* since $\Omega(0)\neq\Omega^*$. Note that under CRC*, the hyperbolic curve of *delta** to *beta** exists as I will discuss in the next section.

The current situation is under either increasing or diminishing returns to capital (IRC or

$$g_k(t)=\frac{\frac{K(t+1)}{L(t+1)}-\frac{K(t)}{L(t)}}{\frac{K(t)}{L(t)}}=\frac{\frac{K(t+1)}{(1+n)L(t)}-\frac{K(t)}{L(t)}}{\frac{K(t)}{L(t)}}=\frac{K(t+1)-(1+n)K(t)}{K(t)}=\frac{1}{1+n}\left(g_K(t)-n\right).$$

Note that the difference, $n\cdot g_k(t)$, between $g_K(t)=g_k(t)(1+n)+n$ and $g_K(t)=g_k(t)+n$ is very trivial.

DRC), where *alpha* $\neq$ *delta*. My model approves the existence of DRC as in the literature, where CRC is realized by offsetting/neutralizing DRC through total factor productivity enhancement. However, for IRC I have a serious question[9]: is IRC expressed under constant returns to scale (under the Cobb-Douglas production function)? For this question, my intuition is that (1) if I mildly define IRC as such condition that the rate of profit increases over time, this mild IRC is expressed in my model and (2) if I strictly define IRC as such condition that both *beta** and *delta** must satisfy each necessary condition, this strict IRC is not expressed in my model. My model under CRC satisfies the first condition, where CRC is realized by offsetting mild IRC by decreasing total factor productivity over time. My model under CRC, however, does not satisfy the second condition of IRC, where both *beta** and *delta** lies within a region surrounded by the hyperbolic curve and the vertical line of *beta** under *delta** = 0, $\beta^*_{\delta=0}$(see Eq. 22). In short, a mild IRC exists under CRC. A strict IRC exists under CRC*, but it is difficult to empirically identify this IRC even under CRC* since the model is under CRS. I indicate that IRC in the literature is limited to the strict IRC under CRC*. I will discuss the strict IRC in the next section.

I will now discuss DRC and the mild IRC using the relationship between *alpha* and *delta* under CRC. Note that there are numerous *delta* each corresponding with *beta* under CRC, but we cannot determine *delta* using a certain calibrated *beta* under CRC.

Under CRC, a horizontal capital-output ratio over time will be confirmed by either $g_y(t) = g_k(t)$ or $(1-\alpha)g_k(t) = g_A(t)$. However, $g_y(t)$ and $g_k(t)$ cannot be measured under the DRC situation. Thus, I have to use $g_A(t) = i_A \cdot k(t)^{\alpha-\delta}$ for expressing the difference between DRC and CRC. If $k(t)\alpha - \delta = 1$, $g_A(t)$ reduces to i_A under CRC: $g_A(t) = g_{A(CRC)}$. A constant value of i_A, however, changes according to the change in *beta*.

1. If *alpha* > *delta* (or if $m_{(CRC)}$ = *alpha* − *delta* > 0), $k(t)^{\alpha-\delta} > 1$ and the rate of technological progress increases over time, where $g_A(t) > g_{A(CRC)}$. Total factor productivity much improves and leads to a mild IRC.[10] It is relatively easier to neutralize the mild IRC and

9) Dr. Bryce Hool, head of the Department of economics and Dr. Debasis Bandyopadhyay, two supervisors at the University of Auckland, stressed the balance between DRC and productivity enhancement in Kamiryo [2003], where I used *alpha* and the critical *alpha*: (1) If *alpha* < the critical *alpha*, the situation is DRC, if *alpha* = the critical *alpha*, the situation is CRC, and if *alpha* > the critical *alpha*, the situation is IRC, using *delta* as a control parameter. The critical *alpha* shows the limit of recursive programming.

10) The reason why I temporarily use "mild IRC" is that this classification does not take into consideration the relationship between *beta**, *beta*, *delta**, and *delta* under CRC* (see Eq. 25 as a final solution).

attain CRC. This situation, if it is set under CRC*, corresponds with weak DRC.

2. If *alpha* = *delta* (or if $m_{(CRC)}$ = *alpha* – *delta* = 0), $k(t)^{\alpha-\delta}=1$ and $g_A(t)=g_{A(CRC)}$. This situation is, at the beginning of the current situation, already under CRC, where $i_A\ /\ k(t)^{\alpha-\delta}$ remains unchanged over time.
3. If *alpha* < *delta* (or if $m_{(CRC)}$ = *alpha* – *delta* < 0), $k(t)^{\alpha-\delta}<1$ and the rate of technological progress decreases over time, where $g_A(t)<g_{A(CRC)}$. This is DRC in the literature: negative total factor productivity aggravates DRC. We need a strong impact on *delta** to neutralize DRC. (15)

In short, the current situation shifts to CRC by neutralizing a mild IRC or DRC only by adjusting *delta*. After formulating the specific equations under CRC* below, I will finalize the relationship between IRC, CRC*, and DRC in 4.1.

*3.2 Specific equations under CRC**

This section, starting with the growth rates arranged in Section 2, expresses various essential equations under the optimum CRC situation (CRC*). I call these equations as "a set of specific equations" under CRC*. I stress here that no literature has clarified the relationship among parameters under the optimum CRC situation.

(1) The rate of technological progress under CRC*, g_A^*:

$g_A(t)=i_A\cdot k(t)^{\alpha-\delta}=i_A^*$, using a sufficient condition, *alpha* = *delta*. (16)

From Eq. 11 $i_A^*=(1-\beta^*)\cdot i$ and $i\equiv s_{STI/Y}+\theta_1\cdot s_{SH/Y}$ (if $\theta_1=1,\ i=s$), and *beta** is specified by the capital-output ratio, Ω^*, that equals the initial capital-output ratio.

(2) The growth rate of per capita output or capital under CRC*, $g_y^*=g_k^*$:

Starting with $g_y(t)=g_A(t)+\alpha\cdot g_k(t)$ (see Eq. 14) and using $g_A^*=i_A^*$ and $g_y^*=g_k^*$,

$$g_y^*=g_k^*=\frac{i_A^*}{1-\alpha}. \tag{17}$$

(3) The capital-output ratio under CRC*, Ω^*:

The capital-output ratio, Ω^*, is now expressed by parameters and constitutes a key equation under CRC*.

First, set "Eq. 17 equals Eq. 12," where replace (1) $A(t)\cdot k(t)^{\alpha-1}$ by $1/\Omega^*$,[11] and (2) i_A^* by $i_A^*\ /\ k(0)^{\delta*}$ since a calibrated *beta** has *delta** ≠ 0:

$$g_k^*=\frac{1}{1+n}\left(\frac{i_K^*}{\Omega^*}-n\right)\text{ equals }g_k^*=\frac{i_A^*}{(1-\alpha)k(0)^{\delta*}},$$

11) $A(t)\cdot k(t)^{\alpha-1}=k(t)^{1-\alpha}\cdot k(t)^{\alpha-1}\ /\ \Omega^*(t)=1\ /\ \Omega^*(t)$.

Thus, $$\Omega^* = \frac{i_K^*}{\dfrac{i_A^*(1+n)}{(1-\alpha)k(0)^{\delta *}}+n} = \frac{\beta^* \cdot i(1-\alpha)k(0)^{\delta *}}{i\cdot(1-\beta^*)(1+n)+n(1-\alpha)k(0)^{\delta *}}. \tag{18}$$

Eq. 18 clarifies an essence of growth structure under both *delta*$^* \neq 0$ and *delta*$^* = 0$. In particular, if *beta** changes with other parameters the capital-output ratio changes with a different vertical asymptote (also see Eq. 23 under *delta*$^* = 0$).

(4) The value of *delta**:

From Eq. 18, $$k(0)^{\delta *} = \frac{\Omega^* \cdot i\ (1-\beta^*)(1+n)}{(1-\alpha)(\beta^* \cdot i - \Omega^* \cdot n)}. \tag{19}$$

Set $m = k(0)^{\delta *}$,[12] and solve for $\delta^* = LN(m)/LN(k(0))$. (20)

For confirmation, $$\beta^*_{\delta\neq 0} = \frac{\Omega^*\left(n(1-\alpha)k(0)^{\delta *} + i\ (1+n)\right)}{i(1-\alpha)k(0)^{\delta *} + \Omega^* \cdot i(1+n)}. \tag{21}$$

In Eq. 19, *delta** is obtained only when both *beta** and Ω^* are fixed. Also, I can change $\beta^*_{\delta\neq 0}$ without changing $\Omega^*=\Omega(0)$. In this case, the values of *delta** are obtained along with the changes in $\beta^*_{\delta\neq 0}: \delta^*(\beta^*_{\delta *\neq 0})$. I call $\delta^*(\beta^*_{\delta *\neq 0})$ "the hyperbolic curve of *delta** to *beta**" or "the set of *beta** and *delta**" under CRC*. Eq. 21 expresses that *beta** and *delta** are not independent under CRC*.

(5) The value of *beta** with *delta*$^* = 0$: $\beta^*_{\delta=0}$

By setting *delta*$^* = 0$ in Eq. 21, $$\beta^*_{\delta=0} = \frac{\Omega^*\left(n(1-\alpha) + i\ (1+n)\right)}{i(1-\alpha) + \Omega^* \cdot i(1+n)}. \tag{22}$$

Or, using Eq. 18 or 22,

$$\Omega^* = \frac{\beta^*_{\delta=0} \cdot i\ (1-\alpha)}{i\ (1-\beta^*_{\delta=0})(1+n) + n(1-\alpha)} \tag{23}$$

And, finally, $$n = \frac{\beta^* \cdot i(1-\alpha) - \Omega^* \cdot i(1-\beta^*)}{\Omega^*\left(i(1-\beta^*) + (1-\alpha)\right)} \tag{24}$$

Usually, $\beta^*_{\delta\neq 0}$ is obtained by calibration and $\beta^*_{\delta=0}$ is measured using Eq. 22. The difference between the $\beta^*_{\delta\neq 0}$ under *delta*$^* \neq 0$ and the $\beta^*_{\delta=0}$ under *delta*$^* = 0$ is negligible. This implies that $\beta^*_{\delta=0}$ can be a substitute for $\beta^*_{\delta\neq 0}$. Furthermore, I set $\beta^*_{\delta=0}$ "the origin" of CRC*.

12) In the set of specific equation under CRC*, I directly use "*delta*," instead of using "*alpha* less *delta*" for the rate of technological progress under CRC. I find consistency between "*alpha* less *delta*" and "*delta*." For "*alpha* less *delta*", $k(t)^{\alpha-\delta}$ is used, where if $\alpha - \delta = 0$, $k(t)^{\alpha-\delta} = 1$. For "*delta*" in the set of specific equations, $i_A^* / k(0)^{\delta}$ is used, where if $\delta = 0$, $k(0)^{\delta} = 1$. Therefore, $k(t)^{\alpha-\delta} = 1$ corresponds with $k(0)^{\delta} = 1$. In both cases, the rate of technological progress completely similarly changes along with the change in *beta* or *beta**.

In short, both *beta* and *delta* under DRC are each independently calibrated using recursive programming. Despite, *beta** and *delta** under CRC*remain dependent as "the set of *beta** and *delta**." Specific equations in this section present essential relationships between parameters under CRC*. These relationships will make it possible to present some insight into the rate of investment (to output; close to the rate of saving) and the growth rate of population/employed persons, and the unemployment rate.[13] I will briefly clarify some of relationships among parameters using simulation in Appendix. Note that my model in this paper satisfies both the roles of the three financial parameters and eight propositions simulated in Kamiryo [2003], where the specified equations are not formulated.

4. Convergence to CRC* and conditional convergence

*4.1 Convergence from DRC to CRC**

In this section, I first show classification rules for the current situation, DRC, using the hyperbolic curve of *beta** to *delta** (hereunder, the hyperbolic curve) and the vertical line of $\beta^*_{\delta=0}$, and second show some results in the relationship between DRC and CRC* using empirical data.

There are two kinds of convergence in the literature: by country and among countries. An economy has its convergence from DRC to CRC*. This is discussed by country. Among countries, I intends to prove that conditional convergence exists when I use the specific equations or the set of *beta** and *delta** under CRC*.

For these two empirical convergences, I use the national accounts data of Japan 1983–95, the US 1983–95, and the UK 1983–95, each on average, and the data of Japan by year from 1992 to 2000. These data use the OECD statistics and national accounts by country (for data arrangement, see Table 1).[14]

First, I show the classification rules of the DRC situation. These rules are shown in

13) I intend to clarify the relationship between employed persons and unemployment using Eq. 24 in a separate paper in the future.

14) For empirical study, Kamiryo models since 1998 use the data in national accounts and OECD statistics by country to get six basic data: L, K, S_Π, D, W, and S, where n is the growth rate of L, profit Π is the sum of corporate saving S_Π and dividends D, and output Y is the sum of Π and compensation W. Saving, S, equals net investment in capital since S is "net" after deducting changes in inventories and depreciation, and adjusting the surplus of the nation (current external balance) and capital transfers, receivables, and payables. These data are the same as those I used at the University of Auckland and Kamiryo [2002/July].

Table 1 Data, parameters, and variables by country and year

θ_1=0.8 and θ_2=0.7 in each country for comaprison of γ.

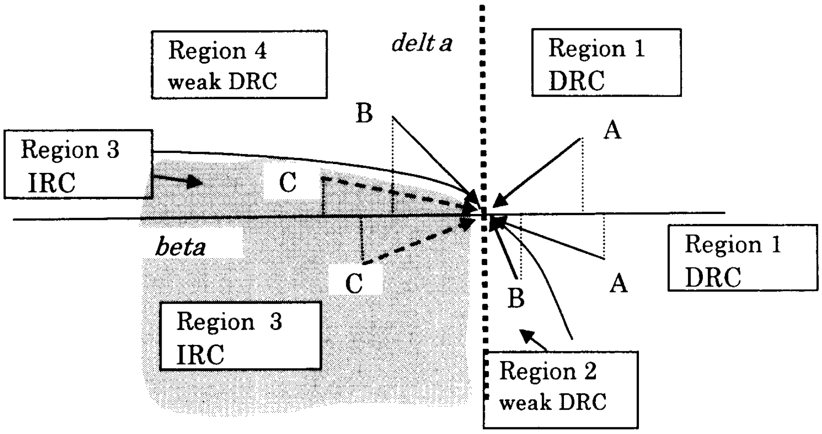
	n	α	s	i	i_K^*	i_A^*	$k(0)$	$\Omega(0)$
Japan	0.00809	0.09483	0.24860	0.20745	0.1733	0.0341	14.46	3.84
the US	0.02268	0.09691	0.09931	0.08575	0.0802	0.0055	86.12	2.78
the UK	0.00668	0.10094	0.10029	0.08754	0.0634	0.0242	23.02	1.89
	$g_{Y\,ave.83\text{-}95}$	$r_{ave.83\text{-}95}$	Ω^*	r^*	$\beta^*_{(\delta\neq0)}$	g_Y^*	conv. speed	$r(0)$
Japan	0.0469	0.02473	3.75	0.02527	0.8354	0.0459	**156**	0.02473
the US	0.0632	0.03491	2.77	0.03500	0.9353	0.0288	**98**	0.03491
the UK	0.0741	0.05334	1.88	0.05381	0.7238	0.0336	**178**	0.05334

Japan	n	α	s	i	i_K^*	i_A^*	$k(0)$	$\Omega(0)$
1992	0.00803	0.05620	0.22819	0.18960	0.1582	0.0314	15.85	3.84
1993	0.00410	0.03770	0.19524	0.15983	0.1317	0.0282	16.30	3.95
1994	0.00042	0.05962	0.17468	0.14778	0.1191	0.0287	16.59	3.86
1995	0.00090	0.06710	0.16309	0.13996	0.1130	0.0269	16.86	3.83
1996	0.00818	0.09042	0.16676	0.14767	0.1251	0.0226	17.20	3.77
1997	0.00641	0.08966	0.15686	0.13978	0.1177	0.0221	17.74	3.83
1998	(0.00978)	0.08244	0.12771	0.11486	0.0851	0.0298	17.85	3.91
1999	(0.00598)	0.10094	0.11573	0.10904	0.0839	0.0251	17.96	3.88
2000	0.00073	0.09894	0.11663	0.10848	0.0887	0.0197	18.14	3.89
Japan	$g_{Y\,ave.83\text{-}95}$	$r_{ave.83\text{-}95}$	Ω^*	r^*	$\beta^*_{(\delta\neq0)}$	g_Y^*	conv. speed	$r(0)$
1992	0.0118	0.01462	3.80	0.01478	0.8344	0.0413	**146**	0.01462
1993	0.0037	0.00954	3.93	0.00959	0.8238	0.0334	**157**	0.00954
1994	0.0422	0.01544	3.85	0.01550	0.8059	0.0309	**161**	0.01544
1995	0.0247	0.01750	3.79	0.01771	0.8076	0.0298	**211**	0.01750
1996	0.0447	0.02396	3.76	0.02404	0.8470	0.0331	**144**	0.02396
1997	0.0218	0.02339	3.81	0.02351	0.8420	0.0307	**167**	0.02339
1998	(0.0223)	0.02110	3.80	0.02172	0.7406	0.0227	**327**	0.02110
1999	0.0068	0.02601	3.84	0.02630	0.7694	0.0220	**288**	0.02601
2000	0.0080	0.02542	3.91	0.02528	0.8180	0.0227	**228**	0.02542

Figure 1 with notes. The current situation is under DRC, but DRC has three different categories in my model: (1) DRC itself found in the literature and (2) two kinds of weak DRC (half-DRC and half-IRC; determined by the differences between "*beta* and $\beta^*_{\delta=0}$" and between "*delta* and *delta**"). In Eq. 15, I showed three situations, mild IRC, CRC, and DRC, using $m_{(CRC)} = alpha - delta > 0$, $m_{(CRC)} = alpha - delta = 0$, and $m_{(CRC)} = alpha - delta < 0$. This classification temporarily holds for numerous CRC situations without introducing the relationship between *beta*, $\beta^*_{\delta=0}$, *delta*, and *delta**. When my model takes into consideration the relationship between *beta** and *delta**, Figure 1 is shown, using Eq. 20, $m = k(0)^{\delta*}$: $m > 1$, $m = 1$, and $m < 1$, which respectively correspond with the above three cases of $m_{(CRC)}$:

1. If $m = k(0)^{\delta*} > 1$ (or *delta*$^* > 0$), total factor productivity already increases over time under DRC. The optimum CRC* situation is attained by neutralizing this half-DRC (or half-IRC). I call this weak DRC. This weak DRC is divided into two categories: (1) *beta* > $\beta^*_{\delta=0}$ and *delta* < *delta**, which I call Region 2 under weak DRC and (2) *beta* < $\beta^*_{\delta=0}$ and

Figure 1 Classification rules for transitional paths: from DRC to CRC



Notes: Explanation of classification rules

	Region 1	Region 2	Region 3	Region 4
$\delta_{\beta^*}-\delta$	minus	plus	plus	minus
$\beta^*_{(\delta=0)}-\beta$	minus	minus	plus	plus
$k(0)$^δ*<1	1: sDRC			4: sDRC
$k(0)$^δ*>1	1: wDRC	2: wDRC	3: IRC	4: wDRC

1. $k(0)$^δ*<1 shows that the current DRC situation is strong.
   So that, "strong" can be added to the front of DRC: sDRC.
2. $k(0)$^δ*>1 shows that the current DRC situation is weak.
   So that, "weak" is added to the front of DRC: weak DRC.
3. The current situation cannot identify IRC due to CRS.
4. $k(0)$^δ* works for attaining CRC by balancing productivity enhancement and DRC.

delta > *delta**, which I call Region 4 under weak DRC.

2. If $m = k(0)^{\delta *} = 1$ (or *delta*$^* = 0$), total factor productivity does not change over time under DRC*. This leads to the optimum CRC* situation, where *beta** equals $\beta^*_{\delta=0}$.
3. If $m = k(0)^{\delta *} < 1$ (or *delta*$^* < 0$), total factor productivity decreases over time under DRC. This situation corresponds with DRC in the literature (see Inada [1963]). The optimum CRC* situation is attained by neutralizing this conventional DRC through productivity enhancement. I call this DRC. This category occupies Region 1. Note that in rare cases Region 1 includes the case of $m = k(0)^{\delta *} > 1$ (or *delta*$^* > 0$). This happens when DRC occurs at a point close to Region 2, which I also call weak DRC. (25)

Using the above classification rules, the hyperbolic curve (of *delta** to *beta**) takes *beta** on the X axis and *delta** on the Y axis in Figure 1. Along with this curve, I pay attention to three *beta** values: (1) *delta*$^* = 0$ (which satisfies $\beta^*_{\delta=0}$), (2) *delta*$^* \neq 0$ (which is close to zero and satisfies $\beta^*_{\delta=0}$), and (3) *delta**=*alpha* just for information. The origin is shown by $\beta^*_{\delta=0}$. Figure 1, at the same time, shows the current DRC situation by replacing *beta** with *beta* and *delta**

with *delta*, where the point of intersection of *beta* and *delta* indicates the current situation of DRC.

The transition paths from DRC to CRC* are shown by the arrow from this intersection to the origin. The difference/distance between $\beta^*_{\delta=0}$ and *beta* and the distance between *delta*$^* = 0$ and d*elta* are most important in terms of convergence. These distances are definitely related to the speed of convergence, which is in turn related to the level of the capital-output ratio under CRC*. In this sense, a condition of *delta*$^* =$ *alpha* is irrelevant of convergence by country.

Figure 1 distinguishes Regions 1, 2, and 4 under DRC and Region 3 under IRC. Region 3 is surrounded by the hyperbolic curve and the vertical line of $\beta^*_{\delta=0}$. Regions 2 and 4 show weak DRC since either *beta** or *delta** is half-DRC (getting rid of Region 1 or conventional DRC). The transitional paths are limited: from Regions 1, 2, or 4 to the origin. Under CRS, the path from Region 3 to the origin is not possible.

Turning to the second issue, Figure 2 and Table 2 show some results in the relationship between DRC and CRC* using empirical data.

1. The vertical line through the origin differs by country: the UK is lowest and the US the highest. The difference between *beta* and $\beta^*_{\delta=0}$ is largest in the UK and smallest in the US. Japan is between the UK and the US.
2. The hyperbolic curve significantly differs by country: the US is highest and most sharp, while the UK and Japan are similar and mild. This difference comes mainly from the difference of the growth rate of population/workers.[15] This result is consistent with that of MRW [1992. p. 433] (see Appendix).[16]
3. For Japan in the 1900s, the vertical line is lowered to some extent. Also, the character of current situation of DRC changes: from a high positive *delta* to a low positive *delta* and sometimes from a high *beta* to an extremely high *beta* (as in 1998).

15) The Kamiryo model uses employed persons for data, but the relationships among population (as in OECD data), employees, and employed persons (= employees + employers) are expressed by equations using the rate of unemployment. Thus, the differences can be absorbed in any model. In this paper, I use, for simplicity, an expression of population.

16) Mankiw, Romer, and Weil [1992, p. 433] state as follows: "population growth also has a larger impact on income per capita than the textbook model indicates. In the textbook model higher population growth lowers income because the available capital must be spread more thinly over the population of workers. In the augmented model human capital also must be spread more thinly, implying that higher population growth lowers measured total factor productivity." In this paper, I formulated Eq. 24 and showed Appendix SF 3 to clarify $n(\beta^*)$or $\beta^*(n)$, where the growth rate per capita output is $(1 - \beta^*)s/(1 - \alpha)$ with no banking costs.

Figure 2 The curve of *delta** to *beta** under CRC and the current situation by country and year

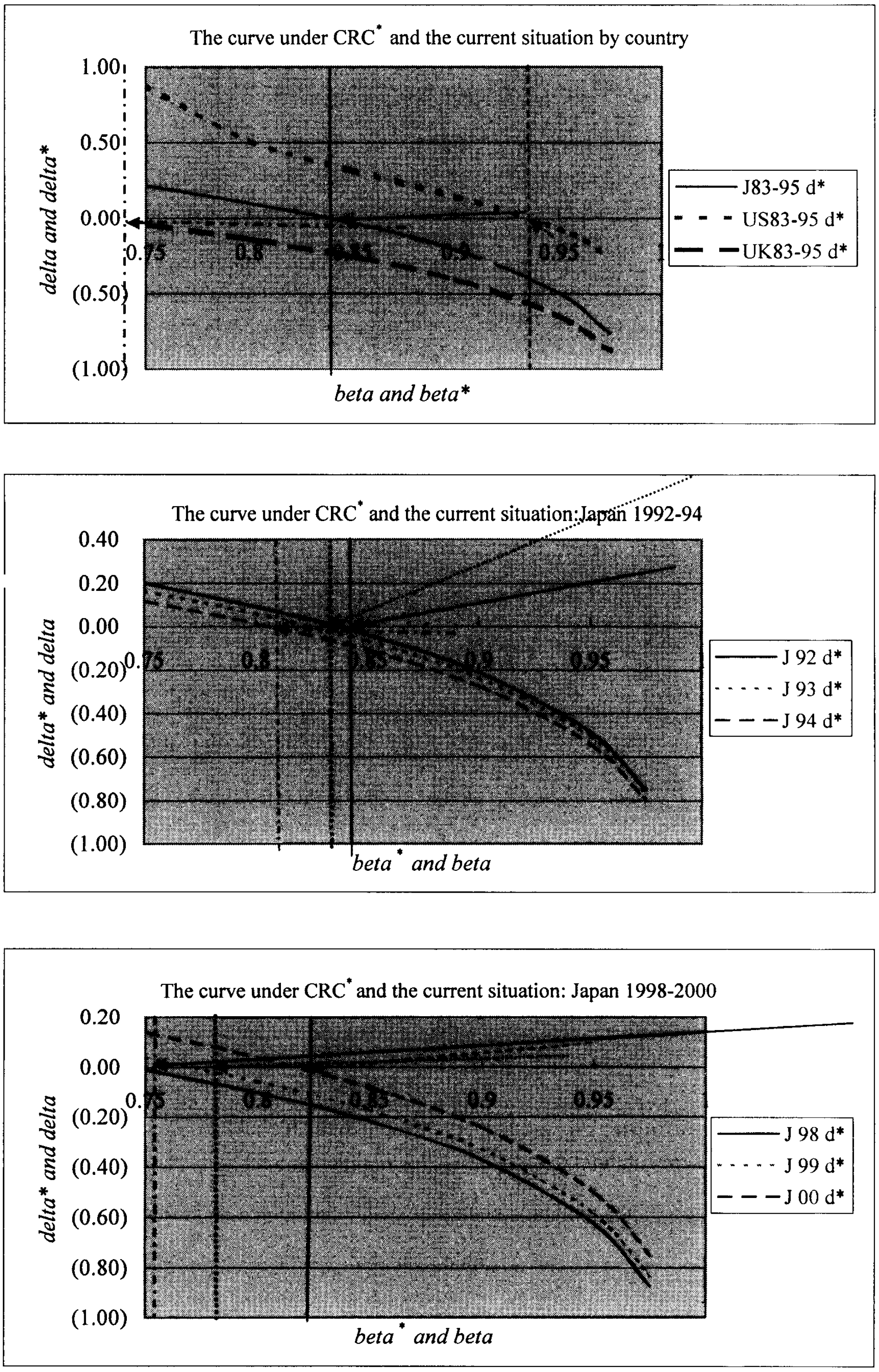


Table 2 Decision-making parameters under the current and CRC* situations by country

	delta	δ^* at β^*	beta	$\beta^*_{(\delta=0)}$	$m=k(0)^{\delta^*}$	$\delta^*_{\beta^*}-\delta$	$\beta^*_{(\delta=0)}-\beta$	Region & level
Japan	0.00623	0.00919	0.93242	0.83867	1.0249	0.003	−0.0938	2:wDRC
the US	−0.06288	0.00274	0.94776	0.9359	1.0123	0.0656	−0.0119	2:wDRC
the UK	−0.01577	0.00309	0.87067	0.72565	1.0097	0.0189	−0.145	2:wDRC
Japan1992	0.23312	0.01084	0.98054	0.83597	1.0304	−0.2223	−0.1446	1:DRC
1993	15.47	0.00195	0.98717	0.8246	1.0055	−15.468	−0.1626	1:DRC
1994	−0.00687	0.00106	0.87699	0.80638	1.003	0.0079	−0.0706	2:wDRC
1995	0.00723	0.00387	0.90213	0.80925	1.011	−0.0034	−0.0929	1:DRC
1996	−0.00886	0.00122	0.91686	0.84739	1.0035	0.0101	−0.0695	2:wDRC
1997	0.04047	0.00181	0.93481	0.84262	1.0052	−0.0387	−0.0922	1:DRC
1998	0.14978	0.00652	1.18174	0.74455	1.019	−0.1433	−0.4372	1:DRC
1999	0.03125	0.00261	0.93737	0.77078	1.0076	−0.0286	−0.1666	1:DRC
2000	0.0726	−0.00236	0.94988	0.81703	0.9932	−0.075	−0.1328	1:sDRC

4. As shown in Table 2, the current situation is weak DRC except for Japan 2000. For Japan in the 1990s, Region 2 disappears after 1996, where both *beta* and *delta* and also the difference between DRC and CRC have not improved.[17]

It is anticipated that the Japanese economy cannot recover without strong government leadership for decreasing *beta* and *delta* through structural reform and deregulation. If Japan cannot get rid of Region 1, Japan must eventually be defeated in international competition. Note that the Japanese economy is *beta*-oriented (structural reform/deregulation-oriented) more than *delta*-oriented (R & D/education-oriented).

4.2 *Conditional convergence among countries*

I tested convergence across countries under the optimum CRC situation, using the same data in the previous section together with additional data from China, Korea, and Taiwan. Tables 3-1 to 3-3 show the results of convergence.

For convergence-simulation, I arranged the following eight cases using four related parameters, n, *alpha*, s, and *beta**, which are essential as shown in Eq. 18:

Case 1.1 Data before simulation.

Case 1.2 Set $n = 0$, $s = 0.1$, and *beta** = 0.8, but *alpha* remain unchanged: convergence.

Case 1.3 Set $n = 0.02$, $s = 0.1$, and *beta** = 0.8, but *alpha* remain unchanged: convergence.

17) According to my divisional analysis, a fundamental reason is that *beta* in the government and household sector is 1.4. This implies that the rate of technological progress in this section is minus 5%, which results in deflation.

Table 3-1 Convergence-simulation of *beta**, *n*, *s*, and *alpha*: Cases 1.1 to 1.4

using θ_1=0.8 and θ_2=0.7, where if θ_i=1, i=s. Root Mean Square Mean (RMSE)

Case 1.1 Before simulation (SIMU.) under the optimum CRC situation by country

t=1000	α	$\beta^*_{(\delta\neq 0)}$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	n	s
J 1996	0.0904	0.8450	0.0331	0.0226	0.0249	3.7607	0.0240	0.0082	0.1668
J 1998	0.0824	0.7435	0.0227	0.0298	0.0325	3.7960	0.0217	**(0.0098)**	0.1277
J 2000	0.0989	0.8180	0.0227	0.0197	0.0219	3.9136	0.0253	0.0007	0.1166
J 83-95ave.	0.0948	0.8354	0.0459	0.0341	0.0378	3.7531	0.0253	0.0081	0.2486
US 83-95ave.	0.0969	0.9353	0.0288	0.0055	0.0061	2.7690	0.0350	0.0227	0.0993
UK 83-95ave.	0.1009	0.7238	0.0336	0.0242	0.0269	1.8758	0.0538	0.0067	0.1003
China 97	0.0981	0.7015	0.0886	0.0706	0.0785	1.8539	0.0529	0.0101	0.2807
Taiwan98	0.0768	0.7744	0.0684	0.0551	0.0598	2.7456	0.0280	0.0086	0.2861
Korea 98	(0.0853)	0.8095	0.0596	0.0556	0.0511	3.9344	(0.0217)	0.0085	0.3907

Case 1.2 SIMU. n=0, s=0.1,*beta*=0.8, but α remains unchanged: convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0
J 1996	0.0904	**0.8**	0.0207	0.0189	0.0207	3.6349	0.0249	305	(0.498)
J 1998	0.0824	**0.8**	0.0202	0.0185	0.0202	3.6672	0.0225	262	(1.840)
J 2000	0.0989	**0.8**	0.0211	0.0190	0.0211	3.6005	0.0275	280	1.714
J 83-95ave.	0.0948	**0.8**	0.0196	0.0177	0.0196	3.6173	0.0262	278	(0.522)
US 83-95ave.	0.0969	**0.8**	0.0191	0.0173	0.0191	3.6090	0.0269	291	(0.927)
UK 83-95ave.	0.1009	**0.8**	0.0194	0.0175	0.0194	3.5927	0.0281	282	(1.209)
China 97	0.0981	**0.8**	0.0204	0.0184	0.0204	3.6039	0.0272	392	(1.329)
Taiwan98	0.0768	**0.8**	0.0207	0.0191	0.0207	3.6899	0.0208	228	0.619
Korea 98	(0.0853)	**0.8**	0.0182	0.0198	0.0182	4.3446	(0.0196)	691	1.135

Case 1.3 SIMU. n=0.02, s=0.1, *beta*=0.8, but α remains unchanged: convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0
J 1996	0.0904	**0.8**	0.0407	0.0189	0.0207	1.8321	0.0494	143	rela.diffi.
J 1998	0.0824	**0.8**	0.0402	0.0185	0.0202	1.8253	0.0452	177	(1.660)
J 2000	0.0989	**0.8**	0.0411	0.0190	0.0211	1.8316	0.0540	129	5.249
J 83-95ave.	0.0948	**0.8**	0.0396	0.0177	0.0196	1.7723	0.0535	193	1.008
US 83-95ave.	0.0969	**0.8**	0.0391	0.0173	0.0191	1.7473	0.0555	128	0.209
UK 83-95ave.	0.1009	**0.8**	0.0394	0.0175	0.0194	1.7536	0.0576	179	(0.388)
China 97	0.0981	**0.8**	0.0404	0.0184	0.0204	1.8005	0.0545	141	(0.662)
Taiwan98	0.0768	**0.8**	0.0407	0.0191	0.0207	1.8567	0.0414	116	0.479
Korea 98	(0.0853)	**0.8**	0.0309	0.0118	0.0109	1.5197	(0.0561)	165	3.872

Case 1.4 Summary: using the capital-output ratio (cf. Table 4 that uses gy*)

	J 1996	J 1998	J 2000	J 83-95	US 83-95	UK 83-95	China 97	Taiwan 98	Korea 98
1.2: n=0.0	3.635	3.667	3.601	3.617	3.609	3.593	3.604	3.690	4.345
Bef. SIMU	3.761	3.796	3.914	3.753	2.769	1.876	1.854	2.746	3.934
difference	0.126	0.129	0.313	0.136	(0.840)	(1.717)	(1.750)	(0.944)	(0.410)
1.7: n & s	4.954	2.657	4.047	4.592	13.012	2.353	2.116	3.166	4.615
1.2: n=0.0	3.635	3.667	3.601	3.617	3.609	3.593	3.604	3.690	4.345
difference	1.319	(1.010)	0.447	0.974	9.403	(1.239)	(1.488)	(0.524)	0.270
1.5: *beta*	5.145	5.964	3.492	3.051	1.629	2.663	2.992	3.150	3.728
1.2: n=0.0	3.635	3.667	3.601	3.617	3.609	3.593	3.604	3.690	4.345
difference	1.510	2.297	(0.109)	(0.566)	(1.980)	(0.930)	(0.612)	(0.539)	(0.616)

Notes:

1. For the initial data including the three actual growth rates, see Supplementary data 5 to 10.
   For Korea, I directly use the data of the Bank of Korea, but with economic difficulties.
2. The speed of convergence is measured by the number of times when Ω^* becomes horizontal.
3. It is almost impossible in simulation to have the Root Mean Square Error (RMSE) equal zero.
   The relative difficulty for attaining onvergence is measured by the value of RMSE in calibration.
4. Even if the capital-output ratio is the same, the speed of convergence differs by relative difficulty.

Table 3-2 Convergence-simulation of *beta**, *n*, *s*, and *alpha*: Cases 1-5 to 1-8

using θ_1=0.8 and θ_2=0.7, where if θ_i=1, i=s. Root Mean Square Mean (RMSE)

Case 1.5 SIMU. *beta*=0.8, but α, n, and s remain unchanged: no convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0:
J 1996	0.0904	**0.8**	0.0246	0.0313	0.0344	5.1452	0.0176	194	(0.715)
J 1998	0.0824	**0.8**	0.0163	0.0239	0.0260	5.9642	0.0138	277	(2.070)
J 2000	0.0989	**0.8**	0.0248	0.0217	0.0241	3.4917	0.0283	207	2.207
J 83-95ave.	0.0948	**0.8**	0.0540	0.0415	0.0459	3.0509	0.0311	81	0.409
US 83-95ave.	0.0969	**0.8**	0.0417	0.0171	0.0190	1.6287	0.0595	143	0.365
UK 83-95ave.	0.1009	**0.8**	0.0262	0.0175	0.0195	2.6630	0.0379	176	(0.936)
China 97	0.0981	**0.8**	0.0627	0.0473	0.0525	2.9924	0.0328	98	(0.918)
Taiwan98	0.0768	**0.8**	0.0616	0.0488	0.0530	3.1504	0.0244	95	0.073
Korea 98	(0.0853)	**0.8**	0.0621	0.0583	0.0536	3.7283	(0.0229)	83	1.824

Case 1.6 SIMU. n=0, s=0.1, α=0.1, but *beta* changes: no convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0:
J 1996	0.1000	**0.9500**	0.0059	0.0053	0.0058	15.6102	0.0064	691	(1.592)
J 1998	0.1000	**0.9700**	0.0037	0.0033	0.0038	24.6569	0.0041	913	(1.944)
J 2000	0.1000	**0.9300**	0.0075	0.0068	0.0075	11.8107	0.0085	685	(0.701)
J 83-95ave.	0.1000	**0.9250**	0.0079	0.0071	0.0078	10.5098	0.0095	522	(1.448)
US 83-95ave.	0.1000	**0.9030**	0.0098	0.0088	0.0097	8.0166	0.0125	485	(1.490)
UK 83-95ave.	0.1000	**0.8380**	0.0154	0.0139	0.0154	4.7334	0.0211	419	(1.382)
China 97	0.1000	**0.8700**	0.0137	0.0123	0.0136	5.8715	0.0170	398	(1.581)
Taiwan98	0.1000	**0.9560**	0.0054	0.0048	0.0053	18.0519	0.0055	682	1.548
Korea 98	0.1000	**0.9800**	0.0028	0.0026	0.0027	37.2539	0.0027	836	1.730

Case 1.7 SIMU. n=0, s=0.1, but α and *beta* remain unchanged: no convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0:
J 1996	0.0904	0.8450	0.0161	0.0146	0.0161	4.9536	0.0183	382	(0.876)
J 1998	0.0824	0.7435	0.0259	0.0238	0.0259	2.6570	0.0310	204	(1.691)
J 2000	0.0989	0.8180	0.0192	0.0173	0.0192	4.0472	0.0244	334	1.358
J 83-95ave.	0.0948	0.8354	0.0161	0.0146	0.0161	4.5917	0.0207	336	(0.819)
US 83-95ave.	0.0969	0.9353	0.0062	0.0056	0.0062	13.0118	0.0074	830	(1.651)
UK 83-95ave.	0.1009	0.7238	0.0269	0.0241	0.0269	2.3534	0.0429	164	(0.833)
China 97	0.0981	0.7015	0.0304	0.0274	0.0304	2.1162	0.0464	204	(0.889)
Taiwan98	0.0768	0.7744	0.0233	0.0215	0.0233	3.1658	0.0243	271	0.524
Korea 98	(0.0853)	0.8095	0.0104	0.0113	0.0104	4.6147	(0.0185)	461	1.141

Case 1.8 SIMU. α=0.1, n=0.02, s=0.1, and *beta*=0.8: no convergence relative difficulty

t=1000	α	$beta^*$	$g_Y^*=g_K^*$	g_A^*	$g_y^*=g_k^*$	$\Omega^*=k^*/y^*$	r^*	cov. speed	For RMSE=0:
J 1996	0.1000	**0.8**	0.0473	0.0245	0.0271	1.6065	0.0622	IRC: 224#	1.106
J 1998	0.1000	**0.8**	0.0553	0.0317	0.0349	1.3733	0.0728	IRC: 243#	-1.698
J 2000	0.1000	**0.8**	0.0417	0.0195	0.0217	1.8090	0.0553	189	5.289
J 83-95ave.	0.1000	**0.8**	0.0223	0.0201	0.0222	3.20431	0.0312	IRC: 196#	-0.451
US 83-95ave.	0.1000	**0.8**	0.0407	0.0186	0.0207	1.6843	0.0594	236	0.243
UK 83-95ave.	0.1000	**0.8**	0.0390	0.0171	0.0190	1.7715	0.0565	229	-0.395
China 97	0.1000	**0.8**	0.0783	0.0523	0.0582	2.3838	0.0419	174#	-0.714
Taiwan98	0.1000	**0.8**	0.0583	0.0345	0.0377	1.3671	0.0731	240#	0.474
Korea 98	0.1000	**0.8**	#NUM!	#NUM!	#NUM!	#NUM!	#NUM!	no conv.	2.725

Notes:

1. See Notes in Supplementary material 1 (1). Mark, #, in Case 1.8 indicates that Ω^* cannot be horizontal.
2. Supplementary data 1 (1) shows convergence and 1 (2), shows no convergnece.
3. Population growth, n, influences the capital-output ratio and, accordingly, the speed of convergence.
4. Case 1.8 never converges since the use of all four parameters makes this impossible.
   See Eq. 22 for clarification of the relationship among parameters under the optimum CRC situation.

Table 3-3 A summary of simulation: to test conditional convergence

Using the difference of the growth rate of per capita output between two cases

Cases	J 1996	J 1998	J 2000	J 83-95	US 83-95	UK 83-95	China 97	Taiwan 98	Korea 98
1.2 & 1.3	0.021	0.020	0.021	0.020	0.019	0.019	0.020	0.021	0.011
Bef. SIMU	0.025	0.033	0.022	0.038	0.006	0.027	0.079	0.060	0.051
difference	0.004	0.012	0.001	0.018	(0.013)	0.007	0.058	0.039	0.040
1.7: n & s	0.016	0.026	0.019	0.016	0.006	0.027	0.030	0.023	0.017
1.2 & 1.3	0.021	0.020	0.021	0.020	0.019	0.019	0.020	0.021	0.011
difference	(0.005)	0.006	(0.002)	(0.003)	(0.013)	0.007	0.010	0.003	0.006
2.5: $beta^*$	0.034	0.026	0.024	0.046	0.019	0.019	0.053	0.053	0.054
1.2 & 1.3	0.021	0.020	0.021	0.020	0.019	0.019	0.020	0.021	0.011
difference	0.014	0.006	0.003	0.026	(0.000)	0.000	0.032	0.032	0.043

Note: The difference between 1.2 and 1.3 is 2% as population growth, which I excluded. I used the same data as Kamiryo [2002/NIRA].

Case 1.4 Summary of convergence: using the growth rate of per capita output.

Case 1.5 Set $beta^* = 0.8$, but *alpha*, n, and s remain unchanged: no convergence.

Case 1.6 Set $n = 0$, $s = 0.1$, and $alpha = 0.1$, but $beta^*$ changes: no convergence.

Case 1.7 Set $n = 0$, $s = 0.1$, but *alpha* and $beta^*$ remain unchanged: no convergence.

Case 1.8 Set $alpha = 0.1$, $n = 0.02$, $s = 0.1$, and $beta^* = 0.8$: with no convergence.

I find in simulation that (1) convergence occurs when three of four parameters are the same (see Table 3-1) and (2) convergence does not occur when one/two of four parameters are the same or when all four parameters are the same (see Table 3-2). The value of $beta^*$ significantly influences convergence, yet $beta^*$ alone is not enough for insuring convergence. If the four parameters are used for simulation, there is no room for calibration to satisfy the OLS method (RMSE = 0), owing to the tight relationship among the four parameters as shown in Eq. 18.

For convergence that uses three parameters (except for *alpha*), I present the following interesting findings (see Cases 1.2 and 1.3):

1. For nine cases (Japan 96, 98, and 2000; Japan, the US, and the UK each in 1983–95 on average; China, Taiwan, and Korea in 1997 to 98), each capital-output ratio becomes significantly similar (except for Korea).
2. The speed of convergence (in Tables 3-1 and 3-2) is accordingly similar, yet with some differences, due to the relative difficulty in getting the RMSE $\rightarrow 0$ (except for Korea).
3. The relative difficulty varies widely across countries due to different combinations of the initial parameters. The use of three parameters out of four makes it impossible to get the RMSE close to zero.

4. Population growth significantly influences the results: (1) if $n = 0$, $\Omega^* = 3.6$ and the speed of convergence is roughly 280 and (2) if $n = 0.02$, $\Omega^* = 1.8$ and the speed of convergence is roughly 140 (one-half of 280).
5. The results of convergence are shown in Table 3-3 (see Cases 1.2 and 1.3) using the growth rate of per capita output, g_y^*.

In short, I proved the existence of conditional convergence across countries by using simulation. However, I must distinguish the existence of conditional convergence with the possibility of its realization. Conditional convergence only holds under an assumption that n, s, and $beta^*$ are the same across countries. In the real world, it is difficult to satisfy this assumption and thus, it is not possible for many countries to realize conditional convergence.[18)]

What is most important for each country to approach the same conditional convergence? Suggested answer to this is: to decrease each country' *beta/delta* to attain $beta^*/delta^*$ as a common target by structural reform and deregulation.

5. Conclusions

My endogenous growth model uses the Cobb-Douglas production function starting with Solow [1956] under constant returns to scale (CRS). I do not introduce human capital (stock) into my model and, instead, I introduce investment in quality for the rate of technological progress and capital (stock) in quality for the level of technology. For this, I introduce the function of the corporate, financial and government sectors and, accordingly, related decision-making parameters in my model. These decision-making parameters are *beta* and *delta*. These distinguish investment in quality from investment in quantity and clarify each accumulation as the level of technology and physical capital.

The idea of this model comes from Kamiryo [2003] that uses the three financial parameters (a financial intermediary parameter, θ_1, a corporate decision-making parameter, θ_2, and a parameter capturing barriers to technology growth, γ) together with a control parameter, *delta*.

18) Jones [1998, p. 62: Note 9] explains "conditional convergence" as follows: conditional convergence reflects the convergence of countries after we control for ("condition on") differences in steady states. It is important to keep in mind what this "conditional convergence" result means. It is simply a confirmation of a result predicted by the neoclassical growth model: that countries with similar steady states will exhibit convergence. It does not mean that all countries in the world converging to the same steady state, only they are converging to their own steady states according to a common theoretical model. I leave a message here: my model has clarified the conditions for conditional convergence.

This model, based on the same financial and corporate function, replaced the three financial parameters by *beta* and also replaced *delta* as a control parameter with *delta* as a decision-making parameter.

By so doing, I finally completed, in this model, equations related to endogenous economic growth (Eqs. 1 to 17) and formulated the set of specific equations under the optimum CRC* situation. The optimum CRC* situation that attains the highest rate of profit differs from numerous CRC situations, where the capital-output ratio converges to a certain value higher than the initial/current value. Under CRC*, *delta** is a function of *beta** and expressed as the hyperbolic curve of *delta** to *beta**, which makes it possible to strictly clarify the difference between IRC, CRC*, and DRC.

My model thus sets the classification rules (see Figure 1) for the current situations, DRC and weak DRC, by fixing the category of CRC*, after temporarily setting the difference between the current situations under CRC. The current situation of DRC is now expressed using the differences both between *beta* and *beta** and between *delta* and *delta** (see Figure 2). I found a new category of weak DRC in Regions 2 and 4, where either *beta* or *delta* is below either the hyperbolic curve or the vertical line of $\beta^*_{\delta=0}$. Why did I find this new category in DRC? This is because I found two kinds of $m = k(0)^{\delta *}$ under DRC (see Eq. 25), where if $m > 1$(or *delta** > 0), this situation enhances productivity over time even under DRC and if $m < 1$(or *delta** < 0), this situation diminishes productivity over time as seen in conventional DRC. Both cases converge to CRC* by offsetting enhancing or diminishing productivity over time and DRC. Thus, to attain CRC, the former is weak DRC and the latter is strong DRC or conventional DRC (in Region 1).

IRC exists in Region 3. I strictly define IRC as the situation, where both *beta* and *delta* are below the hyperbolic curve and the vertical line of $\beta^*_{\delta=0}$. However, I cannot identify IRC due to CRS and thus, the arrow path from IRC to CRC* cannot be indicated. Empirically, the Japanese economy in the 1990s has lost the power to make Region 1 turn to Region 2 or 4 due to an extremely high level of *beta** (particularly due to public excessive investment in quantity).

The set of specific equations under CRC* (Eqs. 18 to 24) is unique in that these equations can clarify the relationship between parameters and variables: in particular, *n*(*beta**), Ω^*(*beta**), and $\Omega^*(n)$, based on Eq.24 (see Appendix). These equations also clarify essential conditions for conditional convergence among countries: I find that the growth rate of per capita output among countries is very close to 2% if $n = 0$, $s = 0.1$, and *beta** = 0.8, but *alpha* remains unchanged among countries (see Case 1.2), where, I indicate, structural reform and deregulation parameter, *beta*, significantly influences on economic growth and also the inclination between *beta** and *delta** significantly differs by country.

Appendix The specific equations under the optimum CRC* situation: similarities and differences compared with the literature

This section clarifies the characteristics of the specific equations, in particular Eqs. 23 and 24, using supplementary tables and figures (ST 2 and 3 and SF 2 and 3 at the end).

1. $\Omega^*(\beta^*_{\delta=0})$: Ω^* is a hyperbolic function of $\beta^*_{\delta=0}$: The curvature of this function depends on the values of the set of combinations of *n*, *alpha*, and *i*. In particular when $\beta^*_{\delta=0}$ becomes higher than 0.9, Ω^* becomes rapidly higher.

2. $i^*_A(\beta^*_{\delta=0})$: Needless to say, i^*_A is a negative linear function of $\beta^*_{\delta=0}$: $i^*_A = (1-\beta^*)\cdot i$, where if $\beta^*_{\delta=0} = 1$, $i^*_A = 0$. Since $g^*_y = i^*_A/(1-\alpha)$, g^*_y is a negative linear function of $\beta^*_{\delta=0}$. Therefore i^*_A is a negative function of Ω^*: $i^*_A(\Omega^*)$.

3. $\beta^*_{\delta=0}$ is a positive linear function of *n*: $\beta^*_{\delta=0}(n)$. The higher *n*, higher $\beta^*_{\delta=0}$ (and accordingly, lower i^*_A). Although *n* does not usually change, the level of *n* determines $\beta^*_{\delta=0}$ and accordingly, i^*_A. This is a striking fact, especially when compared with what Mankiw, Romer, and Weil (MRW) [1992] and Groth and Schou [2002] indicate: that there is a significant influence of population growth on per capita output.

4. (1) g^*_y is a negative linear function of $\beta^*_{\delta=0}$, (2) $\beta^*_{\delta=0}$ is a positive linear function of *n*, and (3) the growth rate of output, g^*_Y, is a slightly positive linear function of $\beta^*_{\delta=0}$ (see SF 2). These linear results show a more definite finding than MRW [1992] that stresses a negative relationship between *n* and g^*_y (compare with SF 3).

5. The rate of saving/investment, *s* or *i*, where s = *i* under no banking costs, has a vertical asymptote to $\beta^*_{\delta=0}$ at *s* = 0 or *i* = 0 (as shown in empirical results). The existence of the vertical asymptote found in $\beta^*_{\delta=0}(s)$ is another striking fact: if *s* = 0, there exists a trap of economic growth, where no convergence/CRC is expected. When the payout ratio is extremely high together with wages fully consumed, an economy may have a possibility to fall into this trap.

6. On the contrary, the relative share of profit, *alpha*, is irrelevant to $\beta^*_{\delta=0}$. *Alpha*, together with *s*, only determines the difference between g^*_y and r^* (see below).

7. Finally, using $\Omega^*(\beta^*_{\delta=0})$, I prove that the initial capital-output maintains a basis for profit-

maximizing. This is because $r^* = \alpha / \Omega^*$, where the lower Ω^* the higher the rate of profit, r^*, and Ω^* (based on $\beta^*_{\delta=0}$) cannot be lower than $\Omega(0)$ under given parameters.

Next, definite differences between *beta**= 1 (with no technological progress) and *beta**< 1 (with technological progress) are summarized in comparisons with the literature as follows:

1. If $\beta^*_{\delta=0} = 1$, $n = s / \Omega^*$, where $s = i = i^*_K$ and $n = g^*_Y$: Harrod [1973, pp. 16–31].
2. If $n = 0$, $\Omega^* = \dfrac{\beta^*(1-\alpha)}{1-\beta^*} = \dfrac{i^*_K(1-\alpha)}{i^*_A} = \dfrac{s}{g^*_y}$, where $\Omega^* = \Omega(0)$ under the optimum CRC situation. This proves that $g^*_y = s / \Omega^*$ holds under $\beta^*_{\delta=0} > 1$ and modifies Harrod's [1973, pp. 16–31], where no knife's edge exists when *beta** changes.
3. If "$s = \alpha$," $n = s / \Omega^* = g^*_Y$ becomes equal to $\alpha / \Omega^* = r^* = g^*_Y$ under $\beta^*_{\delta=0}$= 1, and also $g^*_y = s / \Omega^* = \alpha / \Omega^* = r^*$ under $\beta^*_{\delta=0}$ > 1. O'Connell [1995] reviewed the case of $\beta^*_{\delta=0}$ = 1. I find its generalization under $\beta^*_{\delta=0}$ > 1: If $\alpha > i^*_K, r^* > g^*_y$, if $\alpha = i^*_K, r^* = g^*_y$, and if $\alpha < i^*_K, r^* < g^*_y$. A situation of $s = \alpha$ implies that dividends are equal to saved wages, as Robinson [1957] indicated.
4. If $\beta^*_{\delta=0} > 1$, $g^*_y = i^*_K / \Omega^*$ (see the above 3): This form is similar to MRW's [1992, p. 433] $MPK = \alpha(n+g) / s_k$: under an assumption of a constant exogenous technological progress, the sum of the fraction of income invested in physical capital and the fraction invested in human capital, $s = s_k + s_h$, holds in $Y(t) = K(t)^\alpha \cdot H(t)^\beta \cdot (A(t) \cdot L(t))^{1-\alpha-\beta}$. Kamiryo's model newly finds: If $i^*_K > n \cdot \Omega^*_,\ g^*_y > 0$, if $i^*_K = n \cdot \Omega^*_,\ g^*_y = 0$, and if $i^*_K < n \cdot \Omega^*_,\ g^*_y < 0$, each under an endogenous growth situation. This finding between population growth and quantitative investment implies that an economy should not be closed but open to the world.

The above results are unique in that no literature shows the relationship among parameters under the optimum CRC* situation: some of these results are consistent and others significantly differ. These results are almost consistent with the results simulated in Kamiryo [2003].

References

Arrow, K. J., H. B. Chenery, B. S. Minhas, and R. M. Solow. 1961. Capital-Labor Substitution and Economic Efficiency. *Review of Economics and Statistics* 43(Aug.): 225–250.

Barro, R. J. 1991. Economic Growth in a Cross Section of Countries, *Quarterly Journal of Economics* 106 (May): 407–443.

Barro, R. J., and Xavier Sala-i-Martin. 1991. Convergence across States and Regions. *Brookings Papers on Economic Activity*, No. 1: 107–182.

De Long, J. B., and L. H. Summers. 1991. Equipment Investment and Economic Growth, *Quarterly*

Journal of Economics 106 (May): 445–502.

Domar, E. D. 1948. The Problem of Capital Accumulation. *American Economic Review* 5 (Dec): 777–794.

Dowrick, S., and M. Rogers. 2002. Classical and Technological Convergence: beyond the Solow-Swan Growth Model. *Oxford Economic Papers* 54: 369–385.

Groth, C., and P. Schou. 2002. Can non-renewable resources alleviate the knife-edge character of endogenous growth? *Oxford Economic Papers* 54: 386–411.

Harrod, R. F. 1973. *Economic Dynamics*. London: Macmillan. 195 pp.

Inada, K. 1963. On a Two-Sector Model of Economic Growth: Comments and A Generalization. *Review of Economic Studies* 30 (June): 119–127.

Jones, C. I. 1998. *Introduction to Economic Growth.* New York and London: W. W. Norton. 200 pp.

Jones, C. I. 2002. Sources of U.S. Economic Growth in a World of Ideas. *American Economic Review* 92 (March): 220–239.

Kaldor, N. 1961. Increasing Returns and Technological Progress: A Comment on Professor Hick's Article. *Oxford Economic Papers* 13(Feb): 1–4.

Kahn, R. F. 1931. The relation of Home Investment to Unemployment. *Economic Journal* 31 (June): 173–198.

Kaldor, N. 1978. *Further Essays on Economic Theory.* London: Duckworth. 232 pp.

Kamiryo, H. 1998. *Economic Accounting: a Macro and Micro Common Approach Using National and Corporate Accounts.* Hiroshima: Hiroshima Shudo University. 305 pp. (Annotation: Journal of Economic Literature 37(March), JEL99–0079).

Kamiryo, H. 2000. Growth Accounting: in Discrete Time: a New Approach Using Recursive Programming. Hiroshima: Hiroshima Shudo University. 306 pp. (JEL2001–0349).

Kamiryo, H. 2002. Numerical Relationships between Technological Progress and Structural Reform: to Save the Unprecedented Difficulties in the Japanese Economy. National Institute for Research Advancement. *NIRA Report 2002/July.* 75 pp.

Kamiryo, H. 2003. Furthering the Role of Corporate Finance in Economic Growth. The University of Auckland (submitted to the Graduate Center on the 10th of Nov). 125 pp.

Keynes, J. M. 1936. *The General Theory of Employment Interest and Money*, 113–254. London: Macmillan and Co.

Krugman, P. R. 1998. It's Baaack: Japan's Slump and the return of the Liquidity Trap. *Brooking's Papers on Economic Activities* 2: 137–205.

Lucas, R. E. 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics* 22 (July): 3–42.

Mankiw, N. G., D. Romer, and D. N. Weil. 1992. A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics* 107(May): 407–437.

O'Connell, J. 1995. The Two-one Class Model of Economic Growth. *Oxford Economic Papers* 47: 363–368.

Phelps, E. S. 1961. The Golden Rule of Accumulation: A Fable for Growthmen. *American Economic Review* 51(Sep): 638–643.

Phelps, E. S. 1962. The End of the Golden Age in Solovia: Comment. *American Economic Review* 52 (Dec): 1097–1099.

Ramsey, F. P. 1928. A Mathematical Theory of Saving. *Economic Journal* 38 (Dec): 543–559.

Rebelo, S. 1991. Long-Run Policy Analysis and Long-Run Growth. *Journal of Political Economy* 99 (June): 500–521.

Romer, D. 2000. Keynesian Macroeconomics without the LM Curve. *Journal of Economic Perspec-*

tives 14 (Spring): 149–169.

Romer, P. M. 1986. Increasing Returns and Long-Run Growth. *Journal of Political Economy* 94 (Oct.): 1002–1037.

Romer, P. M. 1990. Endogenous Technological Change. *Journal of Political Economy* 98 (Oct): S71–S102.

Romer, P. M. 1994. The Origins of Endogenous Growth. *The Journal of Perspectives* 8 (Winter): 3–22.

Samuelson, P. A. 1939. Interactions between the multiplier analysis and the principle of acceleration. *Review of Economic Statistics* 21: 75–78.

Samuelson, P. A. 1970. Law of the Conservation of the Capital-output Ratio. Proceedings of the National Academy of Sciences, *Applied Mathematical Science* 67: 1477–79.

Sato, R. 1964. The Harrod-Domar Model vs. The Neo-classical Growth Model. *Economic Journal* 74 (June): 380–387.

Solow, R. M. 1956. A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70 (Feb.) : 65–94.

Solow, R. M. 1957. Technical Change and the Aggregate Production Function. *Review of Economics Statistics* 39 (Aug): 312–320.

Solow, R. M. 1987. Growth Theory and After. *Nobel Lecture*, December 8, 1987.

Solow, R. M. 1992. *Siena Lectures on Endogenous Growth Theory.* Edited by Serena Sordi, Universita, Degli Studi Di Siena. 91 pp.

Solow, R. M. 2000 (2nd ed). *Growth Theory: an Exposition.* Oxford University Press. 190 pp.

Swan, T. W. 1956. Economic Growth and Capital Accumulation. *Economic Record 32* (Nov): 334–361.

Acknowledgement:

I am thankful to Dr. Debasis Bandyopadhyay and Dr. Bryce Hool, Head, at the Department of Economics, the University of Auckland, New Zealand, for their instructions and severe help as supervisors for many years and also to Dr. Kazuo Mino and Juliet Tompkins (now at the Oxford University), together with the Graduate Center. This paper is an extension of my thesis at the University of Auckland and I stress here that this paper does not exist if I did not meet Debasis at the University of Auckland in earlier 1995. The difference between my thesis and this paper is that: (1) My thesis cultivates a whole version of my endogenous growth model, concentrates on the optimum CRC situation, and raises several new propositions compared with the related literature, using *delta** as a control parameter and without introducing *beta**. This paper uses *beta** as a clue and compares the transition from the current situation to the optimum CRC* situation as shown in Figure 1 "Classification rules for transitional paths: from DRC to CRC" on page 63. Empirical results under CRC* of this paper (that uses the root mean square error, RMSE) are the same as those in my thesis (that does not use the RMSE). This is because when *delta** is equal to *alpha* the actual growth rates are irrelevant to the growth rates under CRC*.

I am also thankful to my colleagues at the Hiroshima Shudo University, in particular, Dr. Susumu Tokimasa and Dr. Bob Austenfeld since my starting learning the production function. Also, Dr. Toshimi Fujimoto has helped me greatly to review all the equations in this paper for the last two years. I intend to develop my life-work as warm-hearted Debasis suggested me. Finally, I repeat the words, which I left in my Acknowledgement of my thesis:

The Southern Cross is a beautiful sight, especially in New Zealand. When viewing it there I couldn't help but think how fittingly it symbolized the results of my work which have come from the four points, north, east, south, and west, of that constellation; that is, Japan, the U. S., New Zealand, and the U. K. For me especially, it is a beautiful sight!

ST1 Aggregated amounts and ratios by country and year

(1) Aggregated amounts between 1983 and 1995 by country (in detail, see Supplementary data 7, 8, and 9)

	$L(0)$	$K(0)$	$S_{\Pi}(0)$	$D(0)$	$\Pi(0)$	$W(0)$	$Y(0)$	$S(0)$	$S_H(0)$
Japan 83-95	826462	11949490	133533	161924	295457	2820218	3115675	774566	641033
US 83-95	1451243	124974600	1417015	2945243	4362258	40649600	45011858	4470268	3053253
UK 83-95	331474	7629000	147227	259695	406922	3624390	4031312	404307	257080

(2) Ratios on average

	n	α	$\Omega(0)$	$r(0)$	$k(0)$	$y(0)$	s	s_{Π}	s_H
Japan 83-95	0.00809	0.09483	3.83528	0.02473	14.45861	3.76989	0.24860	0.45195	0.21496
US 83-95	0.02268	0.09691	2.77648	0.03491	86.11556	31.01607	0.09931	0.32484	0.07004
UK 83-95	0.00668	0.10094	1.89244	0.05334	23.01538	12.16177	0.10029	0.36181	0.06619

(2) Ratios on average (continued)

	$g^a{}_Y$	$g^a{}_K$	$g^a{}_y$	*beta*	θ_1	θ_2	γ	$s_{S\Pi/Y}$	$s_{SH/Y}$
Japan 83-95	0.04690	0.06164	0.03881	0.93592	0.80000	0.70000	0.87407	0.04286	0.20574
US 83-95	0.06316	0.04897	0.04048	0.92023	0.80000	0.70000	1.07180	0.03148	0.06783
UK 83-95	0.07407	0.08808	0.06739	0.85222	0.80000	0.70000	0.74091	0.03652	0.06377

SNA Japan from 1992 to 2000

(1) Agri. Amou	$L(0)$	$K(0)$	$S_{\Pi}(0)$	$D(0)$	$\Pi(0)$	$W(0)$	$Y(0)$	$S(0)$	$S_H(0)$
Japan 1992	66368	1052215	9655	5733	15388	258438	273827	62483	52828
Japan 1993	66640	1086129	4994	5368	10362	264484	274846	53661	48667
Japan 1994	66668	1106055	11516	5560	17076	269355	286431	50033	38517
Japan 1995	66728	1125221	13913	5782	19695	273809	293504	47869	33956
Japan 1996	67274	1157092	21870	5854	27724	278897	306621	51131	29261
Japan 1997	67705	1200789	22390	5701	28091	285198	313290	49143	26753
Japan 1998	67043	1196896	19441	5809	25250	281040	306290	39115	19674
Japan 1999	66642.2	1196858	25375	5750	31125	277233	308358	35687	10312
Japan 2000	66691	1209717	23585	7168	30753	280080	310833	*36253*	10960

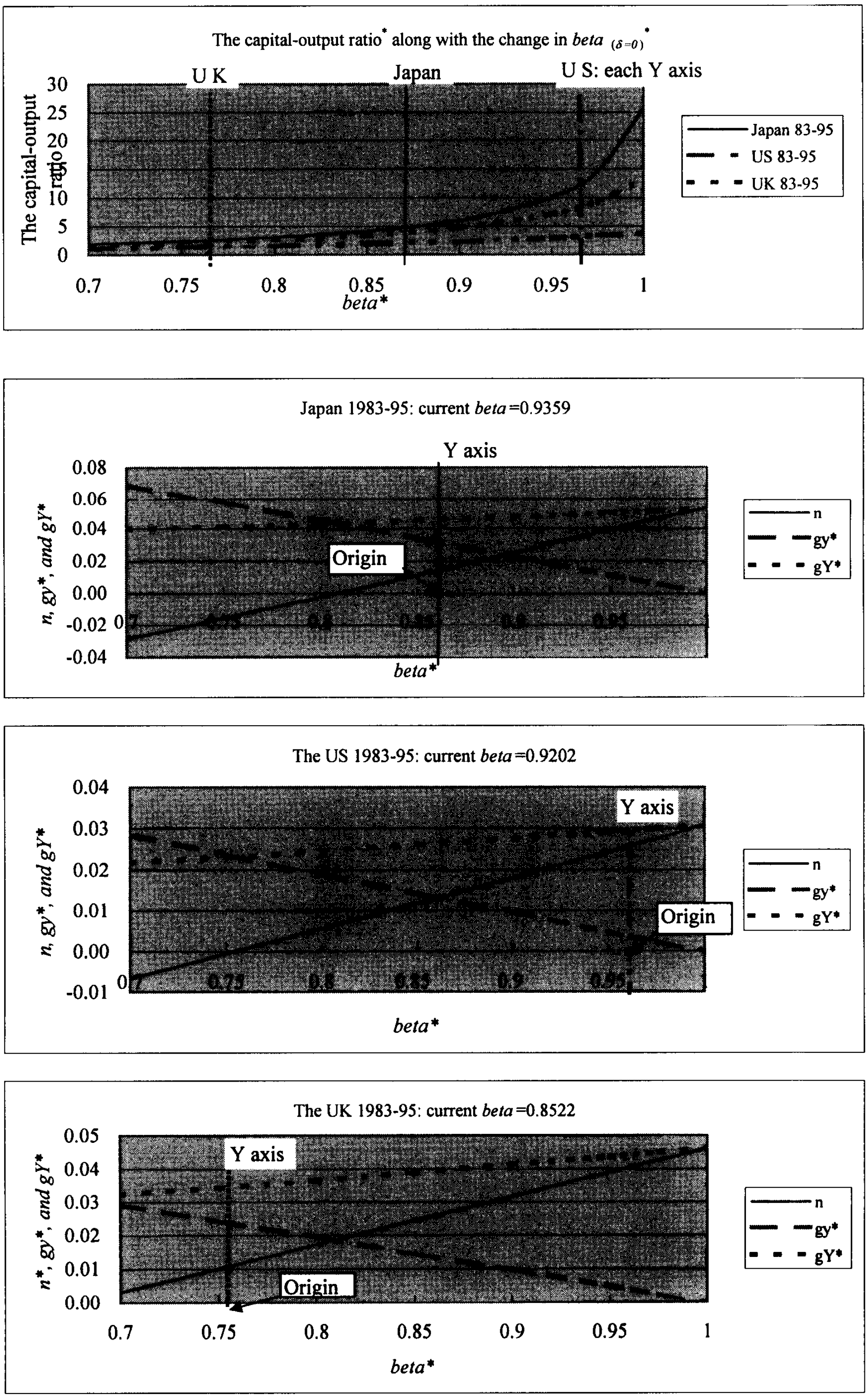
(2) Ratios	n	α	$\Omega(0)$	$r(0)$	$k(0)$	$y(0)$	s	s_{Π}	s_H
Japan 1992	0.00803	0.05620	3.84263	0.01462	15.85425	4.12589	0.22818	0.62742	0.19998
Japan 1993	0.00410	0.03770	3.95177	0.00954	16.29845	4.12435	0.19524	0.48195	0.18035
Japan 1994	0.00042	0.05962	3.86150	0.01544	16.59049	4.29638	0.17468	0.67441	0.14011
Japan 1995	0.00090	0.06710	3.83375	0.01750	16.86280	4.39852	0.16310	0.70641	0.12145
Japan 1996	0.00818	0.09042	3.77369	0.02396	17.19969	4.55779	0.16676	0.78884	0.10276
Japan 1997	0.00641	0.08967	3.83284	0.02339	17.73560	4.62728	0.15686	0.79704	0.09197
Japan 1998	-0.00978	0.08244	3.90772	0.02110	17.85266	4.56856	0.12771	0.76993	0.06859
Japan 1999	-0.00598	0.10094	3.88139	0.02601	17.95946	4.62707	0.11573	0.81526	0.03644
Japan 2000	0.00073	0.09894	3.89186	0.02542	18.13913	4.66079	*0.11663*	0.76692	0.04410

(2) Ratios (con	$g^a{}_Y$	$g^a{}_K$	$g^a{}_y$	*beta*	θ_1	θ_2	γ	$s_{S\Pi/Y}$	$s_{SH/Y}$
Japan 1992	0.01181	0.04820	0.00374	0.98054	0.80000	0.70000	0.86213	0.03526	0.19293
Japan 1993	0.00372	0.03223	-0.00037	0.98717	0.80000	0.70000	0.83972	0.01817	0.17707
Japan 1994	0.04215	0.01835	0.04171	0.87699	0.80000	0.70000	0.84549	0.04021	0.13447
Japan 1995	0.02469	0.01733	0.02377	0.90213	0.80000	0.70000	0.86266	0.04740	0.11569
Japan 1996	0.04469	0.02832	0.03621	0.91686	0.80000	0.70000	0.98425	0.07133	0.09543
Japan 1997	0.02175	0.03776	0.01525	0.93481	0.80000	0.70000	0.99047	0.07147	0.08539
Japan 1998	-0.02234	-0.00324	-0.01269	1.18174	0.80000	0.70000	0.79085	0.06347	0.06423
Japan 1999	0.00675	-0.00003	0.01281	0.93737	0.80000	0.70000	1.37036	0.08229	0.03344
Japan 2000	0.00803	0.01074	0.00729	0.94988	0.80000	0.70000	1.09276	0.07588	0.04076

Note:

1. An amount is shown by BN YEN for Japan, MN US$ for the US, and MN PS for the UK (as in Kamiryo, 2002/NIRA).
2. Each amount is principally based on the data of OECD (Main Aggregates of National Accounts).
3. For the Japanese data, I use Annual Report on National Accounts, ESRI, Cabinet Office, by year.
4. For the number of labour, I principally use employed persons, instead of population or employees.
5. I assume that net saving equals net capital investment, but statiscally, this assumption does not hold:
   Thus, I adjust indirectly the increase in net capital formation (ΔK: net investment) by adjusting saving-side data:
   Basically, net capital formation=net saving after capital formation - incease in inventories - (exports, E_X - imports, I_M).
   And, current external balance (E_X-I_M) = the differences of saving and net investment in private and public sectors.
   Instead of using E_X-I_M, I use the rest of the world as the sum of E_X-I_M and the net capital transfers etc., r. & p
6. Saving in 2000 for Japan (italic) was just adjusted by taking into consideration the statsitical discrepancies.
7. For brevity in this paper, I assume that banking costs parameter, θ_1, is 0.8 and manager's parameter, θ_2, is 0.7.

SF2 Growth structure with related transition dynamics by country



Note: n is a positive function of $beta^*$, and $i_A{}^*$ is a negative function of $beta^*$ under CRC.

ST2 The relatonhsip between *beta*, Ω, *n*, growth rates under CRC*

Under *delta*=0 and $k(t)^{\delta}=1$

Japan 83-95

n	α	*s*	*i*	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	*opt. beta**
0.00809	0.09483	0.24860	0.20745	0.1740	0.0335	3.8353	0.8387	0.8354

$beta^*_{(\delta=0)}$	Ω^*	*n*	g_y^*	g_Y^*	*n*	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
0.7	1.8761	-0.02891	0.0688	0.0399	-0.01	0.7719	0.0473	0.0523
0.75	2.3628	-0.01582	0.0573	0.0415	-0.005	0.7906	0.0434	0.0480
0.8	3.0565	-0.00245	0.0458	0.0434	0	0.8091	0.0396	0.0438
0.85	4.1252	0.01121	0.0344	0.0456	0.005	0.8274	0.0358	0.0396
0.9	5.9854	0.02519	0.0229	0.0481	0.01	0.8456	0.0320	0.0354
0.95	10.0337	0.03947	0.0115	0.0509	0.015	0.8637	0.0283	0.0312
0.975	14.5875	0.04674	0.0057	0.0525	0.02	0.8816	0.0246	0.0271
1	25.6439	0.05409	0.0000	0.0541	0.025	0.8993	0.0209	0.0231
0.8387	3.8353	0.00809	0.0370	0.0451	0.00809	0.8387	0.0335	0.0370

The US 83-95

n	α	*s*	*i*	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	*opt. beta**
0.02268	0.09691	0.09931	0.08575	0.0802	0.0055	2.7765	0.9359	0.9353

$beta^*_{(\delta=0)}$	Ω^*	*n*	g_y^*	g_Y^*	*n*	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
0.7	1.1585	-0.00668	0.0285	0.0218	-0.01	0.6726	0.0281	0.0311
0.75	1.3696	-0.00056	0.0237	0.0232	-0.005	0.7138	0.0245	0.0272
0.8	1.6294	0.00561	0.0190	0.0246	0	0.7546	0.0210	0.0233
0.85	1.9569	0.01184	0.0142	0.0261	0.005	0.7951	0.0176	0.0195
0.9	2.3827	0.01813	0.0095	0.0276	0.01	0.8353	0.0141	0.0156
0.95	2.9585	0.02448	0.0047	0.0292	0.015	0.8752	0.0107	0.0119
0.975	3.3300	0.02767	0.0024	0.0300	0.02	0.9148	0.0073	0.0081
1	3.7809	0.03088	0.0000	0.0309	0.025	0.9541	0.0039	0.0044
0.9359	2.7765	0.02268	0.0061	0.0288	0.02268	0.9359	0.0055	0.0061

The UK 83-95

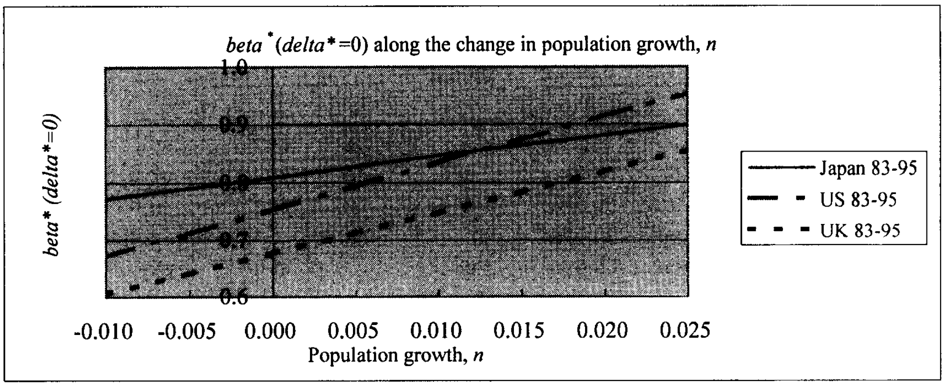
n	α	*s*	*i*	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	*opt. beta**
0.00668	0.10094	0.10029	0.08754	0.0635	0.0240	1.8924	0.7256	0.7238

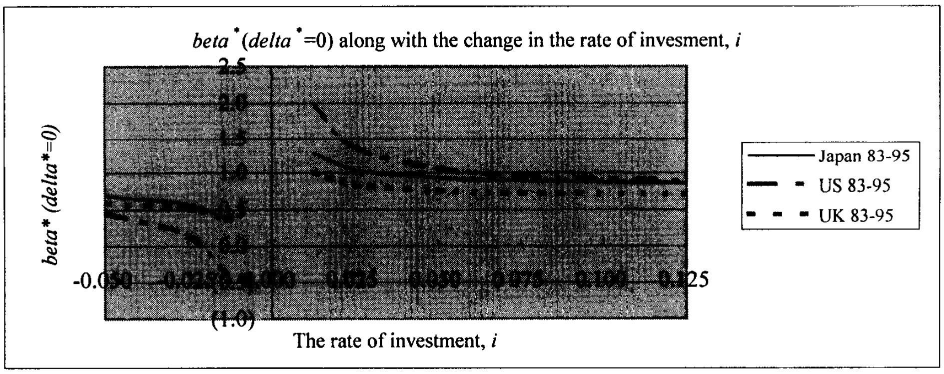
$beta^*_{(\delta=0)}$	Ω^*	*n*	g_y^*	g_Y^*	*n*	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
0.7	1.6983	0.00308	0.0292	0.0323	-0.01	0.6056	0.0345	0.0384
0.75	2.1057	0.01011	0.0243	0.0344	-0.005	0.6419	0.0313	0.0349
0.8	2.6649	0.01720	0.0195	0.0367	0	0.6779	0.0282	0.0314
0.85	3.4806	0.02436	0.0146	0.0390	0.005	0.7137	0.0251	0.0279
0.9	4.7815	0.03159	0.0097	0.0413	0.01	0.7493	0.0219	0.0244
0.95	7.1838	0.03889	0.0049	0.0438	0.015	0.7846	0.0189	0.0210
0.975	9.3526	0.04256	0.0024	0.0450	0.02	0.8196	0.0158	0.0176
1	13.1135	0.04626	0.0000	0.0463	0.025	0.8545	0.0127	0.0142
0.7256	1.8924	0.00668	0.0267	0.0334	0.00668	0.7256	0.0240	0.0267

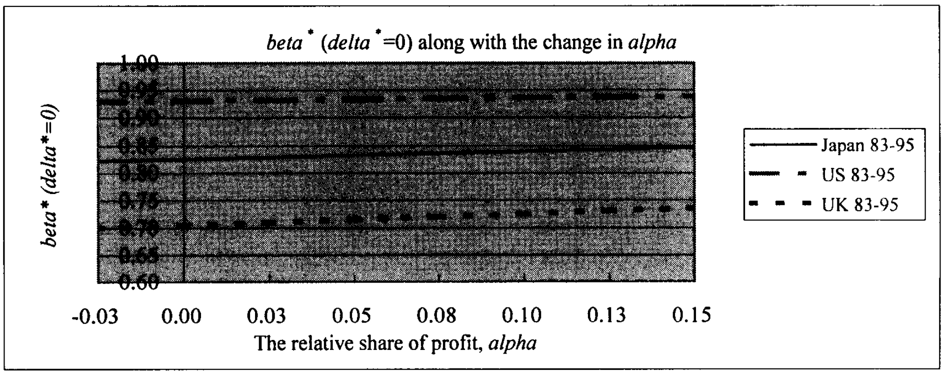
Notes:

1. Equations: $\Omega=(\beta\, i(1-\alpha))/(i(1-\beta)(1+n)+n(1-\alpha))$. $n=(\beta\, i(1-\alpha)-\Omega i(1-\beta))/(\Omega(i(1-b)+(1-\alpha)))$.
   $\beta=(\Omega(n(1-\alpha)+i\,(1+n))/(i(1-\alpha)+\Omega\, i\,(1+n))$.
2. The higher the i, the higher Ω^*, where i differs by θ_I *as* banking cost parameter.
3. The higher the n and Ω^*, the higher the $beta^*$ under CRC.
4. The higher the n, signifcantly lower the g_y^*, but gradually higher the g_Y^* under CRC.

SF3 Changes in population growth, the rate of investment, and *alpha*, to *beta**







Notes:

1. Based on Eq. 23 nad 24 under the optimum CRCsituation and using *delta** =0.
2. With no banking costs, the rate of saving equals the rate of investment: $s=i$.
3. Population growth and the rate of investment influence *beta** and, accordingly, growth rates*.
4. The relative share of profit, *alpha*, does not influence *beta**, in contrast to n and i.
5. Each country has its own growth structure, depending on the initial parameters.

ST3 The changes in *alpha* and *i* in growth structure: simulation by country

Japan 83-95

n	α	s	i	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	opt. beta*
0.00809	0.09483	0.24860	0.20745	0.1740	0.0335	3.8353	0.8387	0.8354

α	$beta^*_{(\delta=0)}$	i_A^*	g_y^*	g_Y^*	i	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
-0.025	0.8218	0.0370	0.0361	0.0442	-0.050	0.6926	-0.0154	-0.0170
0	0.8252	0.0363	0.0363	0.0443	-0.025	0.5749	-0.0106	-0.0117
0.025	0.8287	0.0355	0.0364	0.0445	-0.0125	0.3394	-0.0083	-0.0091
0.05	0.8323	0.0348	0.0366	0.0447	0	#DIV/0!	#DIV/0!	#DIV/0!
0.075	0.8358	0.0341	0.0368	0.0449	0.0125	1.2812	-0.0035	-0.0039
0.1	0.8394	0.0333	0.0370	0.0451	0.025	1.0457	-0.0011	-0.0013
0.125	0.8431	0.0326	0.0372	0.0453	0.05	0.9280	0.0036	0.0040
0.15	0.8467	0.0318	0.0374	0.0455	0.075	0.8888	0.0083	0.0092
0.09483	0.8387	0.0335	0.0370	0.0451	0.125	0.8574	0.0178	0.0197

The US 83-95

n	α	s	i	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	opt. beta*
0.02268	0.09691	0.09931	0.08575	0.0802	0.0055	2.7765	0.9359	0.9353

α	$beta^*_{(\delta=0)}$	i_A^*	g_y^*	g_Y^*	i	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
-0.025	0.9295	0.0060	0.0059	0.0286	-0.050	0.4548	-0.0273	-0.0302
0	0.9308	0.0059	0.0059	0.0286	-0.025	0.1509	-0.0212	-0.0235
0.025	0.9321	0.0058	0.0060	0.0287	-0.0125	-0.4568	-0.0182	-0.0202
0.05	0.9334	0.0057	0.0060	0.0287	0	#DIV/0!	#DIV/0!	#DIV/0!
0.075	0.9347	0.0056	0.0061	0.0287	0.0125	1.9742	-0.0122	-0.0135
0.1	0.9361	0.0055	0.0061	0.0288	0.025	1.3665	-0.0092	-0.0101
0.125	0.9374	0.0054	0.0061	0.0288	0.05	1.0626	-0.0031	-0.0035
0.15	0.9388	0.0052	0.0062	0.0289	0.075	0.9613	0.0029	0.0032
0.09691	0.9359	0.0055	0.0061	0.0288	0.125	0.8803	0.0150	0.0166

The UK 83-95

n	α	s	i	i_K^*	i_A^*	$\Omega(0)$	$beta^*_{(\delta=0)}$	opt. beta*
0.00668	0.10094	0.10029	0.08754	0.0635	0.0240	1.8924	0.7256	0.7238
			0.0875					

α	$beta^*_{(\delta=0)}$	i_A^*	g_y^*	g_Y^*	i	$beta^*_{(\delta=0)}$	i_A^*	g_y^*
-0.025	0.7007	0.0262	0.0256	0.0322	-0.050	0.5984	-0.0201	-0.0223
0	0.7054	0.0258	0.0258	0.0325	-0.025	0.5174	-0.0121	-0.0134
0.025	0.7103	0.0254	0.0260	0.0327	-0.0125	0.3554	-0.0081	-0.0090
0.05	0.7153	0.0249	0.0262	0.0329	0	#DIV/0!	#DIV/0!	#DIV/0!
0.075	0.7203	0.0245	0.0265	0.0331	0.0125	1.0034	0.0000	0.0000
0.1	0.7255	0.0240	0.0267	0.0334	0.025	0.8414	0.0040	0.0044
0.125	0.7307	0.0236	0.0269	0.0336	0.05	0.7604	0.0120	0.0133
0.15	0.7360	0.0231	0.0272	0.0339	0.075	0.7334	0.0200	0.0222
0.10094	0.7256	0.0240	0.0267	0.0334	0.125	0.7118	0.0360	0.0401