# Loci of Transitional Paths from Current to Optimum CRC Situation: Extended Equations with Empirical Results

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## 1. Introduction

This paper intends to extend the equations under convergence in Kamiryo [2004a], and revise the processes in transitional paths together with its characters, showing empirical results using IFSY, IMF. A transitional path starts with decreasing returns to capital (DRC) and normally converges to constant returns to capital (CRC). When an endogenous growth model uses the Cobb-Douglas production function, transitional paths will accept constant returns to scale. In this case, is it possible for a transitional path to have the current situation at increasing returns to capital (IRC)? This paper will give a basic answer to the existence of IRC under constant returns to scale.

Why did my past papers fail in proving the existence of IRC with definite conditions/restrictions?

Kamiryo [2004a] started with Kamiryo [2003], and used *beta* for the current situation and *beta** under convergence (hereunder, *beta* and *beta**), to integrate the two parameters for structural reform in Kamiryo [2003], together with *delta* for the current situation and *delta** under convergence (hereunder, *delta* and *delta**). How are these four parameters (*beta* & *delta* and *beta** & *delta**) formulated each by equation? This has been a serious problem. Kamiryo [2003] calibrated these corresponding parameters directly by recursive programming that used the calculus of variations.[1] Kamiryo [2004a] succeeded in formulating equations under convergence using several initial/current values and ratios available in national accounts data. These equations include *beta**, but not yet each equation for *beta*, *delta*, and *delta**. This paper finalizes each equation for *beta*, *delta*, and *delta**, where *delta* and *delta** guarantee

1) More exactly, I used four financial parameters, $theta_1$ between saving and net investment, $theta_2$ for corporate saving, *gamma* for household saving, and *delta* for neutralizing diminishing returns. Soon later, to express the whole situation, I integrated $theta_2$ and *gamma* into *beta* as a weighted average.

each convergence at CRC. As a result, transitional paths will be shown more definitely and at the same time, the above question for the conditions of IRC will be answered.

Before formulating each of the above parameters in this paper, I will here briefly confirm the relationship between *beta* & *delta* and *beta*$^*$ & *delta*$^*$. Both *beta* and *beta*$^*$ are determined independently from *delta* and *delta*$^*$. Both *beta* & *beta*$^*$ and *delta* & *delta*$^*$ are, however, determined using the same initial parameters, the rate of saving, *s*, and the rate of net investment, *i,* the relative share of returns/rental, *alpha*, the growth rate of population/ employed persons, *n*, the capital-labor ratio, $k(0)$, and the capital-output ratio, $\Omega(0)$ or the rate of rental, $r(0)$. Under any convergence, *delta*$^*$ = *alpha*, holds but this relationship remains one of sufficient conditions, and for an economy to converge optimally, we need another *delta* (which I call the current *delta*, $\delta_{CURRENT}$, but I abbreviate it as δ), with a necessary condition that the initial capital-output ratio equals that under convergence.

In a transitional path, *beta* approaches *beta*$^*$ gradually over time and at the same time, *delta* approaches *alpha* gradually over time: Note that, at time $t = 0$, *delta* = 0 holds with diminishing returns (before being influenced by a neutralizing factor, *delta*). The whole length of time, from $t = 0$ to convergence-time, is measured by the speed of convergence. A coefficient for the speed of convergence is $(\delta - \alpha)n$, as Kamiryo and Fujimoto [2005a] showed in Eq. 33, by using the same approach as Barro Robert J., and Xavier Sala-i-Martin [1995] took. In my endogenous case, it will take much more time for an economy to converge than the time shown by Barro and Sala-i-Martin [1995] in their exogenous case. It is possible, however, for a transitional path to shorten the whole time by a device that replaces *beta* by *beta*$^*$ and *delta* by *alpha* suddenly at $t = 100$ or $t = 200$. In spite of this device, it will take some more time (20 to 30 years) for the economy to converge.

Kamiryo and Fujimoto [2005a] also showed Eq. 47 in Appendix for the value of *delta*, without interpreting that this *delta* should be used only at $t = 1$.[2)] In other words, we could not justify Eq. 33 and Eq. 47 by using the last recursive programming. I have calibrated *delta* using the last recursive programming, separately for the current situation and for the convergence situation. This paper reviewed the last recursive programming and renewed the contents of recursive programming so that the locus of a transitional path gradually reaches convergence over time as above. As a result, I finally confirmed and proved that a transitional path of an economy converged from the initial situation to the optimum CRC

2) This lack of understanding is traced back to Eq.19 in Kamiryo [2004a]), which holds under convergence (also, see note 7).

situation, by using both equations and a new recursive programming.

In more detail, for IRC, Kamiryo [2004a, p. 62] "temporarily" showed the difference between IRC, CRC, and DRC by using Eq. 20 for $m = k(0)^{\delta *}$. This is because I could not finalize the value of the current *delta* that showed the level of diminishing returns inherently expressed in the Cobb-Douglas production function. Kamiryo and Fujimoto [2005a] formulated the value of *delta* as Eq. 47 in Appendix. This paper, by extending equations expressed in Kamiryo [2004a], differently formulated the same result as Eq.47 in Kamiryo and Fujimoto [2005a]. Kamiryo and Fujimoto [2005a] aimed at the measurement of the speed of convergence, using the same approach as Barro and Sala-i-Martin [1995] took, and clarifying the characteristics and elasticity of related parameters. Kamiryo [2004a] aimed at formulating the equations under convergence using per capital values, but now I confirm that both approaches are consistent with each other in every respect.

Finally, this paper cooperates with Kamiryo [2005b][3], which explores a utility function of consumption, formulated from basic ideas of Frank Plumpton Ramsey [1928] and Jan Tinbergen [1956]. For equations under convergence, the relative share of rental is one of sensitive factors. The returns/rental for capital, which I need for equations under convergence is set maximum in production by using the rate of rental, *r*, and the consumption for consumers is set optimum in national disposable income by using the discount rate, *rho*. A utility function that uses (*rho*/*r*) will directly synthesize wages in production with consumption in national disposable income. Kamiryo [2005b] answers this problem to some extent and, for empirical show-up in this paper I use the above function and empirical results in the total economy (without discussing the aggregation problem in the private sector).

## 2. Extended equations under convergence

### 2.1 *Review of delta and alpha under convergence*

As a preliminary discussion, this section clarifies the relationship between *delta* and *alpha* in Kamiryo [2004a] and those in this paper. In my model, net investment is divided into net investment in quantity and net investment in quality, $I_K(t)$ and $I_A(t)$. These are accumulated in each stock, capital, $K(t)$, and the level of technology, $A(t)$. The relationship between each flow and stock is well shown by using per capita capital to output, $i_K(t)$, and per

3) I am much obliged to the advice of Dr. Toshimi Fujimoto for the understanding of the utility function using the calculus of variations.

capita technology to output, $i_A(t)$. These are expressed using *beta*: $i_K(t) = i(t)\cdot beta$ and $i_A(t) = i(t)(1-beta)$, where $i(t) = i\cdot y(t)$. In Kamiryo [2004a], equations under convergence expressed that $delta^*$ is zero under convergence. The purpose of this designation was just to show the origin at $beta^* = 0$ of the X axis and $delta^* = 0$ of the Y axis: in other words, the endogenous rate of technological progress under convergence, $g_A^* = i_A$ holds at the origin. Figure 1 in Kamiryo [2004a], stressed the convergence at the origin.

In this paper, however, I will correctly express the above designation: by showing that $delta^*$ equals *alpha* under convergence (still at the origin). This designation, $\delta^* = \alpha$, is justified by taking into consideration the $k(t)^\alpha$ in $y(t) = A(t)k(t)^\alpha$ into $i_A(t)/k(t)^\delta$, where for the capital-labor ratio, $k(t)^{\alpha-\delta}$ holds and for *delta*, $\delta^* - \alpha = 0$ holds at the origin.

2.2 *Equations of delta and alpha under convergence*

This section formulates *beta* and *delta* at the current situation and $beta^*$ and $delta^*$ under convergence, step by step.

**Equations of *beta* and *beta**:**

The value of *beta* at the current situation has been calibrated using recursive programming up to this paper. However, I find in this paper that the equation of *beta* is obtained by devising the equation of $beta^*$ in Kamiryo [Eq. 21, 2004a].

The equation of $beta^*$ was first formulated as Eq. 22 in Kamiryo [2004a]:

$$\beta^*_{\delta=\alpha} = \frac{\Omega^*\left(n(1-\alpha) + i\,(1+n)\right)}{i(1-\alpha) + \Omega^*\cdot i(1+n)}, \tag{1}$$

where $\beta^*_{\delta=\alpha}$ is now used instead of $\beta^*_{\delta=0}$, as I stated above.

Eq. 22 was derived from Eq. 21 in Kamiryo [2004a] as,

$$\beta^*_{\delta\neq 0} = \frac{\Omega^*\left(n(1-\alpha)k(0)^{\delta*} + i\,(1+n)\right)}{i(1-\alpha)k(0)^{\delta*} + \Omega^*\cdot i(1+n)},$$

where if $delta^* = 0$, $k(t)\delta^*$ reduces to 1.0. Or, similarly,

$$\beta^*_{\delta\neq\alpha} = \frac{\Omega^*\left(n(1-\alpha)k(0)^{\delta*-\alpha} + i\,(1+n)\right)}{i(1-\alpha)k(0)^{\delta*-\alpha} + \Omega^*\cdot i(1+n)}, \tag{2}$$

where if $delta^* - alpha = 0$, $k(t)^{\delta*-a}$ reduces to 1.0 as in this paper.[4)]

Eq. 2 reduces to the above equation 1 when $delta^* = alpha$. Eq. 2 is formulated by

4) Note that $delta^* - alpha^*$ (not "$alpha^* - delta^*$") is used since the denominator of the ratio of net investment in quality (for technological progress) is here shown as $k(t)^\wedge delta$ (for convenience, avoiding the use of minus sign).

assuming that the growth rate of per capita capital expressed by using $i_K(t)$ is equal to that expressed by using $i_A(t)$ (see Eqs. 12 and 17 in Kamiryo [2004a]).

Using Eq. 2, the equation of *beta* at the current situation[5)] is obtained by setting $delta^* = 0$:

$$\beta = \frac{\Omega^* \left( n(1-\alpha)k(0)^{0-\alpha} + i\,(1+n) \right)}{i(1-\alpha)k(0)^{0-\alpha} + \Omega^* \cdot i(1+n)}, \tag{3}$$

where $delta^* = 0$ implies that diminishing returns inherent in the Cobb-Douglas production function holds or the neutralizing factor does not work at the beginning, $t = 0$. For the proof of Eq. 3, I renewed the last recursive programming which I had used since Kamiryo [2002]. I confirm that Eq. 3 is consistent with my new recursive programming in this paper, whose figures I will show in 3.3 below.

**Equation determining the current *delta*:**

If *delta*<*alpha* at the current situation holds, increasing returns to capital (IRC) is shown. This is a crucial issue since constant returns to scale prevails when the Cobb-Douglas production function is used as in my model. How does this case happen and how is it justified under constant returns to scale?

The parameter that neutralizes diminishing returns, *delta*, manages how the current situation can reach convergence smoothly under soft-landing. The value of *delta* at the starting point ($t = 0$) is zero as I stated above. The value of $delta^*$ at the convergence point of time must be equal to *alpha* as shown in Eq. 2. Then, what is the value of *delta* at $t = 1$? I call this *delta* the current *delta* or for simplicity *delta*. In a transitional path, the current *delta* smoothly approaches *alpha* over time.

Kamiryo and Fujimoto [2005a] already showed this *delta* in Appendix, but without finalizing the equation of the current *beta* (see the above Eq. 3). And, the current *delta* was determined without interrupted by the value of *beta*.

Before a final measurement of the current *delta*, I confirm here that the current *delta* is derived by using the same initial parameters and equations used for the $delta^*$ under optimum convergence and also that $delta^* = alpha$ is required for convergence. However, there are numerous cases of convergence under $delta^* = alpha$ and this condition, $delta^* = alpha$, remains one of sufficient conditions. The necessary condition for optimum convergence is that the initial capital-output ratio equals the capital-output ratio under convergence: $\Omega(0) = \Omega^*$. This

5) I have used a few equations in the past, insisting on the use of the actual/current rate of technological progress, but in these cases the values of *beta* by country sometimes show negative. This is inappropriate for my model as a whole, as advised by Michinori Sakaguchi.

is because if $\Omega(0) > \Omega^*$ is allowed the lower the $\Omega^*$ the higher the rate of rental, but this treatment endlessly loses a base required for convergence, where the growth rate of output equals the growth rate of capital.

Then again, why is the current *delta* different from *delta** while using the same initial parameters? This is because (1) I defined *delta* as a parameter for neutralizing diminishing returns and this *delta* starts at time, $t = 1$ and (2) unless the current *delta* becomes equal to *alpha*, convergence is not guaranteed, as proved by comparing equations at the current situation with those under convergence. Under these considerations, the speed of convergence becomes measurable.

If I set *delta* = the current *delta* at $t = 1$ and if this *delta* reduces to *alpha* by spending the whole length of time, whose measurement is done by the speed of convergence,[6)] then this operation is perfect in recursive programming, where the growth rate of (per capita) output becomes completely equal to the growth rate of (per capita) capital. However, a perfect convergence takes much time since its character obeys a hyperbolic curve. For showing transitional paths by graphs, we can arbitrarily shorten the number of periods/years for convergence by shifting the *delta* (very slowly approaching *alpha*) with *alpha*, suddenly in 100 to 200 years from the current situation (see Figure 3). Note in this device that the growth rate of (per capita) output slightly differs from the growth rate of (per capita) capital at a shorten convergence-years.

For formulating the current *delta*, there are two approaches: first, using the current values and second, using values under convergence. First, by using the above Eq.3 for *beta* and the actual rate of technological progress, $g_{A(1)} = (A(1) - A(0)) / A(0),$[7)]

$$\delta = \frac{LN((i\,(1-\beta)/g_{A(1)})}{LNk(0)} + \alpha \text{ is shown.} \tag{4-1}$$

Second, $\delta = \dfrac{n + \alpha(g_A^* - n)}{g_A^*} = \dfrac{n + \alpha(i\,(1-\beta^*_{\alpha=\delta}) - n)}{i\,(1-\beta^*_{\alpha=\delta})}$. (4-2)

The *delta* in the above Eq. 4-1 is equal to that in Eq. 4-2 (see Eq. 46 in Kamiryo and Fujimoto [2005a]). In Kamiryo and Fujimoto [2005a], however, I did not perceive that Eq. 46 was what I was looking for as the current *delta*. The above Eq. 4-2 uses all the initial

6) Note that for optimum convergence, the current *beta* must reduce to *beta** similarly during the whole length of time.

7) Here I cannot use $g_{A(1)} = i_A \cdot k(0)^\alpha$ nor $g_{A(1)} = g_{Y(1)} - \alpha \cdot g_{K(1)} - (1-\alpha)n$. Set $m = k(0)^\delta$ like Eq.20 in Kamiryo [2004a], $\delta = LN(m)/LN(k(0))$.

parameters and thus, is justified for the use at the current situation.

Finally, a coefficient for the speed of convergence is shown as $(\delta - \alpha)n$, which was formulated as Eq. 33 of Kamiryo and Fujimoto [2005a]. This equation is based on the methodology taken by Barro Robert J., and Xavier Sala-i-Martin [pp. 36–38, 1995] for exogenous growth. The number of periods/years for convergence, however, differs from their approach that takes "one-half" of the difference between the initial growth rate and that at convergence. I prefer the use of the whole years that realize a perfect convergence to one-half years:

$$1/(\delta - \alpha)n. \tag{5}$$

The convergence years are simply the inverse number of Eq. 4. For the perfect convergence, it takes much more time, but $g_Y^* = g_K^*$, $g_y^* = g_k^*$, and $g_A^* = g_y^*/(1-\alpha)$ are completely proved. In this proof, there is no difference between endogenous and exogenous convergence in Solow [1956].

The relationship among decreasing, constant, and increasing returns to capital (DRC, CRC, and IRC) is shown using *delta* and *alpha* as follows:

1. If $(\delta - \alpha) > 0$, DRC holds, whose speed of convergence is faster when *delta* is higher.
2. If $(\delta - \alpha) = 0$, CRC holds, whose speed of convergence is impossible.
3. If $(\delta - \alpha) < 0$, IRC holds, whose speed of convergence is slower when *delta* is closer to *alpha* (see the next section below). (6)

In recursive programming, apart from the speed of convergence, I can shorten the whole years for convergence by replacing *delta* by *alpha* at any time/year (note that it takes still more years to converge) although the above perfect convergence does not hold.

### 2.3 *Revised transitional paths by quadrant*

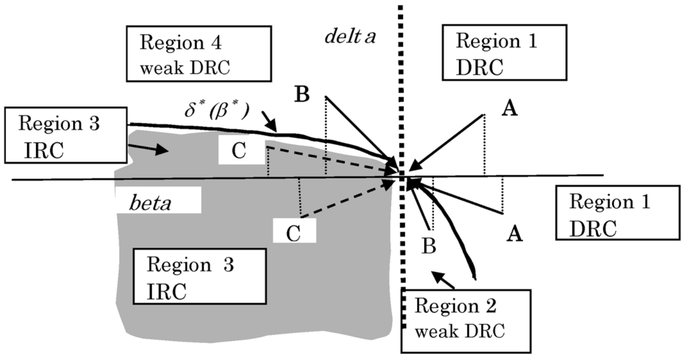
Now, after taking into consideration the above discussion and newly formulated equations, I will revise Figure 1, and show Figure 2.

In Figure 1, the current value of *delta* was not found so that temporarily a concept of $m = k(0)^{\delta*}$ was used instead of the current *delta*, where if $m > 1$ DRC prevails, if $m = 1$ CRC prevails, and if $m < 1$ IRC prevails. My research question is whether IRC theoretically and empirically exists or not. For this question, I extend the above Eq. 4 differently:

$$\frac{\delta}{\alpha} = \frac{n + \alpha \cdot g_A^* - \alpha \cdot n}{\alpha \cdot g_A^*} = 1 + \left(\frac{1-\alpha}{\alpha}\right)\frac{n}{g_A^*}. \tag{7}$$

Similarly to Eq. 6,

1. If $\delta/\alpha > 1$, the situation is under DRC.




Notes: Explanation of classification rules

| | Region 1 | Region 2 | Region 3 | Region 4 |
|---|---|---|---|---|
| $\delta^*_{\beta^*}$ - $\delta$ | minus | plus | plus | minus |
| $\beta^*_{(\delta=0)}$-$\beta$ | minus | minus | plus | plus |
| $k(0)$^$\delta$*<1 | 1: sDRC | 2: sDRC | | 4: sDRC |
| $k(0)$^$\delta$*>1 | 1: wDRC | 2: wDRC | 3: IRC | 4: wDRC |

1. $k(0)$^$\delta$*<1 shows that the current DRC situation is strong.
   So that, "strong" can be added to the front of DRC: sDRC.
2. $k(0)$^$\delta$*>1 shows that the current DRC situation is weak.
   So that, "weak" is added to the front of DRC: weak DRC.
3. The current situation cannot identify IRC due to CRS.
4. $k(0)$^$\delta$* works for attaining CRC by balancing productivity enhancement and DRC.

**Figure 1 Classification rules for transitional paths from DRC to CRC: in Kamiryo [2004a]**

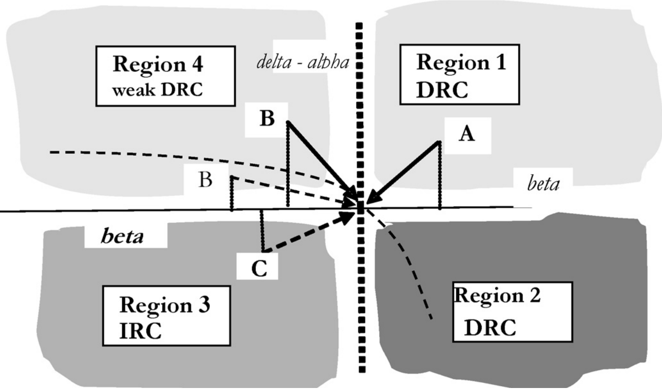



**Figure 2 Revised transitional paths by quadrant**

2. If $\delta/\alpha = 1$, the situation is under a weak CRC (still partly if *beta*>*beta*$^*$) and the condition of $0 = \left(\frac{1-\alpha}{\alpha}\right)\frac{n}{g_A^*}$ will hold. Therefore, the condition is $n = 0$ (I find this situation in Sweden using IMF data).

3. If $\delta/\alpha = 0$, what is this condition? If $\delta/\alpha = 0$, Eq. 7 will have a negative growth rate of population since $-1 = \left(\dfrac{1-\alpha}{\alpha}\right)\dfrac{n}{g_A^*}$ or $\left(\dfrac{\alpha}{1-\alpha}\right) = -\dfrac{n}{g_A^*}$.

4. $0 < \delta/\alpha < 1$, the situation is under a weak IRC (still partly if *beta* > *beta*$^*$), where the growth rate of population must be negative. This condition holds under constant returns to scale. I find this situation in Russia and Italy using IMF data.

The current *delta* cannot below zero due to the use of natural logarithms. And I find this assumption is true as the above relationship between *delta* and the growth rate of population, *n*, shows. Even if I replace *n* by the growth rate of employed persons (using OECD data), the above is true. Therefore, I raise a proposition.

**Proposition 1:** Increasing returns to capital (IRC) occurs only if the growth rate of population[8] is negative under constant returns to capital.

I will confirm this proposition in the next section by showing *delta* with three dimensional figures.

Finally, I replace Figure 1 with Figure 2. Instead of "bold" hyperbolic curve, I use the "dotted" curve (since this curve is independent of the division of the four areas)[9] and add the following explanations.

1. **Area 1 in the first quadrant:** The current situation of any country for the total economy shows almost always this area under DRC.
2. **Area 2 in the second quadrant:** The current situation of the government sector in some countries shows this area under DRC but partly with IRC. This happens in the process to recover from extreme budget deficit. This implies that it is usually difficult for an economy to have *delta* < *alpha* under *beta* > *beta*$^*$.
3. **Area 3 in the third quadrant:** The current situation of the total economy in a few

8) It is possible to replace population by employed persons by reviewing corresponding assumptions in the literature. I will discuss this issue together with the unemployment rate in the future.

9) In Kamiryo [2004a], the hyperbolic curve derived from Eq. 19, $k(0)^{\delta *} = \dfrac{\Omega^* \cdot i\,(1-\beta^*)(1+n)}{(1-\alpha)(\beta^* \cdot i - \Omega^* \cdot n)}$, was wrongly used for the division of four areas that indicate plausible points at the current situation. The value of *beta*$^*$ only exists at *delta*$^*$ = *alpha*. The hyperbolic curve of *delta*$^*$ to *beta*$^*$, neglecting a close relationship between $\Omega$ and *beta*, may exist but it is independent of the division of four areas, which I correct in this paper. Nevertheless, the four initial parameters in the above Eq. 19 ($i$, $n$, $\alpha$, and $\Omega(0) = \Omega^*$) are used commonly for the current and convergence situations. Then, a corresponding curve, $\delta^*(\beta^*)$, may suggest a tight relationship between *beta*$^*$ and *delta*$^*$, but with no relationship to the definition of areas.

countries shows this area under IRC. The IRC situation, however, happens only when the growth rate of population is negative, as proved by the above Eq. 7.

4. **Area 4 in the fourth quadrant:** The current situation in some countries shows this area under DRC but partly with IRC. It may be easier for a sector of an economy to have *beta*<*beta*$^*$ rather than to have *delta*<*alpha*.

The value of *beta* is much more related to investment in quality than *delta*, and *delta* much more related to education and R&D than *beta*. The current situation shows the balance between *beta* and *delta*. I will confirm the above arrangements for transitional paths by dividing the total economy with the government and private sectors in my next paper.

## 3. Characteristics of *delta* and variables: with three dimensional figures

### 3.1 *The characteristics of the current delta*

This section clarifies the characteristics of the current *delta* by showing three dimensional graphs, in particular, with respect to the sign of the growth rate of population (see tables and figures in Appendix).

Let me summarize the characteristics of the current *delta* expressed by Eq. 4, $\delta = \left(n + \alpha(i\,(1-\beta^*_{\alpha=\delta}) - n)\right) / (i\,(1-\beta^*_{\alpha=\delta}))$ and the speed years of convergence expressed by Eq. 5, $1/(\delta-\alpha)n$. When these equations are each expressed using the three dimensional graphs by case, the characteristics of the current *delta* is wholly grasped. For the X axis, I will take the growth rate of population, *n*, and for the Y axis I will take *beta*$^*$, then the shape of the area of *delta* and the coefficient or years for the speed of convergence are each differently shown by the Z axis. A basic calculation is shown in Table A1 using four cases, where Case 1 for *alpha* = 0.1 and *i* = 0.1, Case 2 for *alpha* = 0.1 and *i* = 0.15, Case 3 for *alpha* = 0.1 and *i* = 0.025, and Case 4 for *alpha* = 0.2 and *i* = 0.1. At once, we recognize unique characteristics discriminated by the growth rate of population, *n*, and *beta*$^*$. Both *n* and *beta*$^*$ adversely differentiate the shape of *delta* and the speed of convergence. Both *n* and *beta*$^*$ play important roles.

These characteristics are more clearly shown (see Figure A-2) when each figure is separated by *n* above zero (sigh, plus) and below zero (sigh, minus) and by *beta*$^*$ below 1.0 and above 1.0. In particular, the shape of *delta* under *beta*$^*$ < 1.0 shows why increasing returns to capital (IRC of Area 3 in Figure 2) occurs with a minus *delta* and a minus *n*, resulting in a plus coefficient for the speed of convergence.

3.2 *The characteristics of variables*

This section clarifies the characteristics of such variables as the capital-output ratio, $\Omega^*$, the rate of rental, $r^*$, the growth rate of output, $g_Y^*$, and the growth rate of per capita output, $g_y^*$, each under optimum convergence, where the rate of saving equals the relative share of rental.

Before starting, I will conclusively show related equations (for detail, see Kamiryo [2004a] and Kamiryo [2005b]).

$$\Omega^* = \frac{\beta^*_{\delta=0} \cdot i\ (1-\alpha)}{i\,(1-\beta^*_{\delta=0})(1+n)+n(1-\alpha)}. \tag{8}$$

$$r^* \equiv \frac{\alpha}{\Omega^*} = \alpha\left(\frac{i\,(1-\beta^*_{\delta=\alpha})(1+n)+n(1-\alpha)}{\beta^*_{\delta=\alpha}\cdot i\,(1-\alpha)}\right). \quad \text{Or}$$

$$\frac{r^*}{\alpha} = \frac{i\,(1-\beta^*_{\delta=\alpha})(1+n)}{\beta^*_{\delta=\alpha}\cdot i\,(1-\alpha)} + \frac{n}{\beta^*_{\delta=\alpha}\cdot i}. \tag{9}$$

Since $g_A^* = i\,(1-\beta^*_{\delta=\alpha})$, where $g_A(t) = i_A \cdot k(t)^{\alpha-\delta}$ and $\alpha = \delta$, the growth rate of output is formulated by inserting $g_A^* = i\,(1-\beta^*_{\delta=\alpha})$ into $g_Y^*$:

$$g_Y^* = \frac{g_A^*(1+n)}{1-\alpha} + n = \frac{i\,(1-\beta^*_{\delta=\alpha})(1+n)}{1-\alpha} + n. \tag{10}$$

The relationship between the rate of rental and the growth rate of output under convergence is now derived by using $A = \dfrac{i\,(1-\beta^*_{\delta=\alpha})(1+n)}{1-\alpha}$ for the above $r^*$ in Eq. 9 and $B = \dfrac{i\,(1-\beta^*_{\delta=\alpha})}{1-\alpha}$ for the above $g_Y^*$ in Eq. 10.

$$r^* = \left(\frac{\alpha}{\beta^*_{\delta=\alpha}\cdot i}\right)\cdot g_Y^* \quad or \quad g_Y^* = \left(\frac{\beta^*_{\delta=\alpha}\cdot i}{\alpha}\right)\cdot r^*. \tag{11}$$

**Proposition 2:** If the rate of rental equals the rate of net investment (or the rate of saving) with no technological progress under convergence (where, *beta*$^*$ = 1.0), the rate of rental equals the growth rate of output. This finding is closely related to the utility function of consumption in Kamiryo [2005b].

Using the above equations, each variable is shown under *alpha* = *i* (or *s*). First, the shape of the capital-output ratio is only stable when *n* > 0 and *beta*$^*$ < 0.9. Second, optimum convergence exists when *alpha* = *i* (or *s*) as in the literature, but only when *beta*$^*$ = 1.0 as I clarified in this paper. This is proved by comparing the rate of rental, *r*, with the growth rate of output, $g_Y^*$, by *beta*$^*$ (compare Table/Figure A-3-2 with Table/Figure A-3-3 in Appendix). This is a real interpretation of a golden rule after introducing the change in technology. These results are useful to establish a sustainable growth by country.

3.3 *Empirical results and findings in transitional paths*

Any country has its own transitional path by year, since net investment in quality, education, R&D, the current balance, and budget deficit differ by country. My research question was: Does any country show increasing returns to capital (IRC) at the current situation? Yes, it is possible to have IRC at Area 3, but this case is usually limited to the government sector which I do not discuss in this paper. For the total economy, the results are shown in Table 1. This table shows that countries usually show DRC at the current situation, where DRC differs by the speed of convergence. I just indicate here that the government sector of medium-sized EU countries sometimes show IRC since these countries cannot postpone structural reform. In this respect, Japan has completely failed in economic policies for people: too late and no way but budget reform by reducing public investment and negative primary balance.

For the total economy, a country cannot usually enter into Area 3 of IRC. Sweden is close to CRC with $delta < alpha$, due to $n = 0$, but with $beta > beta^*$. For the total economy, a few countries show partly IRC in Area 2, where *delta* is lower than *alpha*. Nevertheless, there is no country that shows $beta < beta^*$ in Area 4. Generally speaking, the current situation of advanced countries is closer to the origin than Asian countries, but with exception. For example, China now controls the balance between *beta* and *delta* and the speed of convergence is slower than other countries (note that the stronger the DRC the faster the speed of convergence). Nevertheless, the current growth rate of China, 9 to 10%, is too high since the growth rate of output under convergence has decreased to 4 ~ 5% in the last decade. Besides, $beta^*$ and the capital-output ratio has recently become much higher than before. These suggest that effective economic policies should be taken urgently. Any country cannot enjoy prosperity by increasing the capita-output ratio beyond its limit, say around 3 to 3.5. It is inevitable for faster growing countries to quickly face at a destination of the capita-output ratio.

How can an economy effectively maintain a certain level of growth rate? If targets for *beta* and *delta* are connected with effective policies (since both are measurable by 3 to 6 months), sustainable growth is possible by using recursive programming as shown in Figure 3. A clue for economic policies is not to control the current growth rate but to move “transitional path” for the near future to a sustainable direction. Prompt policy execution is required in the world competition.

**Table 1 The current situation in transitional paths: the differences between $\beta - \beta^*$ and $\delta - \alpha$**

| | Japan | Korea | China | India | Brazil | Singapore | Malaysia | Indonesia | Thailand | Philippines |
|---|---|---|---|---|---|---|---|---|---|---|
| **$\beta_{a(d \neq a)} - \beta^*$ for the total economy** | | | | | | | | | | |
| 1996 | 0.1196 | 0.2246 | 0.1036 | 0.0473 | 0.1998 | 0.3934 | 0.4053 | 0.3173 | 0.1957 | 0.0382 |
| 1997 | 0.1124 | 0.2065 | 0.1097 | 0.0525 | 0.2075 | 0.3682 | 0.3873 | 0.3057 | 0.1795 | 0.0380 |
| 1998 | 0.1027 | 0.1695 | 0.1094 | 0.0554 | 0.2084 | 0.3288 | 0.3310 | 0.3387 | 0.1870 | 0.0436 |
| 1999 | 0.1012 | 0.2117 | 0.1018 | 0.0615 | 0.2013 | 0.2767 | 0.2765 | 0.3037 | 0.1592 | 0.0596 |
| 2000 | 0.1028 | 0.2163 | 0.0979 | 0.0685 | 0.2069 | 0.2175 | 0.2571 | 0.3244 | 0.1369 | 0.0742 |
| 2001 | 0.1003 | 0.1985 | 0.0977 | 0.0738 | 0.2110 | 0.2078 | 0.2475 | 0.3200 | 0.1395 | 0.0723 |
| 2002 | 0.1029 | 0.1611 | 0.1002 | 0.0763 | 0.2096 | 0.1986 | 0.2408 | 0.2986 | 0.1384 | 0.0731 |
| 2003 | 0.1055 | 0.1580 | 0.1024 | 0.0748 | 0.2400 | 0.1890 | 0.2443 | 0.2719 | 0.1436 | 0.0702 |
| **$\delta - \delta^* = \delta - \alpha$ for the total economy** | | | | | | | | | | |
| 1996 | 0.0957 | 0.0897 | 0.0697 | 0.1909 | 0.1727 | 0.1446 | 0.0944 | 0.0869 | 0.0620 | 0.1795 |
| 1997 | 0.1032 | 0.1018 | 0.0779 | 0.2154 | 0.1674 | 0.1455 | 0.0951 | 0.0947 | 0.0990 | 0.1607 |
| 1998 | 0.1340 | 0.1881 | 0.0860 | 0.2231 | 0.1748 | 0.1619 | 0.1942 | 0.1020 | 0.1816 | 0.2092 |
| 1999 | 0.1395 | 0.1338 | 0.0950 | 0.2198 | 0.2084 | 0.1887 | 0.2849 | 0.1615 | 0.2444 | 0.2773 |
| 2000 | 0.1209 | 0.1113 | 0.0991 | 0.2213 | 0.2118 | 0.2133 | 0.2264 | 0.1350 | 0.2497 | 0.2456 |
| 2001 | 0.1322 | 0.1172 | 0.0966 | 0.2267 | 0.2209 | 0.1862 | 0.2389 | 0.1351 | 0.2087 | 0.3182 |
| 2002 | 0.1561 | 0.1383 | 0.0891 | 0.2134 | 0.2339 | 0.2332 | 0.2533 | 0.1578 | 0.2292 | 0.3390 |
| 2003 | 0.1329 | 0.1320 | 0.0816 | 0.2060 | 0.0000 | 0.2416 | 0.2633 | 0.1499 | 0.2084 | 0.3488 |

| | The US | Canada | Russia | Australia | NewZealand | The UK | Sweden | Germany | France | Italy |
|---|---|---|---|---|---|---|---|---|---|---|
| **$\beta_{a(d \neq a)} - \beta^*$ for the total economy** | | | | | | | | | | |
| 1996 | 0.0572 | 0.0658 | 0.1023 | 0.0877 | 0.2177 | 0.0613 | 0.1426 | 0.0902 | 0.1013 | 0.0941 |
| 1997 | 0.0592 | 0.0701 | 0.1006 | 0.0944 | 0.2159 | 0.0618 | 0.1490 | 0.0895 | 0.1044 | 0.0934 |
| 1998 | 0.0615 | 0.0712 | 0.1101 | 0.0925 | 0.1993 | 0.0647 | 0.1512 | 0.0901 | 0.0963 | 0.0917 |
| 1999 | 0.0611 | 0.0722 | 0.1892 | 0.0890 | 0.2024 | 0.0638 | 0.1510 | 0.0787 | 0.0744 | 0.0746 |
| 2000 | 0.0620 | 0.0767 | 0.1758 | 0.0927 | 0.2043 | 0.0644 | 0.1527 | 0.0808 | 0.0769 | 0.0736 |
| 2001 | 0.0621 | 0.0793 | 0.1955 | 0.0920 | 0.1934 | 0.0643 | 0.1461 | 0.0834 | 0.0784 | 0.0785 |
| 2002 | 0.0589 | 0.0794 | 0.1997 | 0.0935 | 0.1999 | 0.0631 | 0.1442 | 0.0839 | 0.0749 | 0.0783 |
| 2003 | 0.0591 | 0.0809 | 0.2072 | 0.0913 | 0.1971 | 0.0630 | 0.1391 | 0.0830 | 0.0678 | 0.0763 |
| **$\delta - \delta^* = \delta - \alpha$ for the total economy** | | | | | | | | | | |
| 1996 | 0.5724 | 0.4405 | (0.0250) | 0.2417 | 0.2244 | 0.1672 | 0.0841 | 0.0781 | 0.1374 | 0.0435 |
| 1997 | 0.5534 | 0.3152 | (0.0355) | 0.2105 | 0.1483 | 0.1769 | 0.0442 | 0.0518 | 0.1356 | 0.0359 |
| 1998 | 0.4978 | 0.3060 | (0.0540) | 0.1950 | 0.2298 | 0.1414 | 0.0000 | 0.0313 | 0.1507 | 0.0204 |
| 1999 | 0.4642 | 0.2969 | (0.1893) | 0.1818 | 0.1623 | 0.1410 | 0.0000 | 0.0173 | 0.1232 | 0.0158 |
| 2000 | 0.4437 | 0.2819 | 0.0403 | 0.1923 | 0.1842 | 0.1556 | 0.0000 | 0.0172 | 0.1172 | 0.0049 |
| 2001 | 0.4870 | 0.2612 | (0.0737) | 0.2260 | 0.2493 | 0.1502 | 0.0000 | 0.0233 | 0.1259 | (0.0099) |
| 2002 | 0.5830 | 0.2574 | (0.0913) | 0.1977 | 0.1702 | 0.1634 | 0.0398 | 0.0235 | 0.1625 | (0.0195) |
| 2003 | 0.5537 | 0.2471 | (0.1028) | 0.1793 | 0.1600 | 0.1595 | 0.0446 | 0.0300 | 0.1881 | (0.0313) |

Note:

1. Each country has its own character, but for small population countries, it is difficult to improve both *beta* and *delta*.
   Advanced countries are more quality-investment-oriented for *beta*, with education and R&D for *delta*.
2. Almost all countries show DRC except for a few cases, but this is limited to the total economy.
   Cases of CRC: Brazil 2003, and Sweden 1998-2001, cases of IRC: Russia except for 2000 and Italy 2001–2003.

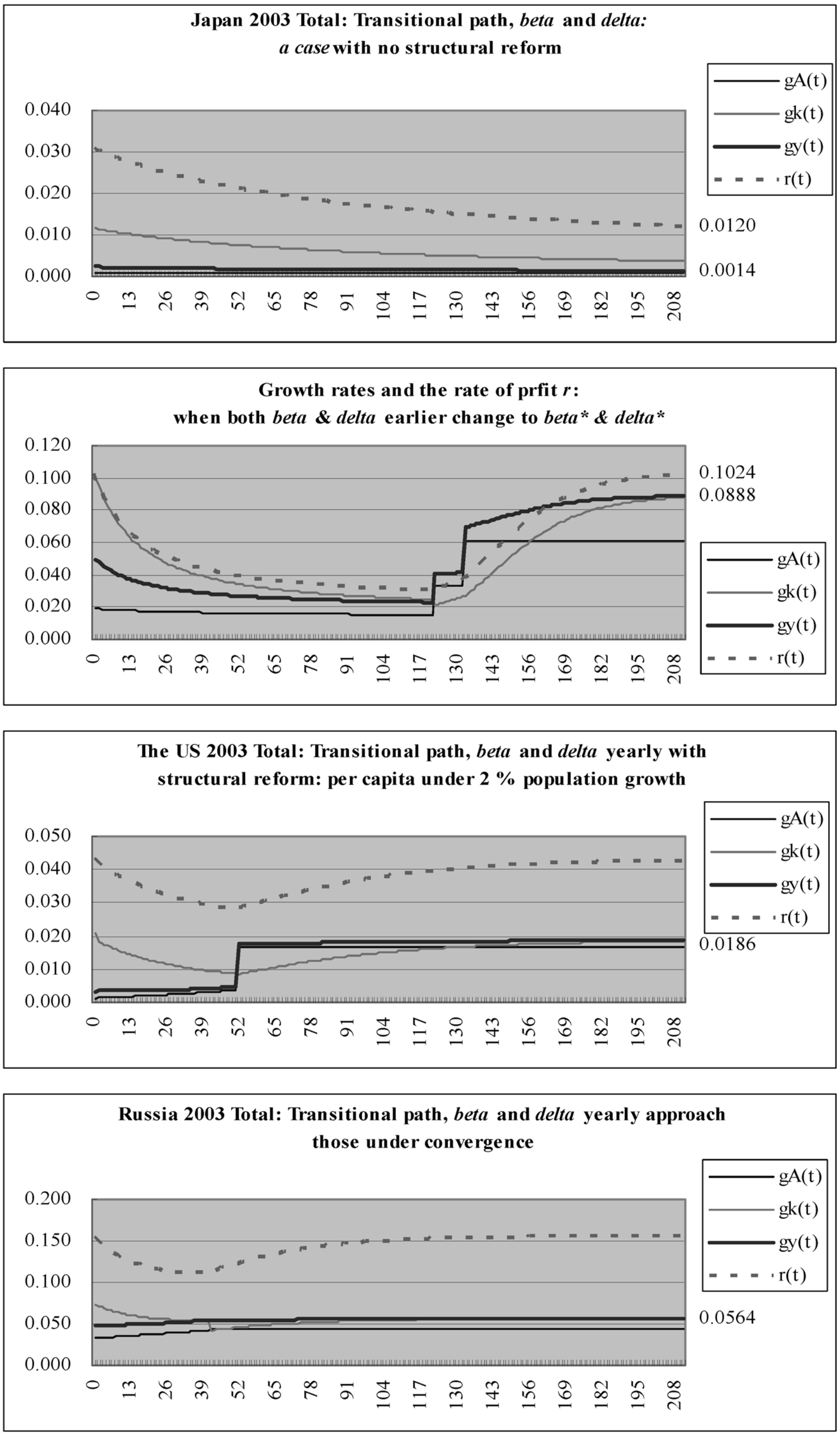



**Figure 3 The current and convergence situations in transitional paths: with and without policies**

## 4. Conclusions

This paper shows the basics of transitional paths, clarifying typical processes of transitional paths by revising Figure 1 in Kamiryo [2004a] (see Figure 2) and defining the speed of convergence as $1/(\delta-\alpha)n$ based on the coefficient for the speed of convergence in Kamiryo and Fujimoto [2005a]. This paper, in particular, finalizes the measurement of the current *delta* which has not been well formulated. And, this paper proves that increasing returns to capital (IRC) under constant returns to scale only exists when both "the current *delta* minus *alpha*" and the growth rate of population/employed persons, *n*, are negative, where I find that the speed of convergence is still positive.

The loci of transitional paths, however, must be tested by empirical results using both equations and recursive programming by country, sector, and year. Empirical results really depend on appropriate data, in particular for wages and returns/rental. It is true that the sum of saving and consumption is national disposable income and the sum of wages and rental constitutes production. Kamiryo [2005b] discussed the measurement of wages and rental by formulating a utility function of consumption, *c*(*rho*/*r*), which well connects consumption with wages, using the discount rate, *rho*, for consumption in income, and the rate of rental, *r*, for capital and production/output, under an assumption that a modified output equals income. When I take into consideration the difference between saving and net investment by sector, using current balance (the balance of payment) = budget surplus/deficit + the difference between saving and net investment in the private sector, the utility function of consumption will be much closer to the real world. This device is not difficult by introducing a parameter of *i*/*s* by sector. I will discuss this final stage in my coming paper, by dividing the total economy into the government sector and the private sector. In this case, I must apply a technology-golden rule to the private sector, where I need to discuss the character of this rule in terms of the so-called aggregation problem.

For empirical analysis, this paper showed the case of the total economy by country, applying the above function to the data available in IFSY, IMF. I find that if I am versed in the current economic circumstances by country,[10)] I could get better results since I can more

10) Or alternatively, if I take into consideration more carefully the differences between my initial (estimated) growth rate of output and the actual initial growth rate of NDI in statistics, the results may be more accurate. I expect, however, that this adjustment alone does not change the results ↗

definitely manipulate the values of $c(rho/r)$ and ratio of the rental rate to the wage rate, $r/w$, which is another important factor for estimating capital (see Kamiryo [2004c]).

In any way, it is vital for an economy first to have *beta* closer to $beta^*$, through structural reform. However, it is noted that even if $beta^*$ is within reach the growth rate under convergence may be extremely low as shown in Japan (see Figure 3). This indicates that the total economy will be significantly influenced by extreme budget deficit, which makes $beta^*$ much higher than that in other countries. Comparison of the government sector by country will be discussed in Kamiryo [2006] at Helsinki, IARIW.

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## Appendix

**Table A1-1 The relationship among _delta_, _n_, and _beta_[*], each by the level of _alpha_ and _i_**

| | _alpha_ | _i_ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $\delta=(n+\alpha(i(1-\beta^*)-n)/i(1-\beta^*)$ | | | | | _beta*_ |
| _n_ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.6400 | 0.7750 | 1.0000 | 1.4500 | 2.8000 | | -2.6000 | -1.2500 |
| 0.025 | 0.5500 | 0.6625 | 0.8500 | 1.2250 | 2.3500 | | -2.1500 | -1.0250 |
| 0.02 | 0.4600 | 0.5500 | 0.7000 | 1.0000 | 1.9000 | | -1.7000 | -0.8000 |
| 0.015 | 0.3700 | 0.4375 | 0.5500 | 0.7750 | 1.4500 | | -1.2500 | -0.5750 |
| 0.01 | 0.2800 | 0.3250 | 0.4000 | 0.5500 | 1.0000 | | -0.8000 | -0.3500 |
| 0.005 | 0.1900 | 0.2125 | 0.2500 | 0.3250 | 0.5500 | | -0.3500 | -0.1250 |
| 0 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | | 0.1000 | 0.1000 |
| -0.005 | 0.0100 | -0.0125 | -0.0500 | -0.1250 | -0.3500 | | 0.5500 | 0.3250 |
| -0.01 | -0.0800 | -0.1250 | -0.2000 | -0.3500 | -0.8000 | | 1.0000 | 0.5500 |

The Z axis: for _delta_ The X ais is for _n_ , and the Y axis is for _beta*_ .

| | _alpha_ | _i_ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.15** | $\delta=(n+\alpha(i(1-\beta^*)-n)/i(1-\beta^*)$ | | | | | _beta*_ |
| _n_ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.4600 | 0.5500 | 0.7000 | 1.0000 | 1.9000 | | -1.7000 | -0.8000 |
| 0.025 | 0.4000 | 0.4750 | 0.6000 | 0.8500 | 1.6000 | | -1.4000 | -0.6500 |
| 0.02 | 0.3400 | 0.4000 | 0.5000 | 0.7000 | 1.3000 | | -1.1000 | -0.5000 |
| 0.015 | 0.2800 | 0.3250 | 0.4000 | 0.5500 | 1.0000 | | -0.8000 | -0.3500 |
| 0.01 | 0.2200 | 0.2500 | 0.3000 | 0.4000 | 0.7000 | | -0.5000 | -0.2000 |
| 0.005 | 0.1600 | 0.1750 | 0.2000 | 0.2500 | 0.4000 | | -0.2000 | -0.0500 |
| 0 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | | 0.1000 | 0.1000 |
| -0.005 | 0.0400 | 0.0250 | 0.0000 | -0.0500 | -0.2000 | | 0.4000 | 0.2500 |
| -0.01 | -0.0200 | -0.0500 | -0.1000 | -0.2000 | -0.5000 | | 0.7000 | 0.4000 |

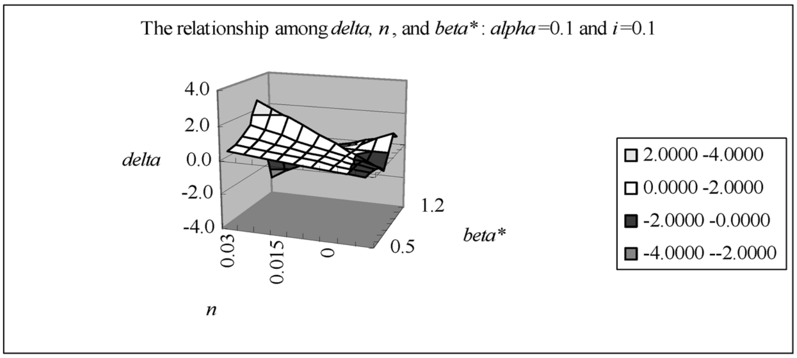
The Z axis: for _delta_ The X ais is for _n_ , and the Y axis is for _beta*_ .

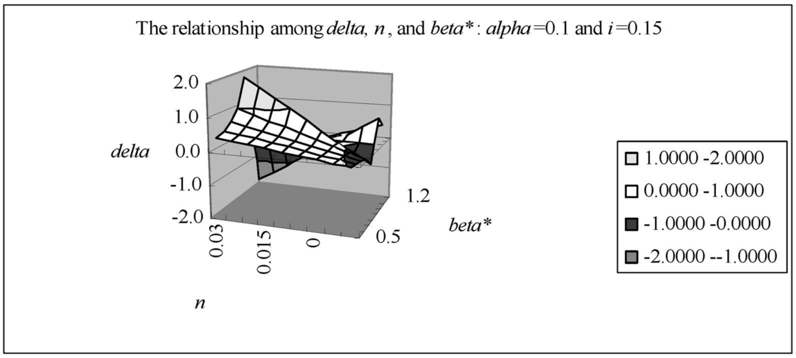
| | _alpha_ | _i_ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.025** | $\delta=(n+\alpha(i(1-\beta^*)-n)/i(1-\beta^*)$ | | | | | _beta*_ |
| _n_ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 2.2600 | 2.8000 | 3.7000 | 5.5000 | 10.9000 | | -10.7000 | -5.3000 |
| 0.025 | 1.9000 | 2.3500 | 3.1000 | 4.6000 | 9.1000 | | -8.9000 | -4.4000 |
| 0.02 | 1.5400 | 1.9000 | 2.5000 | 3.7000 | 7.3000 | | -7.1000 | -3.5000 |
| 0.015 | 1.1800 | 1.4500 | 1.9000 | 2.8000 | 5.5000 | | -5.3000 | -2.6000 |
| 0.01 | 0.8200 | 1.0000 | 1.3000 | 1.9000 | 3.7000 | | -3.5000 | -1.7000 |
| 0.005 | 0.4600 | 0.5500 | 0.7000 | 1.0000 | 1.9000 | | -1.7000 | -0.8000 |
| 0 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | 0.1000 | | 0.1000 | 0.1000 |
| -0.005 | -0.2600 | -0.3500 | -0.5000 | -0.8000 | -1.7000 | | 1.9000 | 1.0000 |
| -0.01 | -0.6200 | -0.8000 | -1.1000 | -1.7000 | -3.5000 | | 3.7000 | 1.9000 |

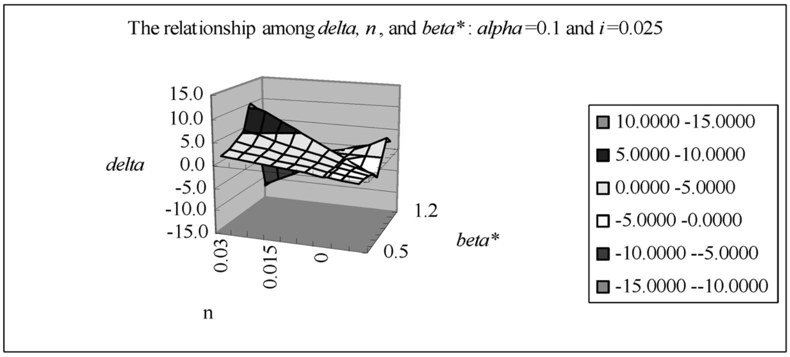
The Z axis: for _delta_ The X ais is for _n_ , and the Y axis is for _beta*_ .

| | _alpha_ | _i_ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.2** | **0.1** | $\delta=(n+\alpha(i(1-\beta^*)-n)/i(1-\beta^*)$ | | | | | _beta*_ |
| _n_ | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.6800 | 0.8000 | 1.0000 | 1.4000 | 2.6000 | | -2.2000 | -1.0000 |
| 0.025 | 0.6000 | 0.7000 | 0.8667 | 1.2000 | 2.2000 | | -1.8000 | -0.8000 |
| 0.02 | 0.5200 | 0.6000 | 0.7333 | 1.0000 | 1.8000 | | -1.4000 | -0.6000 |
| 0.015 | 0.4400 | 0.5000 | 0.6000 | 0.8000 | 1.4000 | | -1.0000 | -0.4000 |
| 0.01 | 0.3600 | 0.4000 | 0.4667 | 0.6000 | 1.0000 | | -0.6000 | -0.2000 |
| 0.005 | 0.2800 | 0.3000 | 0.3333 | 0.4000 | 0.6000 | | -0.2000 | 0.0000 |
| 0 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | 0.2000 | | 0.2000 | 0.2000 |
| -0.005 | 0.1200 | 0.1000 | 0.0667 | 0.0000 | -0.2000 | | 0.6000 | 0.4000 |
| -0.01 | 0.0400 | 0.0000 | -0.0667 | -0.2000 | -0.6000 | | 1.0000 | 0.6000 |

The Z axis: for _delta_ The X ais is for _n_ , and the Y axis is for _beta*_ .



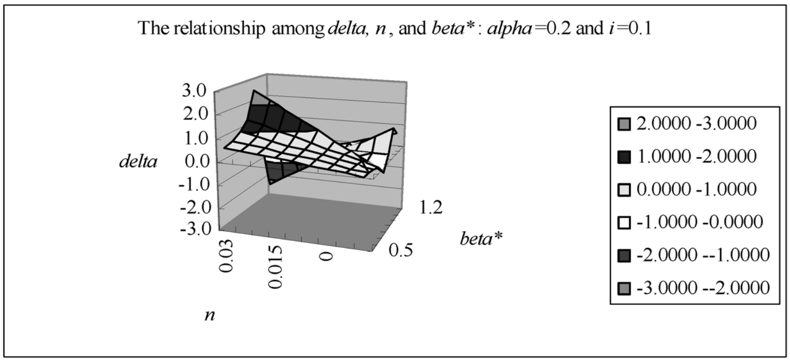




**Figure A1-1　The relationship among *delta, n*, and *beta*$^*$, each by the level of *alpha* and *i***

**Table A1-2 The relationship among the convergence coefficient, *n*, & *beta**, each by *alpha* and *i***

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $\zeta=(\delta-\alpha)n$ | | | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.0162 | 0.0203 | 0.0270 | 0.0405 | 0.0810 | | -0.0810 | -0.0405 |
| 0.025 | 0.0113 | 0.0141 | 0.0188 | 0.0281 | 0.0563 | | -0.0563 | -0.0281 |
| 0.02 | 0.0072 | 0.0090 | 0.0120 | 0.0180 | 0.0360 | | -0.0360 | -0.0180 |
| 0.015 | 0.0041 | 0.0051 | 0.0068 | 0.0101 | 0.0203 | | -0.0203 | -0.0101 |
| 0.01 | 0.0018 | 0.0023 | 0.0030 | 0.0045 | 0.0090 | | -0.0090 | -0.0045 |
| 0.005 | 0.0005 | 0.0006 | 0.0008 | 0.0011 | 0.0023 | | -0.0023 | -0.0011 |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| -0.005 | 0.0005 | 0.0006 | 0.0008 | 0.0011 | 0.0023 | | -0.0023 | -0.0011 |
| -0.01 | 0.0018 | 0.0023 | 0.0030 | 0.0045 | 0.0090 | | -0.0090 | -0.0045 |

The Z axis: the speed of convergenc The X ais is for *n* , and the Y axis is for *beta** .

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.15** | $\zeta=(\delta-\alpha)n$ | | | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.0108 | 0.0135 | 0.0180 | 0.0270 | 0.0540 | | -0.0540 | -0.0270 |
| 0.025 | 0.0075 | 0.0094 | 0.0125 | 0.0188 | 0.0375 | | -0.0375 | -0.0188 |
| 0.02 | 0.0048 | 0.0060 | 0.0080 | 0.0120 | 0.0240 | | -0.0240 | -0.0120 |
| 0.015 | 0.0027 | 0.0034 | 0.0045 | 0.0068 | 0.0135 | | -0.0135 | -0.0068 |
| 0.01 | 0.0012 | 0.0015 | 0.0020 | 0.0030 | 0.0060 | | -0.0060 | -0.0030 |
| 0.005 | 0.0003 | 0.0004 | 0.0005 | 0.0008 | 0.0015 | | -0.0015 | -0.0008 |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| -0.005 | 0.0003 | 0.0004 | 0.0005 | 0.0008 | 0.0015 | | -0.0015 | -0.0008 |
| -0.01 | 0.0012 | 0.0015 | 0.0020 | 0.0030 | 0.0060 | | -0.0060 | -0.0030 |

The Z axis: the speed of convergenc The X ais is for *n* , and the Y axis is for *beta** .

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.025** | $\zeta=(\delta-\alpha)n$ | | | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.0648 | 0.0810 | 0.1080 | 0.1620 | 0.3240 | | -0.3240 | -0.1620 |
| 0.025 | 0.0450 | 0.0563 | 0.0750 | 0.1125 | 0.2250 | | -0.2250 | -0.1125 |
| 0.02 | 0.0288 | 0.0360 | 0.0480 | 0.0720 | 0.1440 | | -0.1440 | -0.0720 |
| 0.015 | 0.0162 | 0.0203 | 0.0270 | 0.0405 | 0.0810 | | -0.0810 | -0.0405 |
| 0.01 | 0.0072 | 0.0090 | 0.0120 | 0.0180 | 0.0360 | | -0.0360 | -0.0180 |
| 0.005 | 0.0018 | 0.0023 | 0.0030 | 0.0045 | 0.0090 | | -0.0090 | -0.0045 |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| -0.005 | 0.0018 | 0.0023 | 0.0030 | 0.0045 | 0.0090 | | -0.0090 | -0.0045 |
| -0.01 | 0.0072 | 0.0090 | 0.0120 | 0.0180 | 0.0360 | | -0.0360 | -0.0180 |

The Z axis: the speed of convergenc The X ais is for *n* , and the Y axis is for *beta** .

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.2** | **0.1** | $\zeta=(\delta-\alpha)n$ | | | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 0.0144 | 0.0180 | 0.0240 | 0.0360 | 0.0720 | | -0.0720 | -0.0360 |
| 0.025 | 0.0100 | 0.0125 | 0.0167 | 0.0250 | 0.0500 | | -0.0500 | -0.0250 |
| 0.02 | 0.0064 | 0.0080 | 0.0107 | 0.0160 | 0.0320 | | -0.0320 | -0.0160 |
| 0.015 | 0.0036 | 0.0045 | 0.0060 | 0.0090 | 0.0180 | | -0.0180 | -0.0090 |
| 0.01 | 0.0016 | 0.0020 | 0.0027 | 0.0040 | 0.0080 | | -0.0080 | -0.0040 |
| 0.005 | 0.0004 | 0.0005 | 0.0007 | 0.0010 | 0.0020 | | -0.0020 | -0.0010 |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| -0.005 | 0.0004 | 0.0005 | 0.0007 | 0.0010 | 0.0020 | | -0.0020 | -0.0010 |
| -0.01 | 0.0016 | 0.0020 | 0.0027 | 0.0040 | 0.0080 | | -0.0080 | -0.0040 |

The Z axis: the speed of convergenc The X ais is for *n* , and the Y axis is for *beta** .

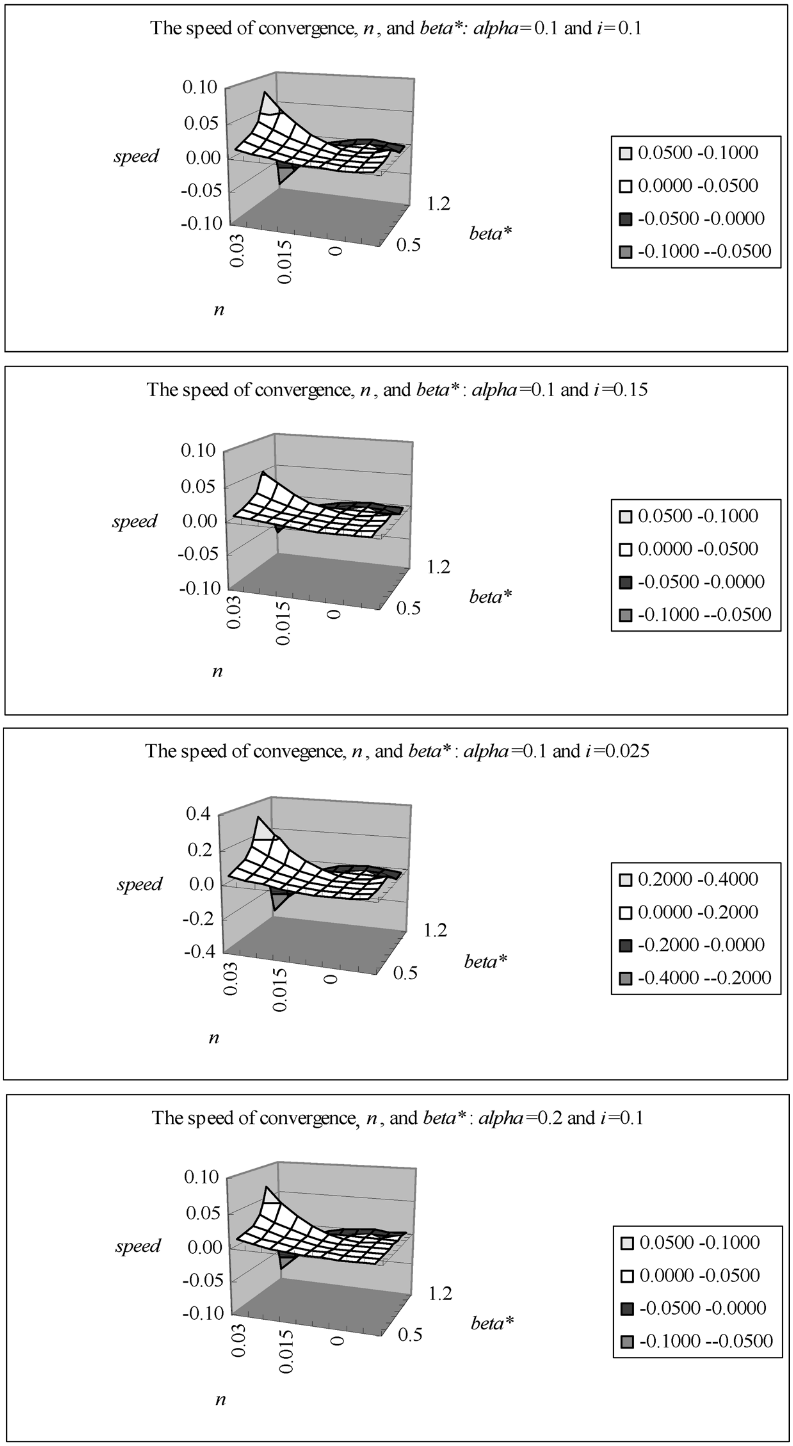



**Figure A1-2 The relationship among the convergence coefficient, *n*, & *beta**, each by *alpha* & *i***

**Table A1-3 The relationship among the years of convergence, *n*, and *beta**, by the level of each *alpha* and *i***

| | *alpha* | *i* | | e^-x | conv. years | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $\zeta=(\delta-\alpha)\,n$ | **1.0** | $=1/\zeta$ | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 61.7 | 49.4 | 37.0 | 24.7 | 12.3 | | -12.3 | -24.7 |
| 0.025 | 88.9 | 71.1 | 53.3 | 35.6 | 17.8 | | -17.8 | -35.6 |
| 0.02 | 138.9 | 111.1 | 83.3 | 55.6 | 27.8 | | -27.8 | -55.6 |
| 0.015 | 246.9 | 197.5 | 148.1 | 98.8 | 49.4 | | -49.4 | -98.8 |
| 0.01 | 555.6 | 444.4 | 333.3 | 222.2 | 111.1 | | -111.1 | -222.2 |
| 0.005 | 2222.2 | 1777.8 | 1333.3 | 888.9 | 444.4 | | -444.4 | -888.9 |
| 0 | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | | #DIV/0! | #DIV/0! |
| -0.005 | 2222.2 | 1777.8 | 1333.3 | 888.9 | 444.4 | | -444.4 | -888.9 |
| -0.01 | 555.6 | 444.4 | 333.3 | 222.2 | 111.1 | | -111.1 | -222.2 |

The Z axis: convergence *years.* The X ais is for *n* , and the Y axis is for *beta* *.

| | *alpha* | *i* | | e^-x | conv. years | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.15** | $\zeta=(\delta-\alpha)\,n$ | **1.0** | $=1/\zeta$ | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 92.6 | 74.1 | 55.6 | 37.0 | 18.5 | | -18.5 | -37.0 |
| 0.025 | 133.3 | 106.7 | 80.0 | 53.3 | 26.7 | | -26.7 | -53.3 |
| 0.02 | 208.3 | 166.7 | 125.0 | 83.3 | 41.7 | | -41.7 | -83.3 |
| 0.015 | 370.4 | 296.3 | 222.2 | 148.1 | 74.1 | | -74.1 | -148.1 |
| 0.01 | 833.3 | 666.7 | 500.0 | 333.3 | 166.7 | | -166.7 | -333.3 |
| 0.005 | 3333.3 | 2666.7 | 2000.0 | 1333.3 | 666.7 | | -666.7 | -1333.3 |
| 0 | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | | #DIV/0! | #DIV/0! |
| -0.005 | 3333.3 | 2666.7 | 2000.0 | 1333.3 | 666.7 | | -666.7 | -1333.3 |
| -0.01 | 833.3 | 666.7 | 500.0 | 333.3 | 166.7 | | -166.7 | -333.3 |

The Z axis: *convergence years.*

| | *alpha* | *i* | | e^-x | conv. years | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.025** | $\zeta=(\delta-\alpha)\,n$ | **1.0** | $=1/\zeta$ | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 15.4 | 12.3 | 9.3 | 6.2 | 3.1 | | -3.1 | -6.2 |
| 0.025 | 22.2 | 17.8 | 13.3 | 8.9 | 4.4 | | -4.4 | -8.9 |
| 0.02 | 34.7 | 27.8 | 20.8 | 13.9 | 6.9 | | -6.9 | -13.9 |
| 0.015 | 61.7 | 49.4 | 37.0 | 24.7 | 12.3 | | -12.3 | -24.7 |
| 0.01 | 138.9 | 111.1 | 83.3 | 55.6 | 27.8 | | -27.8 | -55.6 |
| 0.005 | 555.6 | 444.4 | 333.3 | 222.2 | 111.1 | | -111.1 | -222.2 |
| 0 | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | | #DIV/0! | #DIV/0! |
| -0.005 | 555.6 | 444.4 | 333.3 | 222.2 | 111.1 | | -111.1 | -222.2 |
| -0.01 | 138.9 | 111.1 | 83.3 | 55.6 | 27.8 | | -27.8 | -55.6 |

The Z axis: *convergence years.*

| | *alpha* | *i* | | e^-x | conv. years | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.2** | **0.1** | $\zeta=(\delta-\alpha)\,n$ | **1.0** | $=1/\zeta$ | | | *beta** |
| *n* | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 | 1.1 | 1.2 |
| 0.03 | 69.4 | 55.6 | 41.7 | 27.8 | 13.9 | | -13.9 | -27.8 |
| 0.025 | 100.0 | 80.0 | 60.0 | 40.0 | 20.0 | | -20.0 | -40.0 |
| 0.02 | 156.3 | 125.0 | 93.8 | 62.5 | 31.3 | | -31.3 | -62.5 |
| 0.015 | 277.8 | 222.2 | 166.7 | 111.1 | 55.6 | | -55.6 | -111.1 |
| 0.01 | 625.0 | 500.0 | 375.0 | 250.0 | 125.0 | | -125.0 | -250.0 |
| 0.005 | 2500.0 | 2000.0 | 1500.0 | 1000.0 | 500.0 | | -500.0 | -1000.0 |
| 0 | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | | #DIV/0! | #DIV/0! |
| -0.005 | 2500.0 | 2000.0 | 1500.0 | 1000.0 | 500.0 | | -500.0 | -1000.0 |
| -0.01 | 625.0 | 500.0 | 375.0 | 250.0 | 125.0 | | -125.0 | -250.0 |

The Z axis: *convergence years.*

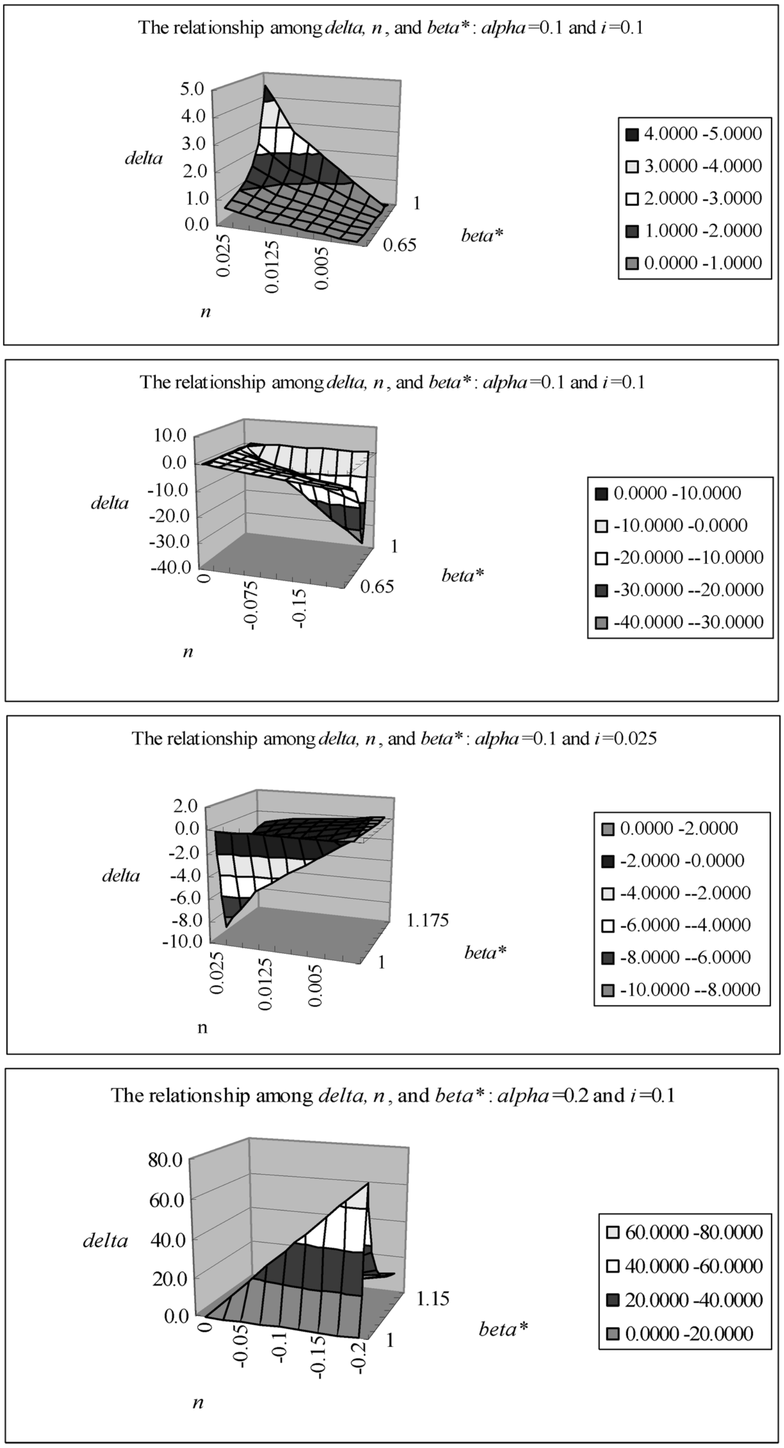



**Figure A2-1 The relationship among *delta, n*, and *beta*$^*$: separately by plus and minus *n* and *beta*$^*$**

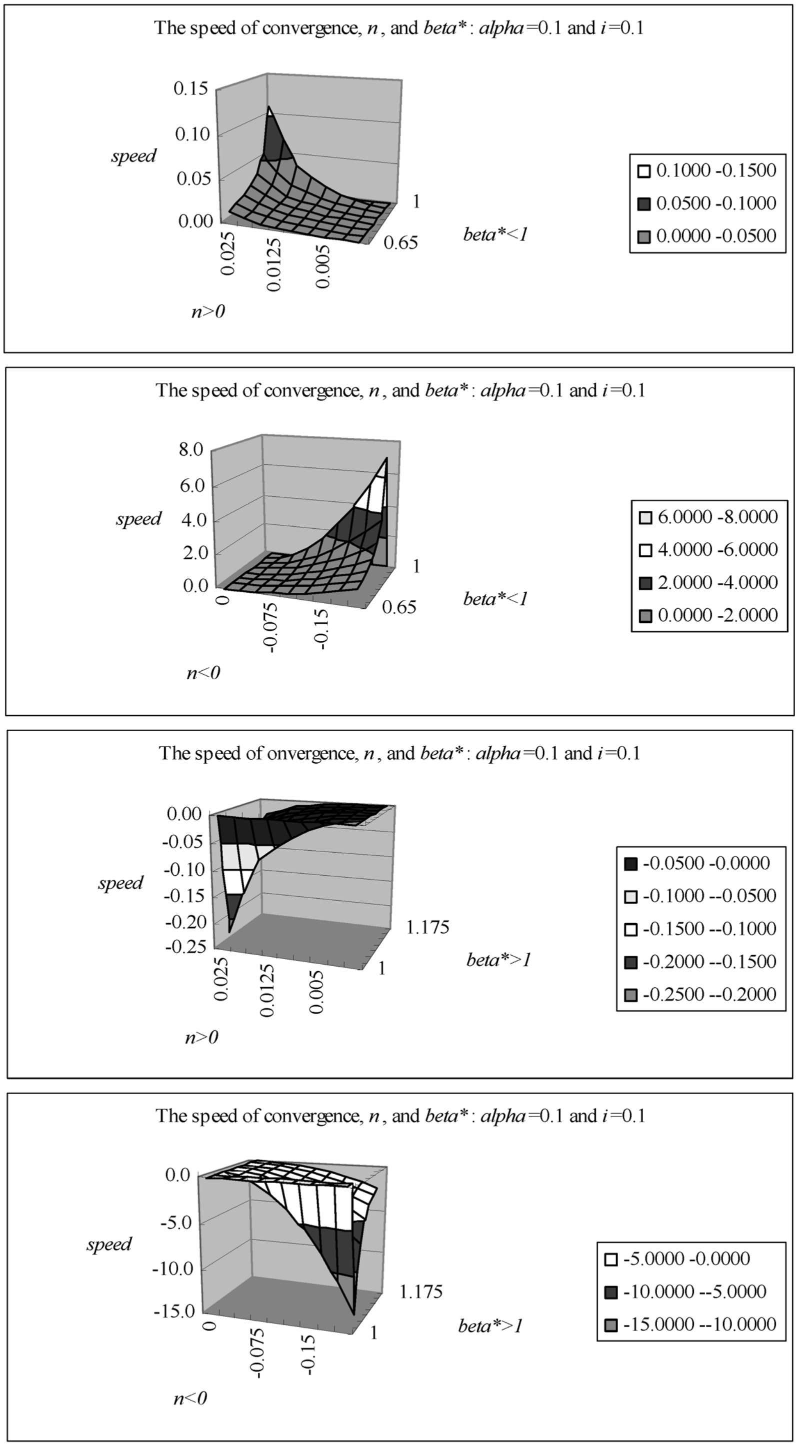



**Figure A2-2 The relationship among the speed, *n*, and *beta*$^*$, separately by plus and minus *n* and *beta*$^*$**

**Table A3-1 The capital-output ratio, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$**

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $\Omega=(\beta^*(i(1-\alpha))/(i(1-\beta^*)(1+n)+n(1-\alpha))$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0.025 | 1.0021 | 1.1831 | 1.4026 | 1.6744 | 2.0198 | 2.4733 | 3.0950 | 4.0000 |
| 0.02 | 1.0894 | 1.2963 | 1.5517 | 1.8750 | 2.2973 | 2.8723 | 3.7013 | 5.0000 |
| 0.015 | 1.1933 | 1.4334 | 1.7363 | 2.1302 | 2.6632 | 3.4249 | 4.6030 | 6.6667 |
| 0.0125 | 1.2530 | 1.5135 | 1.8462 | 2.2857 | 2.8936 | 3.7895 | 5.2414 | 8.0000 |
| 0.01 | 1.3191 | 1.6031 | 1.9708 | 2.4658 | 3.1677 | 4.2408 | 6.0854 | 10.0000 |
| 0.0075 | 1.3924 | 1.7039 | 2.1135 | 2.6766 | 3.4991 | 4.8143 | 7.2534 | 13.3333 |
| 0.005 | 1.4745 | 1.8182 | 2.2785 | 2.9268 | 3.9080 | 5.5670 | 8.9764 | 20.0000 |
| 0.0025 | 1.5668 | 1.9490 | 2.4714 | 3.2287 | 4.4252 | 6.5988 | 11.7728 | 40.0000 |
| 0 | 1.6714 | 2.1000 | 2.7000 | 3.6000 | 5.1000 | 8.1000 | 17.1000 | #DIV/0! |

The Z axis: for *Omega* The X ais is for *n* , and the Y axis is for *beta** .

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.1** | $\Omega=(\beta^*(i(1-\alpha))/(i(1-\beta^*)(1+n)+n(1-\alpha))$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0 | 1.6714 | 2.1000 | 2.7000 | 3.6000 | 5.1000 | 8.1000 | 17.1000 | #DIV/0! |
| -0.025 | 5.0323 | 9.3333 | 36.0000 | -24.0000 | -9.7143 | -6.3529 | -4.8511 | -4.0000 |
| -0.05 | -4.9787 | -3.8182 | -3.1765 | -2.7692 | -2.4878 | -2.2817 | -2.1242 | -2.0000 |
| -0.075 | -1.6655 | -1.5849 | -1.5211 | -1.4694 | -1.4266 | -1.3906 | -1.3598 | -1.3333 |
| -0.1 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 |
| -0.125 | -0.7145 | -0.7304 | -0.7448 | -0.7579 | -0.7698 | -0.7807 | -0.7908 | -0.8000 |
| -0.15 | -0.5558 | -0.5753 | -0.5934 | -0.6102 | -0.6258 | -0.6403 | -0.6539 | -0.6667 |
| -0.175 | -0.4548 | -0.4746 | -0.4932 | -0.5106 | -0.5271 | -0.5427 | -0.5575 | -0.5714 |
| -0.2 | -0.3849 | -0.4038 | -0.4219 | -0.4390 | -0.4554 | -0.4709 | -0.4858 | -0.5000 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.1** | $\Omega=(\beta^*(i(1-\alpha))/(i(1-\beta^*)(1+n)+n(1-\alpha))$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0.025 | 4.0000 | 4.6270 | 5.4388 | 6.5316 | 8.0816 | 10.4516 | 14.5263 | 23.1781 |
| 0.02 | 5.0000 | 5.9709 | 7.3256 | 9.3478 | 12.6923 | 19.2857 | 38.3333 | 705.0000 |
| 0.015 | 6.6667 | 8.4151 | 11.2166 | 16.4331 | 29.5522 | 124.6154 | -60.0000 | -24.8094 |
| 0.0125 | 8.0000 | 10.5806 | 15.2727 | 26.4615 | 88.0000 | -72.0000 | -26.2857 | -16.3478 |
| 0.01 | 10.0000 | 14.2471 | 23.9241 | 67.8947 | -90.0000 | -27.9310 | -16.8293 | -12.1902 |
| 0.0075 | 13.3333 | 21.8021 | 55.1825 | -120.0000 | -29.7744 | -17.3262 | -12.3767 | -9.7186 |
| 0.005 | 20.0000 | 46.4151 | -180.0000 | -31.8519 | -17.8378 | -12.5581 | -9.7872 | -8.0802 |
| 0.0025 | 40.0000 | -360.0000 | -34.2081 | -18.3630 | -12.7331 | -9.8480 | -8.0938 | -6.9146 |
| 0 | #DIV/0! | -36.9000 | -18.9000 | -12.9000 | -9.9000 | -8.1000 | -6.9000 | -6.0429 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.1** | **0.1** | $\Omega=(\beta^*(i(1-\alpha))/(i(1-\beta^*)(1+n)+n(1-\alpha))$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0 | #DIV/0! | -36.9000 | -18.9000 | -12.9000 | -9.9000 | -8.1000 | -6.9000 | -6.0429 |
| -0.025 | -4.0000 | -3.6992 | -3.4521 | -3.2453 | -3.0698 | -2.9189 | -2.7879 | -2.6730 |
| -0.05 | -2.0000 | -1.9472 | -1.8995 | -1.8561 | -1.8165 | -1.7802 | -1.7468 | -1.7160 |
| -0.075 | -1.3333 | -1.3214 | -1.3102 | -1.2997 | -1.2899 | -1.2806 | -1.2719 | -1.2636 |
| -0.1 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 | -1.0000 |
| -0.125 | -0.8000 | -0.8044 | -0.8086 | -0.8126 | -0.8165 | -0.8203 | -0.8239 | -0.8274 |
| -0.15 | -0.6667 | -0.6727 | -0.6786 | -0.6844 | -0.6899 | -0.6953 | -0.7005 | -0.7056 |
| -0.175 | -0.5714 | -0.5781 | -0.5847 | -0.5911 | -0.5973 | -0.6034 | -0.6093 | -0.6150 |
| -0.2 | -0.5000 | -0.5069 | -0.5136 | -0.5202 | -0.5266 | -0.5329 | -0.5391 | -0.5451 |

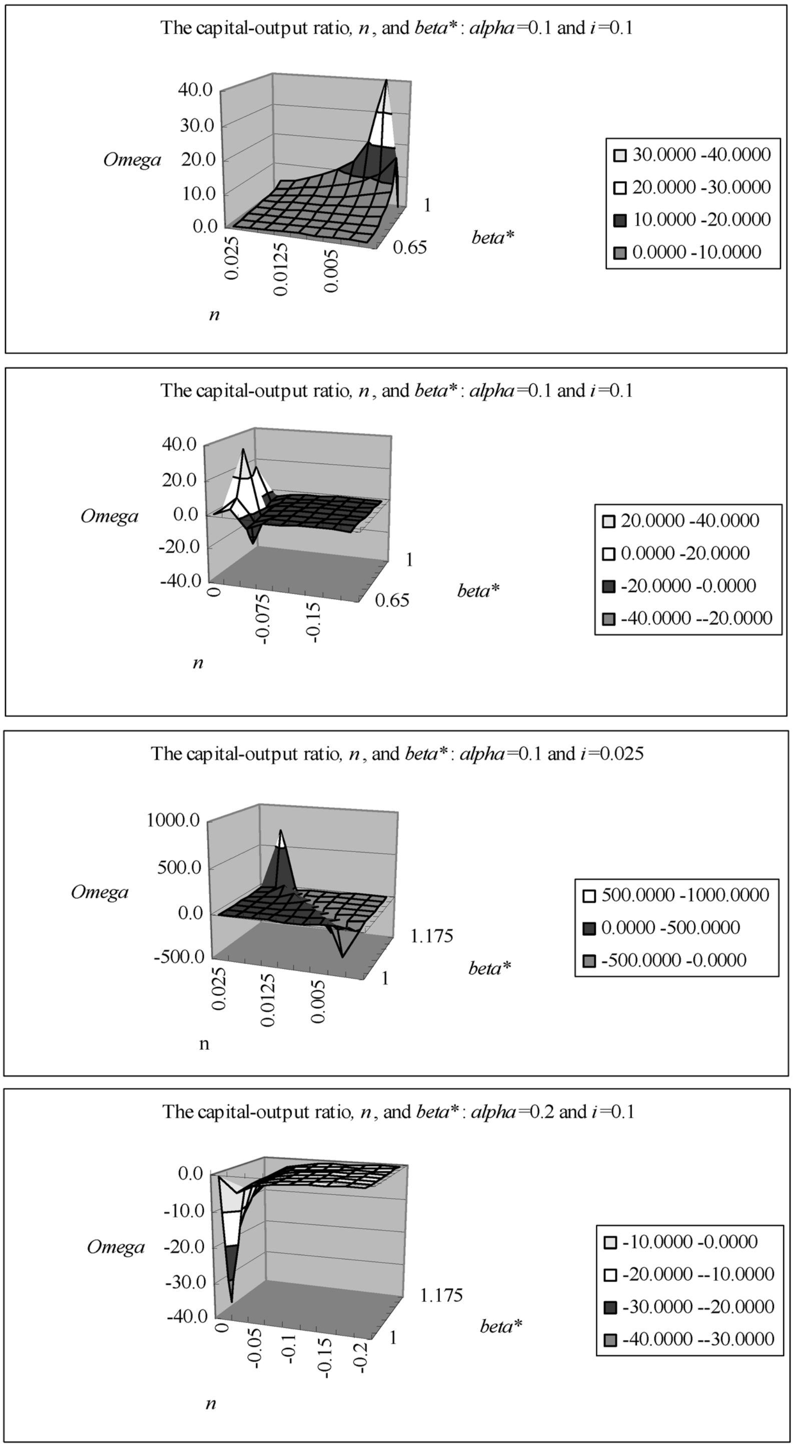



**Figure A3-1 The capital-output ratio, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$**

**Table A3-2 The rate of rental, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$**

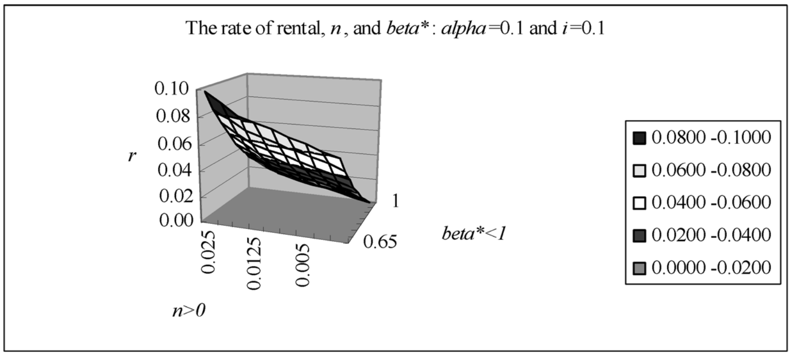
| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $r = \alpha/\Omega$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0.025 | 0.0998 | 0.0845 | 0.0713 | 0.0597 | 0.0495 | 0.0404 | 0.0323 | 0.0250 |
| 0.02 | 0.0918 | 0.0771 | 0.0644 | 0.0533 | 0.0435 | 0.0348 | 0.0270 | 0.0200 |
| 0.015 | 0.0838 | 0.0698 | 0.0576 | 0.0469 | 0.0375 | 0.0292 | 0.0217 | 0.0150 |
| 0.0125 | 0.0798 | 0.0661 | 0.0542 | 0.0438 | 0.0346 | 0.0264 | 0.0191 | 0.0125 |
| 0.01 | 0.0758 | 0.0624 | 0.0507 | 0.0406 | 0.0316 | 0.0236 | 0.0164 | 0.0100 |
| 0.0075 | 0.0718 | 0.0587 | 0.0473 | 0.0374 | 0.0286 | 0.0208 | 0.0138 | 0.0075 |
| 0.005 | 0.0678 | 0.0550 | 0.0439 | 0.0342 | 0.0256 | 0.0180 | 0.0111 | 0.0050 |
| 0.0025 | 0.0638 | 0.0513 | 0.0405 | 0.0310 | 0.0226 | 0.0152 | 0.0085 | 0.0025 |
| 0 | 0.0598 | 0.0476 | 0.0370 | 0.0278 | 0.0196 | 0.0123 | 0.0058 | #DIV/0! |

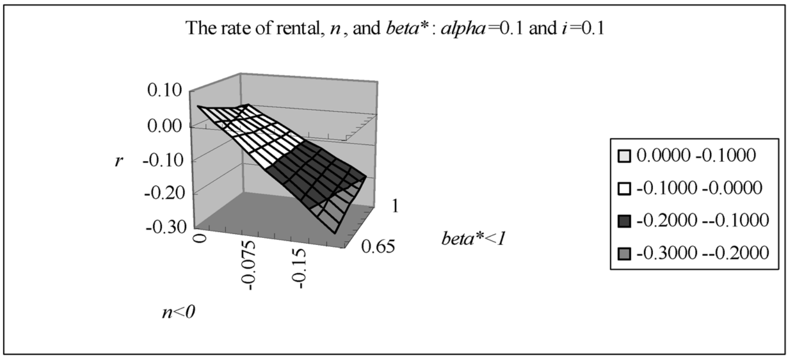
The Z axis: for *r* The X ais is for *n* , and the Y axis is for *beta** .

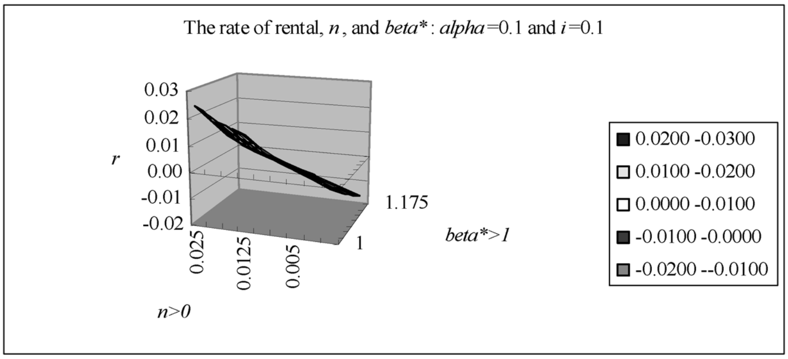
| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.1** | $r = \alpha/\Omega$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0 | 0.0598 | 0.0476 | 0.0370 | 0.0278 | 0.0196 | 0.0123 | 0.0058 | #DIV/0! |
| -0.025 | 0.0199 | 0.0107 | 0.0028 | -0.0042 | -0.0103 | -0.0157 | -0.0206 | -0.0250 |
| -0.05 | -0.0201 | -0.0262 | -0.0315 | -0.0361 | -0.0402 | -0.0438 | -0.0471 | -0.0500 |
| -0.075 | -0.0600 | -0.0631 | -0.0657 | -0.0681 | -0.0701 | -0.0719 | -0.0735 | -0.0750 |
| -0.1 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 |
| -0.125 | -0.1400 | -0.1369 | -0.1343 | -0.1319 | -0.1299 | -0.1281 | -0.1265 | -0.1250 |
| -0.15 | -0.1799 | -0.1738 | -0.1685 | -0.1639 | -0.1598 | -0.1562 | -0.1529 | -0.1500 |
| -0.175 | -0.2199 | -0.2107 | -0.2028 | -0.1958 | -0.1897 | -0.1843 | -0.1794 | -0.1750 |
| -0.2 | -0.2598 | -0.2476 | -0.2370 | -0.2278 | -0.2196 | -0.2123 | -0.2058 | -0.2000 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.1** | $r = \alpha/\Omega$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0.025 | 0.0250 | 0.0216 | 0.0184 | 0.0153 | 0.0124 | 0.0096 | 0.0069 | 0.0043 |
| 0.02 | 0.0200 | 0.0167 | 0.0137 | 0.0107 | 0.0079 | 0.0052 | 0.0026 | 0.0001 |
| 0.015 | 0.0150 | 0.0119 | 0.0089 | 0.0061 | 0.0034 | 0.0008 | -0.0017 | -0.0040 |
| 0.0125 | 0.0125 | 0.0095 | 0.0065 | 0.0038 | 0.0011 | -0.0014 | -0.0038 | -0.0061 |
| 0.01 | 0.0100 | 0.0070 | 0.0042 | 0.0015 | -0.0011 | -0.0036 | -0.0059 | -0.0082 |
| 0.0075 | 0.0075 | 0.0046 | 0.0018 | -0.0008 | -0.0034 | -0.0058 | -0.0081 | -0.0103 |
| 0.005 | 0.0050 | 0.0022 | -0.0006 | -0.0031 | -0.0056 | -0.0080 | -0.0102 | -0.0124 |
| 0.0025 | 0.0025 | -0.0003 | -0.0029 | -0.0054 | -0.0079 | -0.0102 | -0.0124 | -0.0145 |
| 0 | #DIV/0! | -0.0027 | -0.0053 | -0.0078 | -0.0101 | -0.0123 | -0.0145 | -0.0165 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.1** | **0.1** | $r = \alpha/\Omega$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0 | #DIV/0! | -0.0027 | -0.0053 | -0.0078 | -0.0101 | -0.0123 | -0.0145 | -0.0165 |
| -0.025 | -0.0250 | -0.0270 | -0.0290 | -0.0308 | -0.0326 | -0.0343 | -0.0359 | -0.0374 |
| -0.05 | -0.0500 | -0.0514 | -0.0526 | -0.0539 | -0.0551 | -0.0562 | -0.0572 | -0.0583 |
| -0.075 | -0.0750 | -0.0757 | -0.0763 | -0.0769 | -0.0775 | -0.0781 | -0.0786 | -0.0791 |
| -0.1 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 | -0.1000 |
| -0.125 | -0.1250 | -0.1243 | -0.1237 | -0.1231 | -0.1225 | -0.1219 | -0.1214 | -0.1209 |
| -0.15 | -0.1500 | -0.1486 | -0.1474 | -0.1461 | -0.1449 | -0.1438 | -0.1428 | -0.1417 |
| -0.175 | -0.1750 | -0.1730 | -0.1710 | -0.1692 | -0.1674 | -0.1657 | -0.1641 | -0.1626 |
| -0.2 | -0.2000 | -0.1973 | -0.1947 | -0.1922 | -0.1899 | -0.1877 | -0.1855 | -0.1835 |



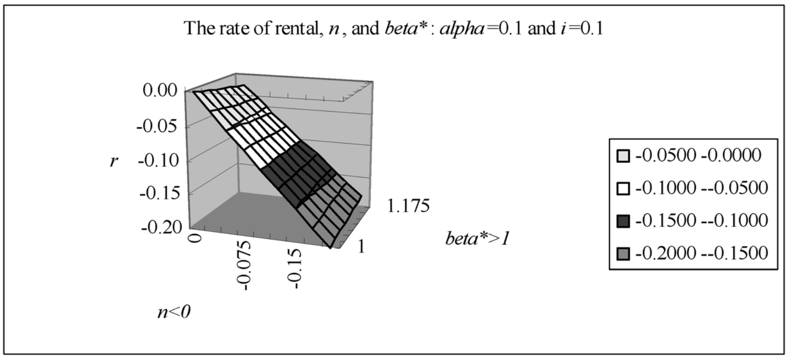




**Figure A3-2 The rate of rental, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$**

### Table A3-3 The growth rate of output, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 1** | **0.1** | **0.1** | $g_Y{}^* = r(i\ beta^*)/\alpha$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0.025 | 0.0649 | 0.0592 | 0.0535 | 0.0478 | 0.0421 | 0.0364 | 0.0307 | 0.0250 |
| 0.02 | 0.0597 | 0.0540 | 0.0483 | 0.0427 | 0.0370 | 0.0313 | 0.0257 | 0.0200 |
| 0.015 | 0.0545 | 0.0488 | 0.0432 | 0.0376 | 0.0319 | 0.0263 | 0.0206 | 0.0150 |
| 0.0125 | 0.0519 | 0.0463 | 0.0406 | 0.0350 | 0.0294 | 0.0238 | 0.0181 | 0.0125 |
| 0.01 | 0.0493 | 0.0437 | 0.0381 | 0.0324 | 0.0268 | 0.0212 | 0.0156 | 0.0100 |
| 0.0075 | 0.0467 | 0.0411 | 0.0355 | 0.0299 | 0.0243 | 0.0187 | 0.0131 | 0.0075 |
| 0.005 | 0.0441 | 0.0385 | 0.0329 | 0.0273 | 0.0218 | 0.0162 | 0.0106 | 0.0050 |
| 0.0025 | 0.0415 | 0.0359 | 0.0303 | 0.0248 | 0.0192 | 0.0136 | 0.0081 | 0.0025 |
| 0 | 0.0389 | 0.0333 | 0.0278 | 0.0222 | 0.0167 | 0.0111 | 0.0056 | #DIV/0! |

The Z axis: for $g_Y{}^*$ The X ais is for *n*, and the Y axis is for *beta**.

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 2** | **0.1** | **0.1** | $g_Y{}^* = r(i\ beta^*)/\alpha$ | | | | | *beta** |
| *n* | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |
| 0 | 0.0389 | 0.0333 | 0.0278 | 0.0222 | 0.0167 | 0.0111 | 0.0056 | #DIV/0! |
| -0.025 | 0.0008 | 0.0107 | 0.0028 | -0.0042 | -0.0103 | -0.0157 | -0.0206 | -0.0250 |
| -0.05 | -0.0131 | -0.0183 | -0.0236 | -0.0289 | -0.0342 | -0.0394 | -0.0447 | -0.0500 |
| -0.075 | -0.0390 | -0.0442 | -0.0493 | -0.0544 | -0.0596 | -0.0647 | -0.0699 | -0.0750 |
| -0.1 | -0.0650 | -0.0700 | -0.0750 | -0.0800 | -0.0850 | -0.0900 | -0.0950 | -0.1000 |
| -0.125 | -0.0910 | -0.0958 | -0.1007 | -0.1056 | -0.1104 | -0.1153 | -0.1201 | -0.1250 |
| -0.15 | -0.1169 | -0.1217 | -0.1264 | -0.1311 | -0.1358 | -0.1406 | -0.1453 | -0.1500 |
| -0.175 | -0.1429 | -0.1475 | -0.1521 | -0.1567 | -0.1613 | -0.1658 | -0.1704 | -0.1750 |
| -0.2 | -0.1689 | -0.1733 | -0.1778 | -0.1822 | -0.1867 | -0.1911 | -0.1956 | -0.2000 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 3** | **0.1** | **0.1** | $g_Y{}^* = r(i\ beta^*)/\alpha$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0.025 | 0.0250 | 0.0222 | 0.0193 | 0.0165 | 0.0136 | 0.0108 | 0.0079 | 0.0051 |
| 0.02 | 0.0200 | 0.0172 | 0.0143 | 0.0115 | 0.0087 | 0.0058 | 0.0030 | 0.0002 |
| 0.015 | 0.0150 | 0.0122 | 0.0094 | 0.0065 | 0.0037 | 0.0009 | -0.0019 | -0.0047 |
| 0.0125 | 0.0125 | 0.0097 | 0.0069 | 0.0041 | 0.0012 | -0.0016 | -0.0044 | -0.0072 |
| 0.01 | 0.0100 | 0.0072 | 0.0044 | 0.0016 | -0.0012 | -0.0040 | -0.0068 | -0.0096 |
| 0.0075 | 0.0075 | 0.0047 | 0.0019 | -0.0009 | -0.0037 | -0.0065 | -0.0093 | -0.0121 |
| 0.005 | 0.0050 | 0.0022 | -0.0006 | -0.0034 | -0.0062 | -0.0090 | -0.0118 | -0.0145 |
| 0.0025 | 0.0025 | -0.0003 | -0.0031 | -0.0059 | -0.0086 | -0.0114 | -0.0142 | -0.0170 |
| 0 | #DIV/0! | -0.0028 | -0.0056 | -0.0083 | -0.0111 | -0.0139 | -0.0167 | -0.0194 |

| | *alpha* | *i* | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Case 4** | **0.1** | **0.1** | $g_Y{}^* = r(i\ beta^*)/\alpha$ | | | | | *beta** |
| *n* | 1 | 1.025 | 1.05 | 1.075 | 1.1 | 1.125 | 1.15 | 1.175 |
| 0 | #DIV/0! | -0.0028 | -0.0056 | -0.0083 | -0.0111 | -0.0139 | -0.0167 | -0.0194 |
| -0.025 | -0.0250 | -0.0277 | -0.0304 | -0.0331 | -0.0358 | -0.0385 | -0.0413 | -0.0440 |
| -0.05 | -0.0500 | -0.0526 | -0.0553 | -0.0579 | -0.0606 | -0.0632 | -0.0658 | -0.0685 |
| -0.075 | -0.0750 | -0.0776 | -0.0801 | -0.0827 | -0.0853 | -0.0878 | -0.0904 | -0.0930 |
| -0.1 | -0.1000 | -0.1025 | -0.1050 | -0.1075 | -0.1100 | -0.1125 | -0.1150 | -0.1175 |
| -0.125 | -0.1250 | -0.1274 | -0.1299 | -0.1323 | -0.1347 | -0.1372 | -0.1396 | -0.1420 |
| -0.15 | -0.1500 | -0.1524 | -0.1547 | -0.1571 | -0.1594 | -0.1618 | -0.1642 | -0.1665 |
| -0.175 | -0.1750 | -0.1773 | -0.1796 | -0.1819 | -0.1842 | -0.1865 | -0.1888 | -0.1910 |
| -0.2 | -0.2000 | -0.2022 | -0.2044 | -0.2067 | -0.2089 | -0.2111 | -0.2133 | -0.2156 |

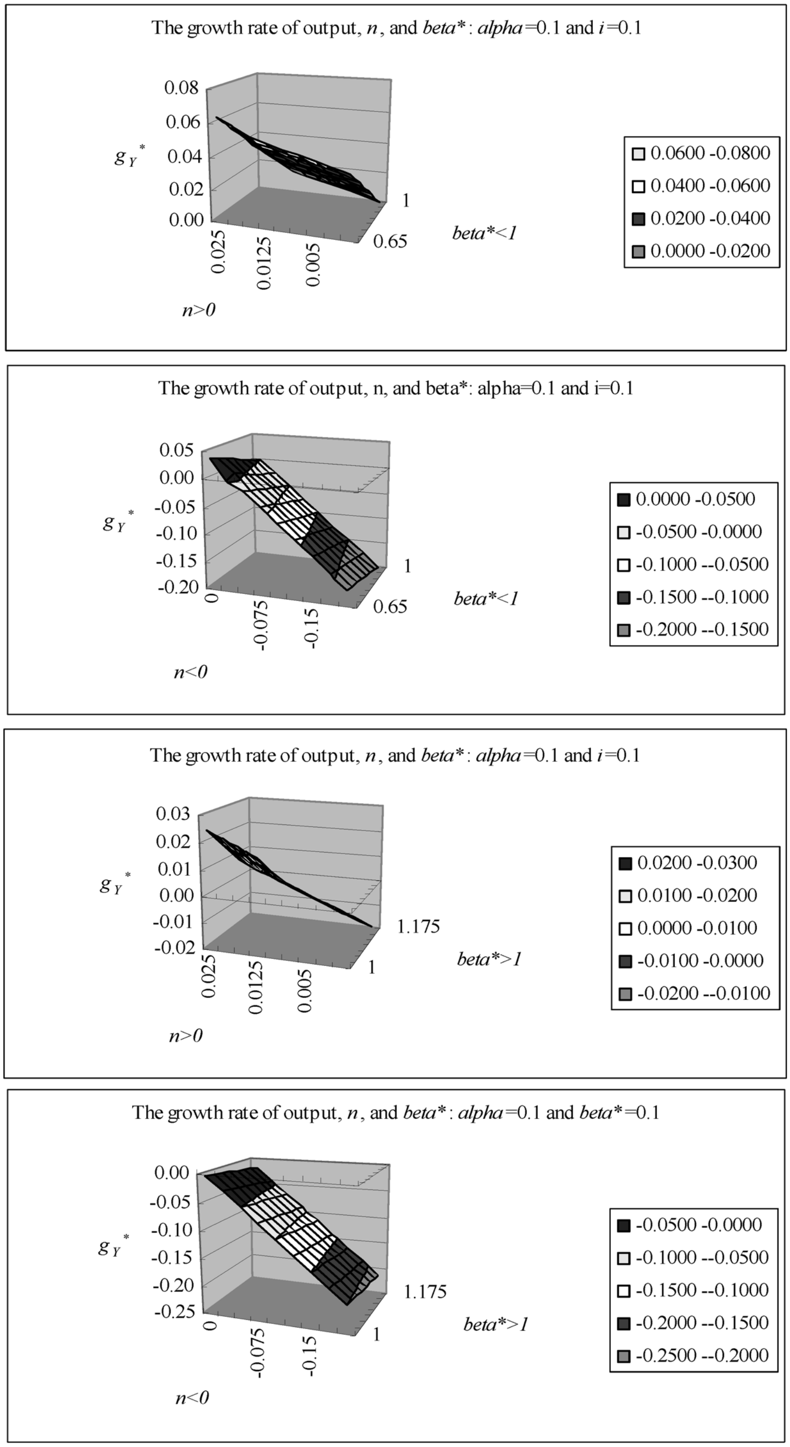



**Figure A3-3 The growth rate of output, *n*, and *beta*$^*$, using $\alpha$ = 0.1 and *i* = 0.1**

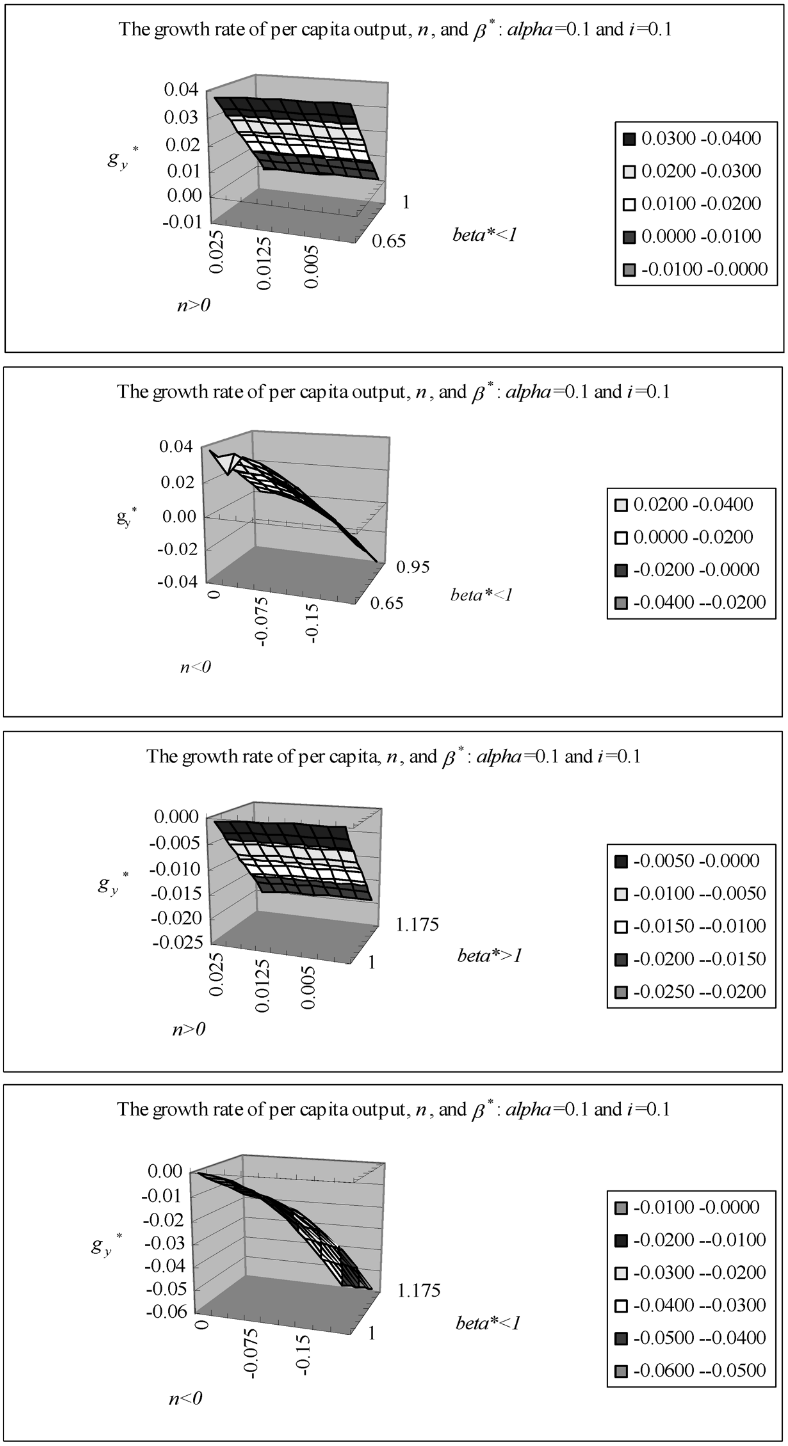
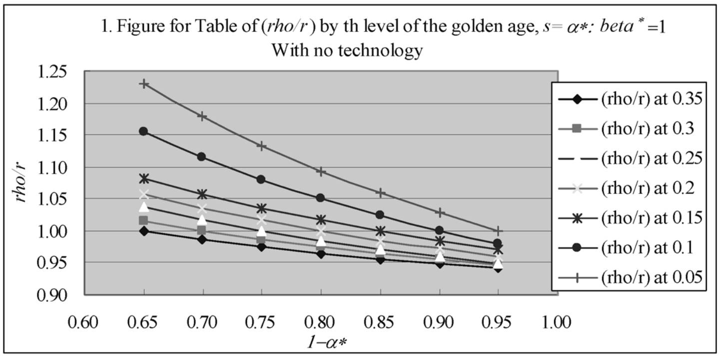
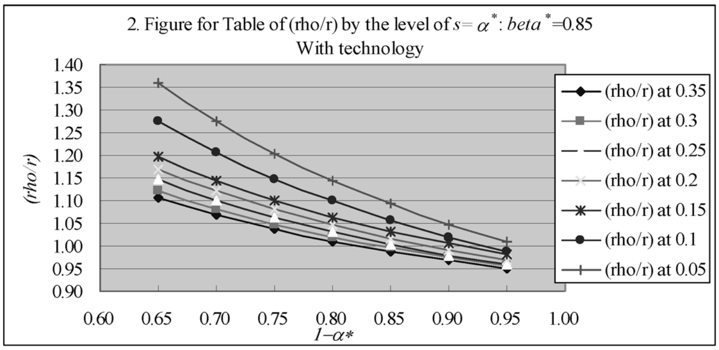
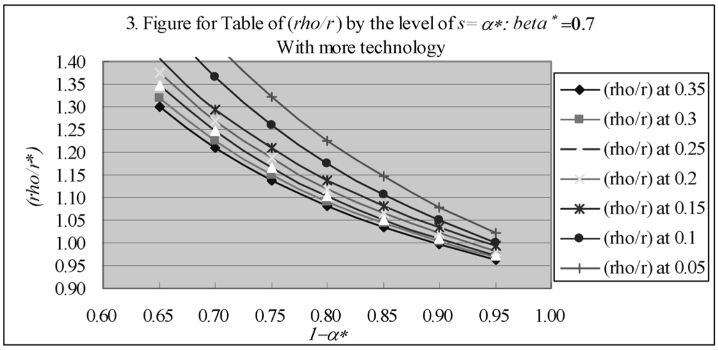



**Figure A3-4 The growth rate of per capita output, *n*, and *beta**, using $\alpha = 0.1$ and $i = 0.1$**



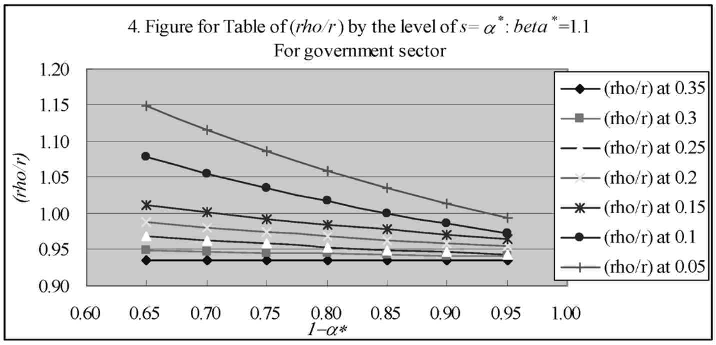




**Figure A4 Figure for Table (*rho*/*r*) by the level of the golden age, *s = alpha*, and by *beta****