

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

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Abstract

This paper explains, with tables and figures in detail, related processes in my simulations used for International Atlantic Economic Conference, Warsaw, on 10 April 2008. The purpose of this paper is that policy-makers by country are able to get useful results in simulations for deficits and debts. Against a common understanding that budget and government's activities produce no earnings, this paper endogenously measures a 'minus' rate of return and a 'minus' relative share of capital in the government sector after government saving becomes minus due to cumulative huge deficits by year. The upper limits of the *EMU* rule of 3% and 60% each to *GDP* are reviewed not exogenously but endogenously. The simulations are composed of three aspects: (1) simulations to De Grauwe's (2005) necessary condition for solvency of debts in the total economy, (2) simulations to the financial degree of solvency, and (3) simulations to the endogenous degree of solvency. These three aspects respect the framework of De Grauwe, P. and the above (1) and (2) follow his intention but the above (3) overcomes his defects and measures numerical levels of solvency, presenting endogenous set of equations whose clue is the cost of capital and the valuation ratio.

The above (1) shows that his condition holds at deficit = 0 in the continuous time. The above (2) and (3) are measured by sector (the government and private sectors) and in the total economy under the rule of sum and, each shows the level of solvency using the magnitude/degree, where (2) introduces the growth rate of output at convergence and (3) introduces both the rate of return and the growth rate of output each at convergence. The relationship between the interest rate of debts and 'the deficit to debt' by year is temporarily clarified in the financial degree of solvency. The degree of solvency is ultimately indicated in the endogenous degree of solvency, by using an endogenous valuation ratio using the cost of capital (i.e., the rate of return less the growth rate of output each at convergence). The horizontal asymptote of the valuation ratio is 1.0 and the vertical asymptote is zero, to the cost of capital. This implies that if the cost of capital is extremely high the valuation ratio reduces close to 1.0 and, if the cost of capital is close to zero, the valuation ratio is maximized. And, these results are all traced back to the rate of technological progress. If the valuation ratio at the government sector is 1.0, the valuation value of capital equals national net wealth, which presents an answer to Barro, R. (1974). For simulations, this paper uses both the data of Japan and the US 1997–2005 in KEWT data-sets (JER 11(2), 2007) and the data in case studies for the developed and developing countries. There is no contradiction between the equation in (1) and those in (2) and (3), where 'the interest rate less the current *GDP* growth rate' in (1) corresponds with the cost of capital in (3).

JEL classification: E62, E22 H62, H54

Click CiNii (NII Scholarly and Academic Information Portal, National Institute of Informatics, Japan) and input Kamiryo for his abstract of papers after 1997.

1. Introduction

The *EMU* rule sets upper limits of deficits 3% and debts 60% each to *GDP*. The author in this paper clarifies a theoretical foundation of this rule and examines this rule, since in the literature there has been found no theoretical solution to the *EMU* rule that was derived empirically. De Grauwe, P. (6th ed., 2005, 225) showed a necessary condition for solvency of national debts, using differential in the continuous time and externally using the interest rate and the growth rate of *GDP*. The author endogenously reviews his forms and applies the author's own equations to this subject. The author uses a set of case studies and simulations, concentrating on deficits and debts and eliminating the influences of the balance of payments.

Consecutive deficits by year result in a minus government saving, similarly to the saving in the balance of payments. This is true. Nevertheless, government returns remain untouched or not measured in national accounts of the *SNA*. The relationship between operating surplus and profits/returns of the private sector, accordingly, remains in vague. De Grauwe (ibid.) takes the relationship between the interest rate and the growth rate of *GDP*, yet both being externally given. Now, when an endogenous growth model is formulated as in Kamiryo, H. (2005b), the interest rate will be replaced by an endogenous rate of return (as the natural rate of interest) and likewise, the growth rate of *GDP* will be replaced by an endogenous growth rate of output (i.e., output as national disposable income that meets the equivalent to three aspects). In this case, how are returns by sector measured? When returns of the government sector are endogenously measured, the *EMU* rule will be interpreted correctly with new facts from simulations. And the fiscal characteristics by country will be clarified so as to justify urgent fiscal and financial policies.

The *EMU* rule has been reviewed by using gross or net investment due to the offset of depreciation, where budget deficit or surplus is shown by the difference of saving and investment. The whole framework between the balance of payments and budget deficits was fundamentally stated by Crowther, G. (1957) and later by Dornbusch, R. (1980).¹⁾ However, the author indicates that their frameworks must be modified in the case of minus saving by sector (the government and private sectors) and that this modification needs, at the macro level, utility measurement and an endogenous model. As a result, if related parameters were measured then

1) Kamiryo, H. (2007b) discusses a whole framework of the balance of payments and budgeting in terms of saving and investment.

theoretical adequacy and its proof to the *EMU* rule will be possible. This is the author's intention in this paper, where the endogenous cause and effect relationship between deficits & debts and related variables are theoretically and empirically shown.

Let the author first review Balassone, F. and Franco, D. (2000, 217–224) that compared the stylized version of the *EMU* rule with the proposal of Modigliani et al (1998), the German model, and the UK model (1997). This comparison is based on the total economy values, where each denominator is *GDP* (without connecting with the government output). The author, however, advocates that the total economy be divided into the government and private sectors under the rule of aggregate/sum and then the analytical results of the government sector will be transformed to a total economy base, by replacing the government output shown in denominators with the output of the total economy, using government share of output. This process is inevitable to obtain endogenous results first at the government sector. Otherwise, the research stops at the calculation of saving, government saving, and private saving or corresponding investment, as shown in the literature. Then government returns remain still unsolved.

To address and satisfy the above requirements, the author uses Kamiryo Endogenous World Table (KEWT) by country and by sector, 1960–2005 (2007b). In this paper, the author first shows the relationship between deficits and debts, after briefly reviewing De Grauwe's framework. Second, the author deepens the relationships between deficits & debts and government saving via government investment. For this purpose, the author simulates twelve cases using Cases 1 to 3 (combining government investment with total-economy investment) and each four sub-cases (four different levels of deficits & debts under a given balance of payments). These twelve cases are separately set for a developed and a developing country, where a given balance of payments and related parameters significantly differ. For these simulations, the original endogenous growth model was devised (without applying the speed of convergence to each time in the transitional path) so that the results of the current situation and those at convergence are directly connected. This direct approach to the situation at convergence is possible by using equations at convergence and simulating related variables by the rate of increase/decrease in the current level of investment.

2. Algorithm to the total economy, the government sector, and the private sector

The fiscal rule will be searched when the total economy is divided into the government and

private sectors, where the rule of aggregate prevails. The author denotes subscript ‘*G*’ for the government sector and subscript ‘*PRI*’ for the private sector. In the current *SNA* (a system of national accounts 1993), the total economy is divided into non-financial corporations, financial corporations, general government, households (including private incorporated enterprises, and private non-profit institutions serving households. In the author’s model, wages paid to households are theoretically divided into the two sectors after redistribution of taxes. The above two-sector model is justified by introducing the labor function to consumption by sector into the model, where wages and returns are theoretical instead of actual and, labor is replaced by wages. $C = C_G + C_{PRI}$: Consumption, where C_G is government consumption and C_{PRI} is consumption of the private sector.

$S = S_G + S_{PRI}$: Saving, where S_G is government saving and S_{PRI} is saving of the private sector.

$I = I_G + I_{PRI}$: Net investment (after depreciation) I , where I_G and I_{PRI} are each net investment of the government and private sectors. Net is used since saving is net; not gross before depreciation.

$Y = C + S$: National disposable income (*NDI*), which is shown by Y , differs from *GDP* and the model uses the $Y = NDI$ base instead of the *GDP* base. Note that *NDI* includes net primary income from abroad if it is available.

$BOP = S - I$: The balance of payments. This presents a base in an open economy.

$\Delta D \equiv S_G - I_G$: budget deficit, where deficit expresses minus. Deficit is replaced by $\Delta D = (S - S_{PRI}) - (I - I_{PRI})$. Similarly, debt D expresses minus.

$W = W_G + W_{PRI}$: Theoretical wages, where W_G is those of the government sector and W_{PRI} is those of the private sector.

$\Pi = \Pi_G + \Pi_{PRI}$: Theoretical returns, where Π_G is those of the government sector and Π_{PRI} is those of the private sector.

$Y = W + \Pi$: Theoretical national disposable income, which equals actual *NDI* by using the wage function of consumption/utility by sector (soon below): $W + \Pi = Y = C + S$. This is expressed reversely using a notion that ‘wages are mostly used for consumption goods while returns are mostly used for capital goods,’ as suggested by Lindahl, E. (1969, 54–57),

$Y = W + \Pi = (W_C + W_S) + (\Pi_C + \Pi_S) = C + S$, where ‘equivalent of three aspects,’ first designed by Meade, J. E. and Stone, J. R. N. (1969, 344–345), now exactly holds.

$Y_G = W_G + \Pi_G$: Government output and its theoretical wages and returns.

$Y_{PRI} = W_{PRI} + \Pi_{PRI}$: Private output and its theoretical wages and returns.

Theoretical wages and returns are measured as follows: For the total economy,

$W = C / \left(\frac{rho}{r} \right)$ or $1 - \alpha = c / \left(\frac{rho}{r} \right)$ is used under $1 - \alpha = W / Y$ and $c = C / Y$, where rho is the discount rate of consumption and r is the discount rate of saving, which equals the rate of return. At the macro level by year, utility social welfare is shown each as the present value of consumption and that of saving each in infinite time, where utility social welfare of consumption equals that of saving.²⁾ As a result, $\Pi = Y - W$ is obtained by offsetting both utility social welfares. The wage function of consumption in the US 1960–2005 is shown as $\left(\frac{rho}{r} \right) = 13.301c^2 - 22.608c + 10.566$. However, the above measurement of wages and returns must be consistent with theoretical capital measurement, which will be justified by the matching test and the smoothening test.³⁾

For the government sector, the wage function of consumption is neutrally specified by $\left(\frac{rho}{r} \right)_G = 1.0$, where $W_G = C_G$ holds. As a result, $\Pi_G = Y_G - W_G$ is obtained. For the private sector, both wages and returns are obtained each as the difference between income and wages (directly; without using the wage function of consumption): $W_{PRI} = W - W_G$ and $\Pi_{PRI} = Y_{PRI} - W_{PRI}$.

Finally, the current given data in the data-sets are: budget surplus or deficit, $\Delta D = S_G - I_G$, lending or debt D , government consumption, C_G , government net investment, I_G , in addition to the data of the total economy such as the balance of payments, $S - I$, consumption, C , net investment, I , and population, L .

3. Methodology for the endogenous fiscal rule

The author will examine the *EMU* rule for deficits and debts, starting with De Grauwe's framework and shifting his differential to the corresponding difference at the discrete time. His principle does not change regardless of continuous or discrete yet, at the discrete time, new facts are found when an endogenous growth model is introduced into its framework. By so doing, the cost of capital and the valuation ratio are measured at convergence, which will be discussed in the next section. Among others, the most decisive finding in the condition for solvency is that

2) This idea starts with Cass' (1964) 'instantaneous' utility at the micro level.

3) The matching test and the smoothening test by year guarantee the consistency among each item of the data-sets in KEWT so that the capital-output ratio by country and sector is measured theoretically and without revise in the long run. There is no data-set that publishes capital and the capital-output ratio by sector today, except for those of the corporate sector by OECD (see Schreyer, P. (2004a and b), since Pen World Table stopped publishing the capital-labor ratio after 1995 by some reasons (see, PWT 6.2, 2004).

the cost of capital is least or the valuation ratio is maximized when deficits and debts are zero, which is the interpretation of his differential. His difference between deficits and the product of debts and ‘the interest rate of debts less the growth rate of *GDP*’ externally shows the level of insolvency yet, the difference between his interest rate and the rate of return at convergence does not differ significantly. This implies that the market interest rate and an endogenous rate of return do not differ significantly when the neutrality of financial assets is respected by the central bank.

According to De Grauwe, P. (2005, 147–148, 222–225), the relationship between debts, \dot{b} , and deficits, b , are formulated using $\dot{b} = (g - t) + (r - x)b$, where g is the sum of government expenditures (excluding interest paid) and investment, t is taxes, r is the interest rate of debts, and x is the nominal growth rate of *GDP*. This equation holds at the current situation without introducing endogenous contents. The author tests this equation and examines it from the following three points of view: (1) the relationship between deficits and $g - t$, (2) the relationship between deficits and debts, and (3) the relationship between interest paid and theoretical rate of return.

For these examinations, the author changes some of his notations: (1) surplus is plus and deficit is minus, and similarly, lending is plus and debt is minus, where b is expressed as $-b$, and (2) $t - g$ is shown by $g - t$. Also, the author uses notations that are consistent with KEWT data-sets, where his ‘ b ’ is expressed by ‘ d ’ (since B is the qualitative to quantitative investment), his ‘ g ’ by $C_G + I_G$, his ‘ t ’ by t_{AX} , his ‘ r ’ by $r_{(DEBT)}$, his ‘ x ’ by g_Y^* . Thus,

$$\Delta d = (t_{AX} - (C_G + I_G)) + (r_{(DEBT)} - g_Y^*)d. \quad (1)$$

For solvency, his equation shows: $\dot{b} = 0$, then $(r - x)b = t - g$ holds. This comes from $G - T + rB = \frac{dB}{dt} + dM / dt$, where $dM / dt = 0$. His $(r - x)b = t - g$ under $\dot{b} = 0$ is shown by using the author’s notation, $d \equiv D / Y$, where ‘ d ’ shows lending and ‘ $-d$ ’ shows debt.

$$(r_{DEBT} - g_Y^*)(-d) = t_{AX} - (C_G + I_G) \text{ under } \Delta d = 0, \quad (2)$$

where taxes are equal to the sum of expenditures, investment, and interest paid, each of the government sector. Interest paid is $R_{DEBT} = r_{DEBT} \cdot D$.

Eq. 2 implies the following two points where the author cites some of his propositions:

1. If the nominal interest rate exceeds the growth rate of output at convergence, then, the government must make sure that the primary budget, $t_{AX} - (C_G + I_G)$, has a surplus.⁴⁾

4) This statement is consistent with the concept of ‘the present value constraint’ to budget and the balance of payments (see Ahmed, S. & Rogers, J. H. (1995), which the author will comment when the cost of capital at convergence is taken into consideration below).

2. If not, debts will increase without limit.

Eq. 2 will be replaced by the next Eq. 3 when the rate of return at convergence, r^* , is used instead of the nominal interest rate.

$$(r^* - g_Y^*)(-d) = t_{AX} - (c_G + i_G). \quad (3)$$

The difference between the author's Eq. 3 and De Grauwe's Eq. B19.5 is that the author uses an endogenous cost of capital, $r^* - g_Y^*$, instead of 'the nominal interest rate less the nominal growth rate of output at the current situation'. The author calls Eq. 3 the financial degree of solvency. Then, the above proposition of De Grauwe is restated as the endogenous proposition:

1. If the cost of capital at convergence is plus, then, the government must make sure that the primary budget, $t_{AX} - (c_G + i_G)$, has a surplus.
2. If not, debts will increase without limit.

This revised proposition of the author implies that primary balance will be plus if the cost of capital at convergence of the total economy is plus. Neglecting the difference between the current/actual growth rate of *GDP* and the endogenous growth rate of output at convergence, the difference between the above two propositions comes from the difference between the nominal interest rate and the endogenous rate of return at convergence. Furthermore, according to the rule of aggregate, 'the total economy = the government sector + the private sector' holds in the endogenous growth model of KEWT. This is interpreted such that economic activities of the private sector are calculated by those of the total economy determined by the balance of payments less those of the government sector determined by budget (deficit & debts).

It is convenient for the rule of aggregate to use the government share of output. First, in the case of the growth rate of government output at convergence,

$$\begin{aligned} \text{Similarly to } g_Y^* &= i(1 - \beta^*) = (Y^* - Y_0)/Y_0, \\ g_{Y(G)}^* &= i_G(1 - \beta_G^*) = (Y_G^* - Y_{(G)0})/Y_{(G)0}. \end{aligned} \quad (4)$$

To make the rule of aggregate to hold, the denominator of $g_{Y(G)}^*$ must be the output of the total economy, Y , instead of government output, Y_G . Then, the use of the government share of output, Y_G/Y , is inevitable:

$$g_{Y(G)/Y}^* \equiv \frac{\Delta Y_G}{Y} = \frac{\Delta Y_G}{Y_G} \cdot \frac{Y_G}{Y} \text{ and } g_{Y(PRI)/Y}^* \equiv \frac{\Delta Y_{PRI}}{Y} = \frac{\Delta Y_{GPRI}}{Y_{PRI}} \cdot \frac{Y_{PRI}}{Y}. \quad (5)$$

Thus, Eq. 6 holds under the rule of aggregate.

$$g_Y^* = g_{Y(G)/Y}^* + g_{Y(PRI)/Y}^*. \quad (6)$$

Second, the rate of return at convergence is obtained by taking advantage of the Petersburg

coefficient⁵⁾ that connects the rate of return with the growth rate of output at convergence, where the Petersburg coefficient is $\alpha/(i \cdot \beta^*)$ in the total economy and $\alpha_G/(i_G \cdot \beta_G^*)$ in the government sector.

$$r^* = g_Y^* \left(\frac{\alpha}{i \cdot \beta^*} \right) \text{ and } r_G^* = g_{Y(G)}^* \left(\frac{\alpha_G}{i_G \cdot \beta_G^*} \right). \quad (7)$$

$$\text{And, } r_{G/K}^* \equiv \frac{\Pi_G^*}{K} = \frac{\Pi_G^*}{K_G} \cdot \frac{K_G}{K} \text{ and } r_{PRI/K}^* \equiv \frac{\Pi_{PRI}^*}{K} = \frac{\Pi_{PRI}^*}{K_{PRI}} \cdot \frac{K_{PRI}}{K}. \quad (8)$$

where the government and private shares of capital are K_G/K and K_{PRI}/K .

Finally, the cost of capital at convergence, $r^* - g_Y^*$, is similarly shown, avoiding the use of $r_G^* - g_{Y(G)}^*$:

$$r^* - g_Y^* = (r_{G/K}^* - g_{Y(G)/Y}^*) + (r_{PRI/K}^* - g_{Y(PRI)/Y}^*). \quad (9)$$

The cost of capital corresponds with ‘the interest rate less the *GDP* growth rate’ in De Grauwe’s equation.

As a result, the valuation ratio, $v = V/K$, is obtained as the rate of return divided by the corresponding cost of capital at convergence.

$$v = r^* / (r^* - g_Y^*), v_G = r_G^* / (r_G^* - g_{Y(G)}^*), \text{ and } v_G = r_G^* / (r_G^* - g_{Y(G)}^*). \\ v = r^* / (r^* - g_Y^*) = r_{G/K}^* / (r_{G/K}^* - g_{Y(G)/Y}^*) + r_{PRI/K}^* / (r_{PRI/K}^* - g_{Y(PRI)/Y}^*). \quad (10-1)$$

$$v = 1 + \frac{g_Y^*}{r^* - g_Y^*} \text{ and } v = \frac{\alpha}{\alpha - i \cdot \beta^*} = \frac{r^*}{r^* - r^* \left(\frac{i \cdot \beta^*}{\alpha} \right)}, \text{ where } \beta^* \text{ is a key.} \quad (10-2)$$

The horizontal asymptote of the valuation ratio is 1.0 and vertical asymptote crosses the origin, when the X axis shows the cost of capital. The valuation value of capital corresponds with national net wealth in the literature: $E_G = V_G$ (for discussions, see Barro, R. (1974, 1989)). If $v_G = 1.0$, capital equals net wealth in the government sector. If $v_G < 1.0$, where deficits and debts always exist to some extent, net wealth reduces by $K_G - V_G$. ‘A set of Eqs. 7, 8, 9, and 10’ exists as the ‘endogenous’ degree of solvency.

When the valuation ratio is not measured endogenously, the leverage of debts to net equity/wealth, E , of a country is simply measured using $E = K + D$:

$$l_{EV} = -D / (K + E_{ABROAD} + D) = -D / E, \text{ if the present value of net primary income from}$$

5) The related equations were wholly discussed in Kamiryo, H. (2007a), whose earlier ideas came from Durand, D. (1957) and Phelps, E. S. (1961, 1965, and 1966), although both were exogenously discussed, and whose endogenous idea comes from Kamiryo, H. (2004). Here, ‘endogenous’ implies that the data of national accounts and the data for the Cobb-Douglas production function are consistently used.

abroad (E_{ABROAD}) is taken into consideration. Or,

$$\begin{aligned} d_{D/K} &\equiv -D/K \text{ and } l_{EV} \equiv d_{D/K} / (1 - d_{D/K}), \text{ where } E = V = K \text{ if the } BOP/Y = 0. \\ l_{EV(G)} &= -D/(K_G + D) = -D/E_G \text{ for the government Sector.} \end{aligned} \quad (10)$$

The relationship between the capital-output ratio and the ratio of money (M2) to output (Marshall's k) is essential to the test of the neutrality of financial assets from real assets. This will be discussed with related equations in a separate paper.

4. Preliminary findings using Japan and the US and case study of fiscal rule

The above methodology is first applied to the data-sets of Japan and the US, 1997–2005 in KEWT and second, to the case study to fiscal rule. The data-sets of KEWT are derived by using the equations to parameters and variables at convergence and recursive programming to the transitional path after measuring the speed of convergence. The comparison of Japan and the US takes data from the data-sets of KEWT. However, for the case study of fiscal rule that uses the percentage changes in net investments of the total economy and the government sector, the author simplifies the algorithm that necessitates the speed of convergence, which the author calls the simplified algorithm.

4.1 Empirical results using Japan and the US

For empirical findings in deficits and debts, this section tests the condition for solvency by using Eq. 3, $(r_{DEBT} - g_Y^*)(-d) = t_{AX} - (c_G + i_G)$, in the financial degree of solvency. In this case, the interest rate remains externally given but an endogenous growth rate of output is measured at convergence.

The purpose of the above analysis is to test (1) the difference between deficits ($\Delta d = \Delta D/Y$), and taxes less government expenditures and net investment, $t_{AX} - (c_G + i_G)$, (2) the difference between deficits and the condition for solvency to primary balance, $(r_{DEBT} - g_Y^*)(-d)$, and (3) the difference between $t_{AX} - (c_G + i_G)$ and $(r_{DEBT} - g_Y^*)(-d)$. The rate of increase/decrease of debts, is shown by $\pi \equiv \Delta D/D$. This rate connects ΔD with D in the discrete time and is used for confirmation of related results of Eq. 2. Assuming that deficit is constant over the infinite years, $\pi \equiv \Delta D/D$ shows a kind of the internal rate of return and thus, is able to compare with the interest rate of debts. The author stresses that $\pi \equiv \Delta D/D$ is meaningful in showing the trend of fiscal policy in the long run. The results are shown in **Figures 2 and 3** (in detail, see **Tables A1, A2, A3, and A4** in Appendix). These tables and figures use the data-

sets of KEWT 1.07, whose original data come from *International Financial Statistics Yearbooks* and *Government Finance Statistics Yearbooks*, International Monetary Fund.

The author uses ‘four cases,’ to test the levels of equal-relationships among three items (the growth rate of output at convergence, g_Y^* , $\pi \equiv \Delta d / d$, and the interest rate, $r_{(DEBT)}$). Four cases are: Case 1 XXX shows $g_Y^* \neq \pi \neq r_{(DEBT)}$. Case 2 OOX shows $g_Y^* = \pi \neq r_{(DEBT)}$. Case 3 XOO shows $g_Y^* \neq \pi = r_{(DEBT)}$. Case 4 OOO shows $g_Y^* = \pi = r_{(DEBT)}$. Each case has its four sub-cases.⁶⁾

Here, the author will use additional equations that show some constraints to deficit and the above $\pi \equiv \Delta D / D$. Taxes are equal to government output, since $\Delta D = Tax - (C_G + I_G + R_{(DEBT)})$ and thus, $T_{AX} = Y_G = (S_G - I_G) + C_G + I_G + R_{(DEBT)}$, where output includes interest paid. For primary balance, $\Delta D_{PRIMARY} = C_G + S_G$ holds, where government expenditures do not include interest paid. Thus, using, Y_G / Y ,

$$\frac{T_{AX}}{Y} = Y_G / Y, \text{ where} \quad (11)$$

The government share of output is a sort of tax rate whose taxes include indirect tax.

Then, a base for ‘the present-value constraint to deficit’ in the literature⁷⁾ will be:

$$D = \Delta D / r_{(DEBT)}, \quad (12)$$

where $D = \Delta D \sum_{t=0}^{\infty} \left(\frac{1}{1+r_{(DEBT)}} \right)$ and, the deficit and interest rate are assumed to be constant in infinite time. The above $\pi \equiv \Delta D / D$ is another expression of the interest rate, where $r_{(DEBT)} = \pi$ holds if deficit is constant over the infinite years.

Back to the above tests, (1), (2), and (3); conclusively, first for (1), deficit always equals ‘taxes less government expenditures and net investment.’ This implies that taxes equal consumption and saving of the government sector but exclude interest paid. Then, if taxes reduce, do people enjoy a small government? Assuming that government expenditures and investment remain unchanged, the decrease in taxes equals the increase in deficit. In another words, the size of government is determined by the sum of government expenditures and investment.

Then, the above tests (2) and (3) become the same. (2) or (3) tests the difference between

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- 6) Supplementary four cases are: Case 2 OOX shows $g_Y^* = \pi \neq r_{(DEBT)}$, yet using debt = 290000 instead of debt = 250000 in Case 2. Case 1-2 XXX shows $g_Y^* \neq \pi \neq r_{(DEBT)}$, yet primary-based as deficit less interest. Case 1-3 XXX shows $g_Y^* \neq \pi \neq r_{(DEBT)}$, yet assuming that taxes are the same as those in non-primary deficit. Case 4 OOO shows $g_Y^* = \pi = r_{(DEBT)}$, with no deficit.
 - 7) The literature tests both the balance of payments and deficits simultaneously from the viewpoint of sustainability. For example, see Ahmed, S. & Rogers, J. H. (1995). The paper uses the PVC test in econometrics while the author uses theoretical equations at convergence. For the comments on the above paper, see case study in the next section that discusses the cost of capital.

deficit and the condition for solvency. Before this test, let the author compare the value of the condition for solvency, $(r_{DEBT} - g_Y^*)(-d)$, by year, by case, and by country. In Japan, this value had been minus earlier in 1997 and slightly plus after 1997. In the US, this value has been minus since 1997 yet, gradually less minus or turning to plus later. Exception is Case 000 as a benchmark, where there is neither deficit nor surplus and $(r_{DEBT} - g_Y^*)(-d) = 0$ holds. This implies that solvency is attained at $\Delta d = 0$. This result corresponds with De Grauwe's differential of 'if $\dot{b} = 0$, then $(r - x)b = t - g$ holds.'

Then, the difference between Δd and $(r_{DEBT} - g_Y^*)(-d)$ has shown minus in Japan, and gradually more minus except for that in 2005. In the US, this difference has been minus and plus since 1997 yet, the range of this difference is comparatively much narrow. Recall that Japan's debts to output are the worst case among countries. The above results imply that the level of the difference of $\Delta d - (r_{DEBT} - g_Y^*)(-d)$ is useful for policy-makers. A wrong idea of 'with growth first, primary solvency will follow' is denied. The test of the above difference is, nevertheless, not enough since the interest rate, $r_{(DEBT)}$, may be arbitrary due to a strong fiscal policy to decrease the burden of interest payment. An idea that the central bank should be absolutely neutral to fiscal and financial policies will be denied, as shown in Japan. To compare $r_{(DEBT)}$ with an endogenous rate of return at convergence, r^* , will be inevitable, which will be discussed soon below.

In short, the author compared Japan with the US from the viewpoint of De Grauwe's condition for solvency in this section, by using the growth rate of output at convergence, which is more stable than that at the current situation. Nevertheless, it is expected that the interest rate at the market principle should be examined by the rate of return at convergence.

4.2 Case study of fiscal rule: a developed country versus developing country

For empirical findings in deficits and debts, this section tests Eq. 3, $(r^* - g_Y^*)(-d) = t_{AX} - (c_G + i_G)$, by presenting two typical economies; a developed country and a developing country⁸⁾ and simulating the changes in net investments of the total economy and the government sector. The rates of increase/decrease in net investment of the total economy are eight: 0.1, 0.2, 0.3, 0.4, -0.1, -0.2, -0.3, -0.4. Each simulation is denoted as simu 1, simu 2, simu 3, simu 4, simu -1, simu -2, simu -3, or simu -4. The rates of increase/decrease in net investment of the

8) Each item of the two typical economies uses close-to-actual data, after testing BRICs and other countries in KEWT 1.07, 1960–2005, so that the results are typical in the comparison between a developed country and a developing one.

government sector are also eight: 0.2, 0.4, 0.6, 0.8, -0.2, -0.4, -0.6, -0.8. Each simulation is similarly denoted as simu 1, simu 2, simu 3, simu 4, simu -1, simu -2, simu -3, or simu -4. For each simulation, three cases are distinguished by the combination of the changes in net investments between the total economy and the government sector: Case 1 only simulates the change in net investment of the total economy (where, a given net investment of the government sector remains unchanged). Case 2 only simulates the change in net investment of the government sector (where, a given net investment of the total economy remains unchanged). Case 3 simulates both changes in net investments of the total economy and the government sector. For each case, four sub-cases aiming at the influence on test ratios by the level of Δd and d are set up: The sub-case ‘-1’ shows the condition of $\Delta d = -0.03$ and $d = -0.6$. The ‘-2’ shows the condition of $\Delta d = -0.015$ and $d = -0.3$. The ‘-3’ shows the condition of $\Delta d = 0$ and $d = 0$. And, the ‘-4’ shows the condition of $\Delta d = 0.03$ and $d = 0.1$. As a result, Case 1 is divided into 1-1, 1-2, 1-3, and 1-4. Case 2 is divided into 2-1, 2-2, 2-3, and 2-4. And, Case 3 is divided into 3-1, 3-2, 3-3, and 3-4.

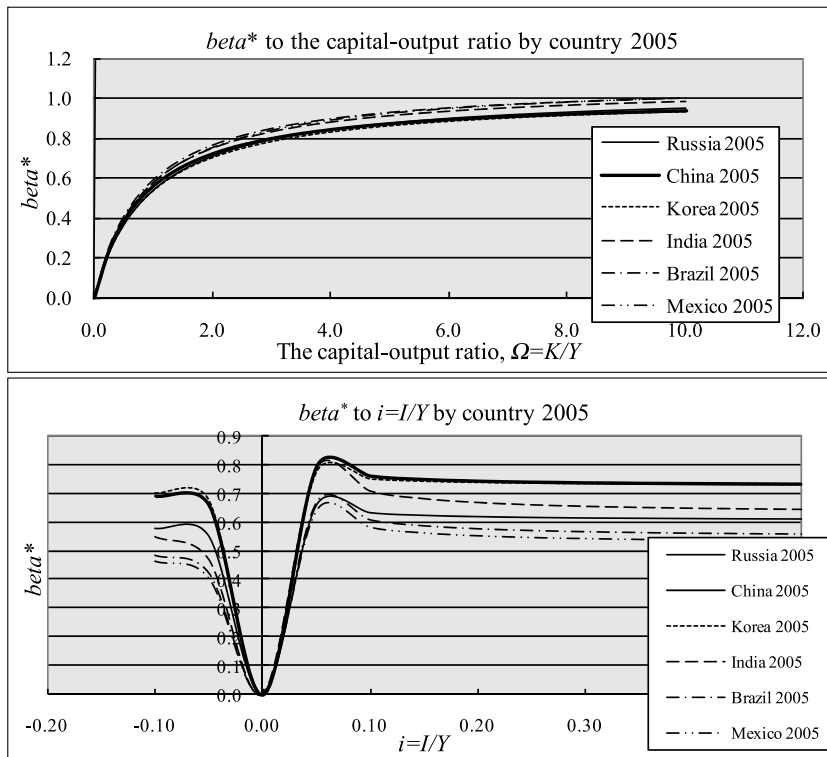
The above cases, furthermore, compare a ‘developed’ country with a ‘developing’ country. Before simulation, the BOP/Y of the former is -0.0655 while the BOP/Y of the latter is 0.0551. The i of the former is 0.1 in Cases 1 and 2 and, 0.075 in Case 3, while the i of the latter is 0.1335 for all three cases. The utility coefficient of the former is 1.13151 for all three cases, while the utility coefficient of the latter is 0.9 in Cases 1 and 2 and 0.8 in Case 3. To make it easy to compare, the growth rate of population/labor in Cases 1 and 2 is set 0.01203 in both countries, while that of Case 3 is set 0.0 in both countries. The growth rate of labor of the government sector in Cases 1 and 2 is set 0.01543 in both countries, while that of Case 3 is set 0.0 in both countries. Note that at $n = 0$ the per capita output growth rate equals the output growth rate, which is more convenient for comparison.

The purpose of the above simulations is to find: (1) How does the growth rate of output at convergence differ by the change in net investment and by the level of deficits and debts? (2) How does the relationship at convergence between the rate of technological progress and $\alpha = \Omega^* \cdot r^*$ differ by the change in net investment and by the level of deficits and debts? (3) How do the condition for solvency, the cost of capital, the valuation ratio, the Petersburg coefficient, and the government share of output differ by the change in net investment and by the level of deficits and debts? The results are shown in **Tables 1, 2, 3, and 4**, with **Figures 4, 5, 6, and 7**.⁹⁾

9) In more detail, see Tables A5 to A8 in Appendix, where Case 3 is shown with algorithm.

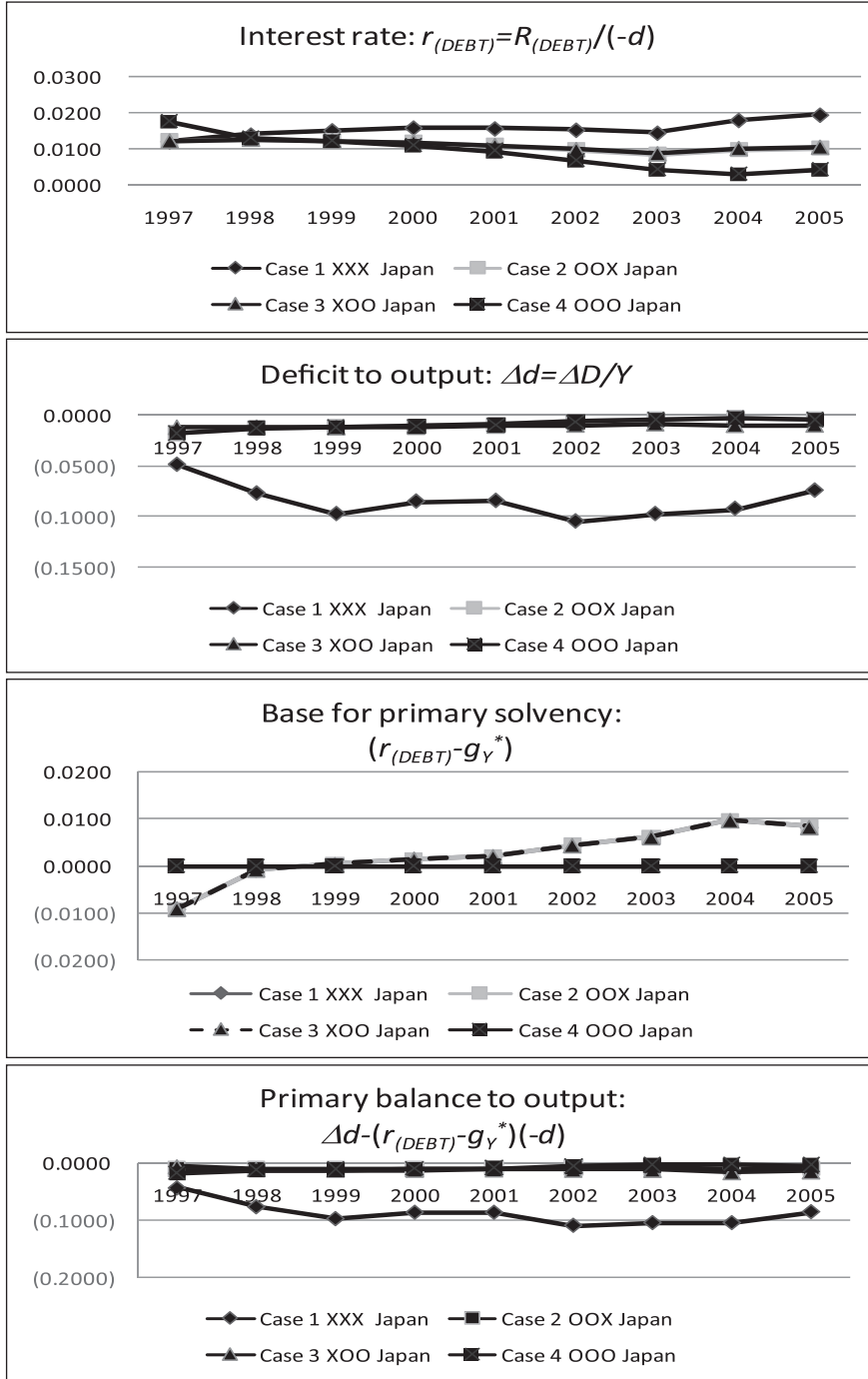
These results will be useful to the interpretation of the *EMU*'s fiscal rule and contribute to improve underlying principles in fiscal and financial policies.

First of all, let the author find the characteristics of the growth rate of output at convergence, g_Y^* , in an open economy. The rate of technological progress as the base of g_Y^* is determined as the product of the ratio of net investment to output, i , and $1 - \beta^*$. The balance of payments to output, once saving is initially given, changes with the rate of increase/decrease of a given net investment, resulting in the change in g_Y^* . At the same time, the capital–output ratio, $\Omega \equiv K/Y$, is tightly related to $1 - \beta^*$. **Figure 1** showed the elasticity of β^* with respect



Note: There are slight differences of elasticity values between developed and developing countries. The above results suggest each optimum range of the capital-output ratio and the ratio of net investment to output. The upper limit of the capital-output ratio of the total economy among countries is roughly 2.5 and that of the private sector is roughly 2.0 among countries due to global competition. The level of the ratio of net investment to output among developing countries is much higher than that among developed countries. When the ratio of net investment to output is less than 0.1, the value of β^* is unstable, which is a strong reason why poor countries cannot get rid of their situation.

Figure 1 Elasticity values of β with respect to the capital-output ratio and the ratio of net investment to output



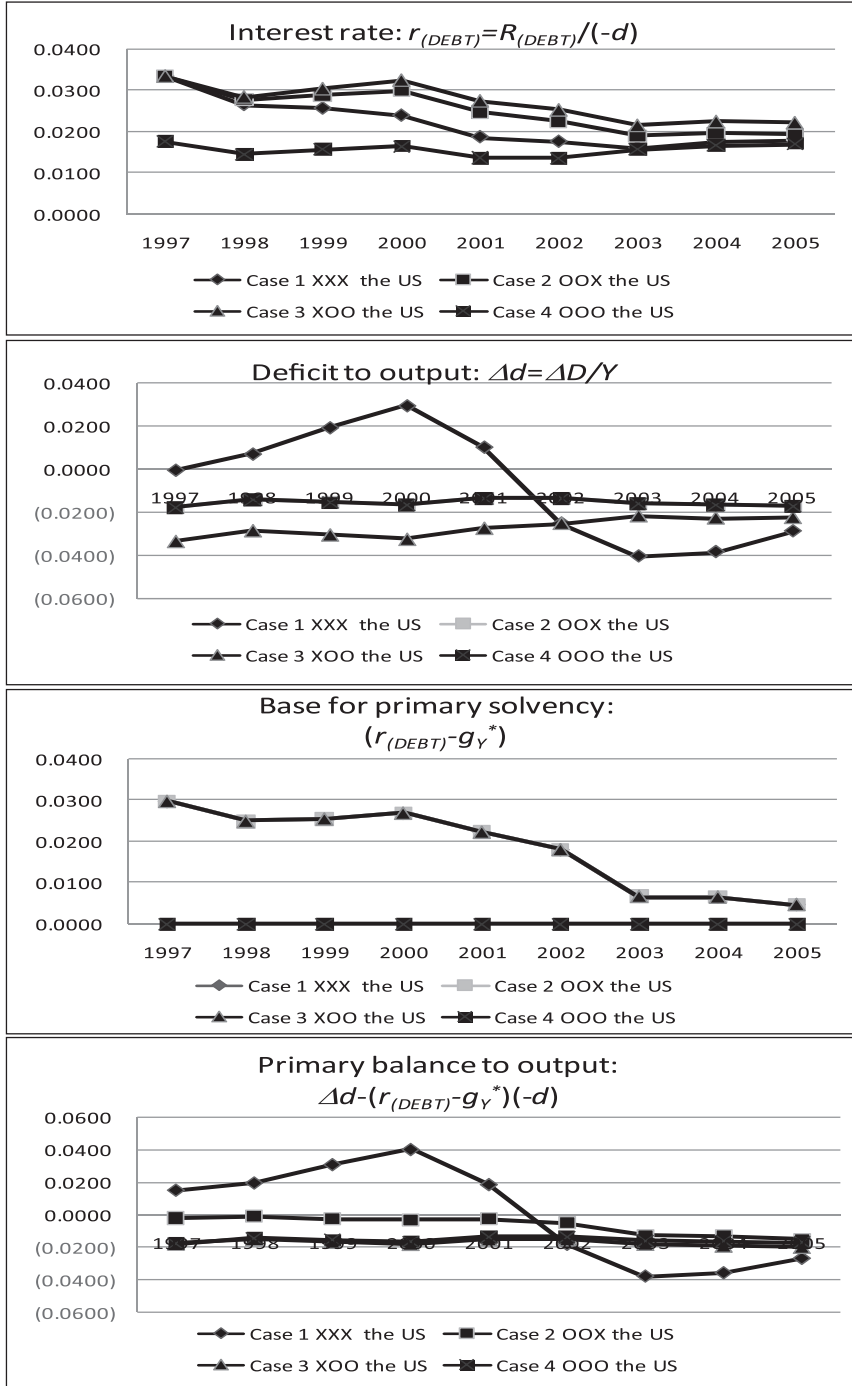
Note: Case XXX shows the actual situation by year, where $g_Y^* \neq \Delta d / d \neq r_{(DEBT)}$.

Figure 2 De Grauwe's (2005) condition for primary solvency: Japan 1997–2005

to the capital-output ratio. If the capital-output ratio of the government sector is extremely high as in Japan, the capital-output ratio of the total economy of a country will be much higher than those of other developed countries. This implies that the rate of technological progress of the total economy have to be lower than those of other developed countries. Besides, if deficits and debts are huge, government saving is extremely minus, which causes the crowding-out of net investment in the private sector. For example, the current rate of increase in net investment of the private sector remains minus in Japan. Therefore, the more plus the balance of payments is the higher the growth rate of output at convergence yet, the increase in the capital-output must be carefully examined.

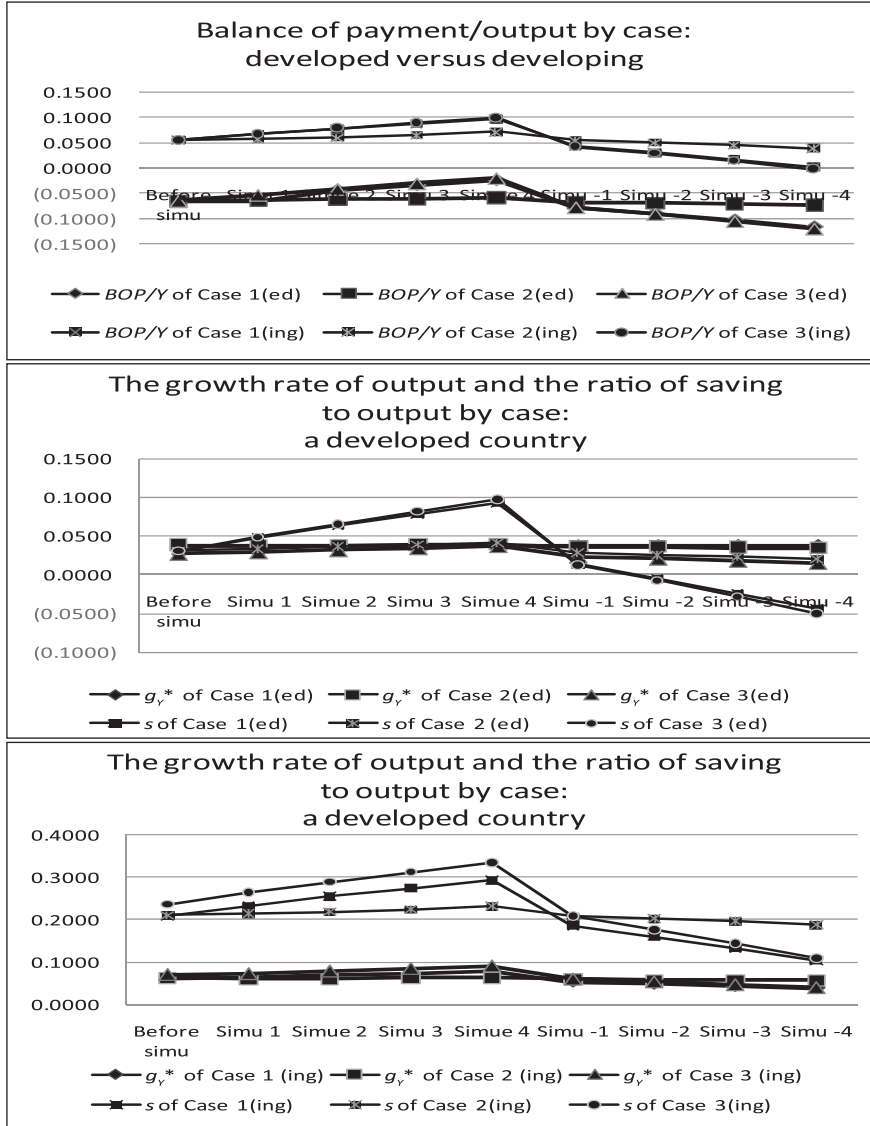
The top of **Figure 4** shows that the growth rate of output at convergence, g_Y^* , is much higher in a developing country than that in a developed country. This is due to the differences of four items; the balance of payments, i , $\Omega \equiv K/Y$, and $1 - \beta^*$. The middle and bottom of Figure 4 compare g_Y^* with the ratio of saving to output, $s = S/Y$, for both a developed country and a developing country. Under a current given *BOP*, the higher the change in net investment is, the higher g_Y^* and s . It is found, however, that the value of s is influenced much more than g_Y^* by the change in net investment. Also, even if the ratio of saving is significantly minus under a minus *BOP*, the growth rate of output decreases rapidly with the decrease in net investment yet it remains always plus. The value of $s = 0$, therefore, will guarantee the corresponding minimum g_Y^* and has its meaning towards a limit to $\Delta d = \Delta D/Y$. Furthermore, $d = D/Y$ decreases with the positive change in net investment and $d = D/Y$ increases with the negative change in net investment. In this respect, investment is neutral to saving and not an evil (under the condition of $1 - \beta_G^* > 1 - \beta^*$) and, investment has its own range of optimum for sustainability.

Now In detail, **Table 1** shows the simulations by case, where observing ratios remain unchanged for sub-cases of $\Delta d = -0.03$ and $d = -0.6$, $\Delta d = -0.015$ and $d = -0.3$, $\Delta d = 0$ and $d = 0$, and $\Delta d = 0.03$ and $d = 0.1$. Observing ratios are five; the growth rate of output, $1 - \beta^*$, the capital-output ratio, the relative share of capital, and the rate of return, each at convergence. For comparison, the author exceptionally added $1 - \beta_G^*$ (that varies by the level of Δd and d) to five ratios. A reason is that if $(1 - \beta_G^*) < (1 - \beta^*)$, government investment should be reduced and vice versa. Table 1 shows that the higher the rate of increase of net investment, the higher g_Y^* , alpha, and the rate of return, and vice versa, and that the higher the rate of increase of net investment the lower Ω and $1 - \beta^*$, and vice versa. The above results for a developed country are similar to those for a developing country. These imply that the above five ratios are determined only by the balance of payments. Note that the balance of payments will be influenced by a level of defi-



Note: Case XXX shows the actual situation by year, where $g_Y^* \neq \Delta d / d \neq r_{(DEBT)}$.

Figure 3 De Grauwe's (2005) condition for primary solvency: the US 1997–2005



Note: If investment increases (by either Case 1, 2, or 3) under a fixed balance of payments, BOP and saving increase by the increase of net investment: $S = (S - I - \Delta I) + I + \Delta I$. This is because the current Y and C each increases by the increase in the growth rate of output at convergence that occurs by the increase in investment.

The higher positively the BOP , the higher is the g_Y^* at convergence and, vice versa, depending on the capital-output ratio of the total economy which determines $1 - \beta^*$.

Figure 4 Simulation of the growth rate of output and the ratio of saving to output, in the balance of payments and budget: Cases 1, 2, and 3

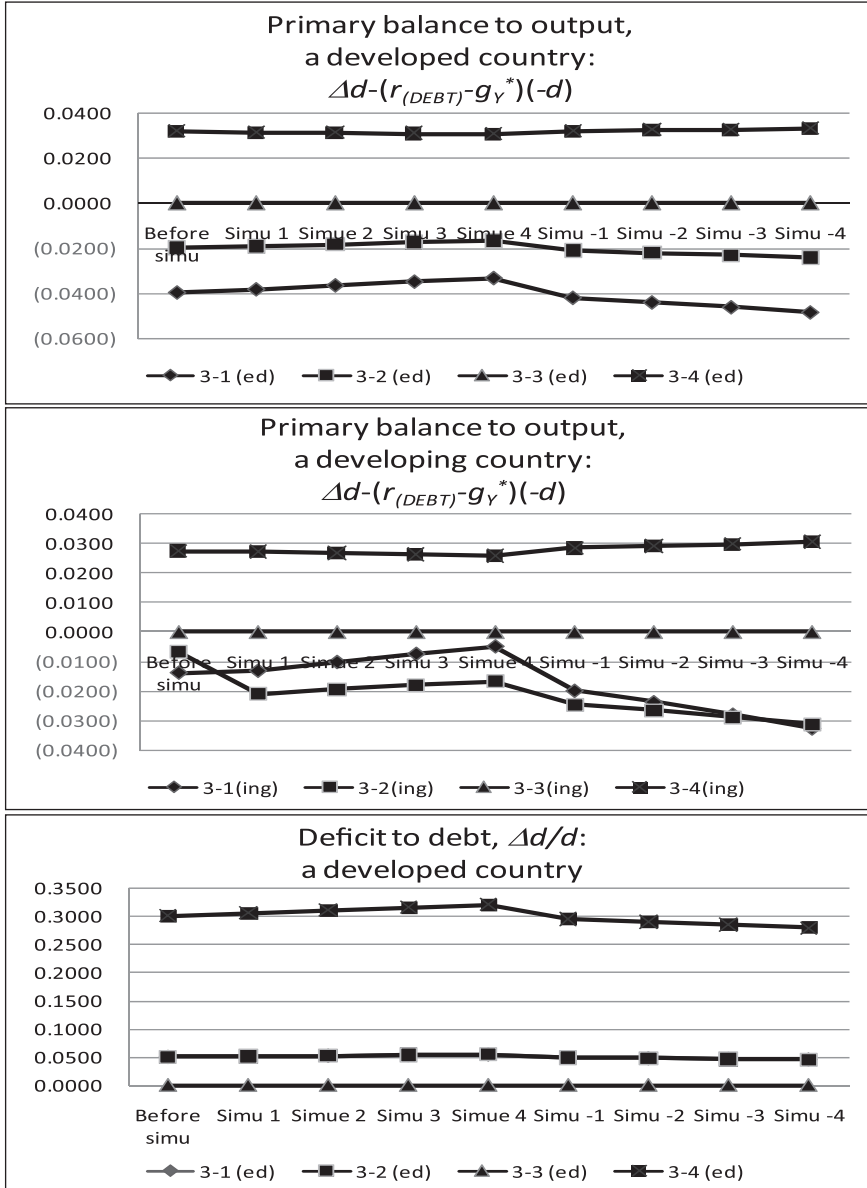
cits and debts if this level is extremely minus (in other words, if the saving of the total economy decreases with a significant minus government saving).

The results of the simulations to Cases 1 and 2 are moderate: the changes in net investment of either the total economy or the government sector do not influence the above five ratios, much less than the results to Case 3, where the net investment of both the total economy and the government sector changes at the same time. As a result, the author uses Case 3 to find new facts in the fiscal rule hereafter.

Table 2 shows, using Case 3, deficits, debts, and the condition for solvency, and each component of $\alpha_G = \Omega_G^* \cdot r_G^*$ in the government sector. The purpose of this table is to find how severely deficits and debts influence each of $\alpha_G = \Omega_G^* \cdot r_G^*$. If $\alpha_G = \Omega_G^* \cdot r_G^*$ is miserable, then $\alpha_G = \Omega^* \cdot r^*$ of the total economy will severely hit, resulting in a low growth rate of output at convergence as in Japan. Table 2 also compares a developed country that has a high level of the government capital-output ratio by sub-case with a developing country that has a low level of the government capital-output ratio by sub-case. In the sub-case 3-1 (i.e., $\Delta d = -0.03$ and $d = -0.6$, as the limit of the EMU rule), the government capital-output ratio is much higher than that of sub-case 3-3 of $\Delta d = 0$ and $d = 0$ as a benchmark. This implies that fiscal decisions directly influence the rate of technological progress via the capital-output ratio. Furthermore, both α_G and r_G^* in $\alpha_G = \Omega_G^* \cdot r_G^*$ become severely minus along with the aggravation of deficits and debts (when Case 3-3 turns to Case 3-1). This implies that when the condition for solvency is hopeless it is difficult for the government to get rid of the aggravation of $\alpha_G = \Omega_G^* \cdot r_G^*$.

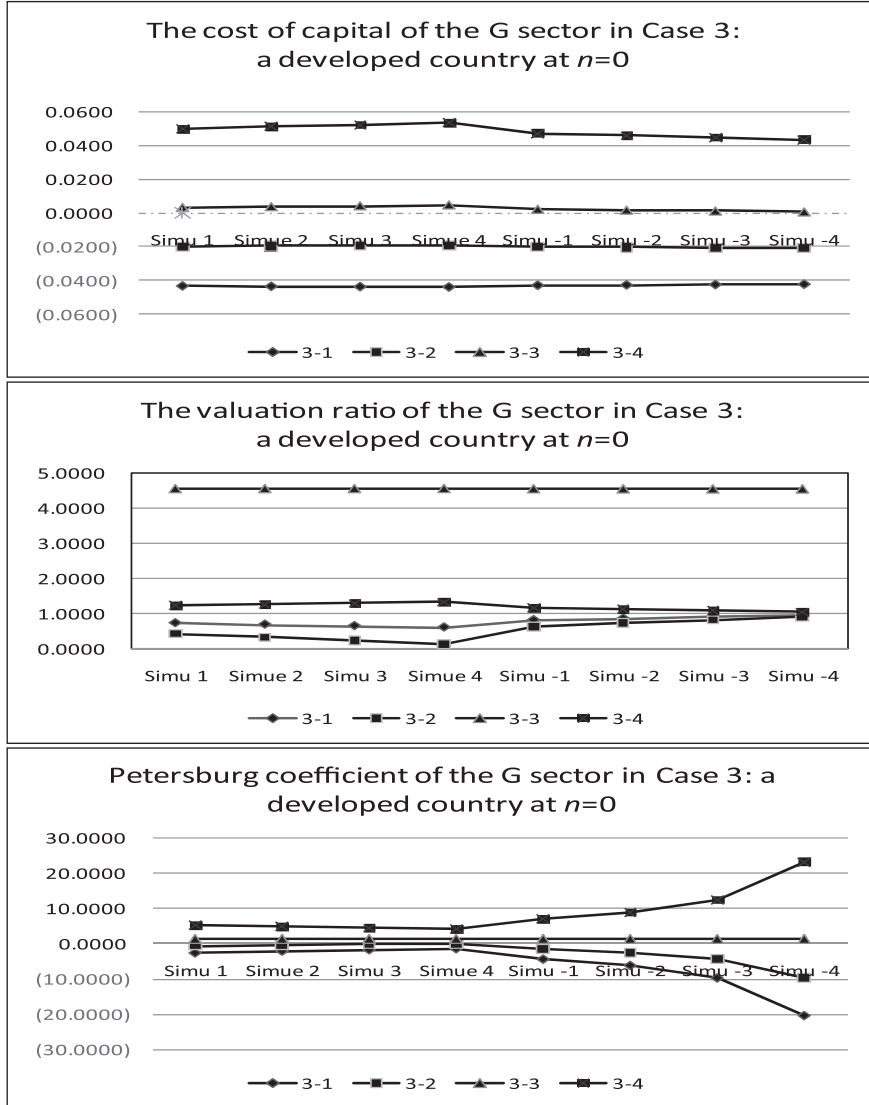
Figure 5 shows the influences of deficits and debts on primary balance to output, by using $\Delta d - (r_{(DEBT)} - g_Y^*)(-d)$. The higher negatively this value, the riskier the condition for primary solvency. This is typical in Case 3-1 in a developed and developing countries, where $\Delta d = -0.03$ and $d = -0.6$.

Table 3 and **Figure 6** show the cost of capital, the valuation ratio, and the Petersburg coefficient, $\alpha_G / (i_G \cdot \beta_G^*)$, to connect the rate of return with the growth rate of output both at convergence, in the government sector. Since the growth rate of output at convergence remains unchanged between Cases 3-1, 3-2, 3-3, and 3-4, the rate of return and/or the Petersburg coefficient must change. With the aggravation of deficits and debts (when Case 3-3 turns to Case 3-1), the cost of capital becomes more minus in both developed and developing countries. This finding is slightly accelerated when the rate of increase/decrease in net investment increases, and vice versa. As a result, the valuation ratio is consistent with the above findings. The farther negatively the condition for solvency becomes the less the valuation ratio is below zero. The



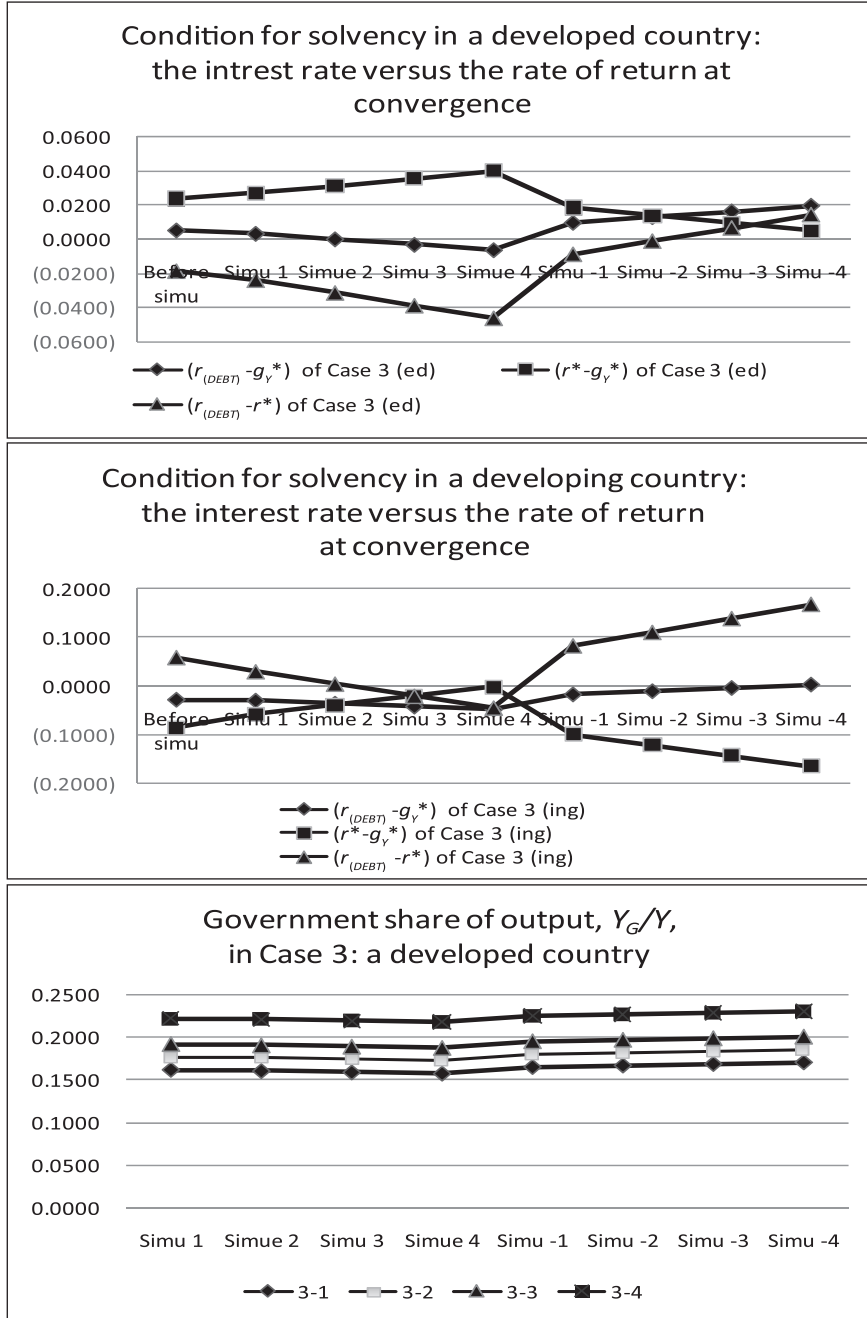
Note: The primary balance $S_G - I_G$ (excluding interest) changes more significantly at Cases 3-1 and 3-2 under deficits than those at Cases 3-3 and 3-4 under surplus. At Case 3-1 ($\Delta d = -0.03$ and $d = -0.6$), why does the primary balance improves with more investment or aggravates with less investment? This is because positive investment enhances g_Y^* and r^* and improves the *BOP* and saving under a fixed $\Delta d = -0.03$.

Figure 5 Simulation of primary balance to output and deficits and debts: Case 3



Note: Case 3-3 has neither deficit nor debt and presents a base for deficit/surplus and debt/lending. Case 3-1 shows the worst and Case 3-4 shows the best, where Case 3 changes net investment of both the total economy and the government sector at the same time. The above results indicate that an optimum situation exists under Case 3-3, where $\Delta d = d = 0$ and the valuation ratio is the highest with the least cost of capital. For the wage function of consumption, $\left(\frac{rho}{r}\right) = 13.301c^2 - 22.608c + 10.566$ is used.

Figure 6 Simulation of the cost of capital, the valuation ratio, and the Petersburg coefficient, each at convergence: Case 3



Note: $(r_{(DEBT)} - g_Y^*)$ and $(r^* - g_Y^*)$ are independent of Δd and d . Yet, if the interest rate is determined so as to minimize $r_{(DEBT)} - r^*$, the neutrality of money is guaranteed.

Figure 7 The neutrality of the interest rate and the government share of output by simulation in Case 3

more the rate of increase/decrease in net investment increases, slightly less the valuation ratio becomes.

Finally, **Table 4** for Case 3 shows the condition for solvency by comparing De Grauwe's interest rate with the author's rate of return at convergence: $(r_{(DEBT)} - g_Y^*)(-d)$ versus $(r^* - g_Y^*)(-d)$ and $(r_{(DEBT)} - g_Y^*)$ and $(r^* - g_Y^*)$. This is also shown in **Figure 7**, followed by the government share of output by the level of Δd and d . The higher negatively the level of Δd and d are, the more the government share of output.

These implies that the cost of capital, the valuation ratio, and the government sector's share of output are consistent with not only the condition for solvency but also the endogenous growth model as a whole, compared with De Grauwe's external condition for solvency alone.

Furthermore, the author indicates some interpretation to the present-value constraint to deficit, or *PVC* (for its origin, see Eq. 12 under $g_Y^* = 0$), particularly citing Ahmed, S. & Rogers, J. H. (1995, 353–355). According to the paper, there are two different results of the *PVC* test under stochastic and non-stochastic conditions, assuming that if $PVC = 0$, deficit is on a sustainable path: (1) the real interest rate $>$ the growth rate of economy holds (which is called 'under dynamic efficiency'), as in 'much of the literature,' and (2) the real interest rate $<$ the growth rate of economy holds, as shown in the tests of Wilcox (1989) and Abel et al (1989). The above (1) corresponds with the author's cost of capital at convergence > 0 and the above (2) corresponds with the author's cost of capital at convergence < 0 ; both by assuming that $r^* = r_{(DEBT)}$ and the balance of payments $= 0$. According to the author's point of view, the difference comes from the Petersburg coefficient, $\alpha / i \cdot \beta^*$, depending on the relationship between the relative share of capital, the ratio of net investment to output, and the quantitative to total investment, β^* . The author concludes that the valuation ratio is more important than the *PVC* test that varies by the above Petersburg coefficient.

5. Examination of the EMU' fiscal rule: towards endogenous fiscal rule

First the author will examine and interpret the contents of the *EMU*'s fiscal rule and second, propose the author's endogenous fiscal rule. According to the upper limit of the *EMU* rule, the ratio of deficit to output, $\Delta d \equiv \Delta D / Y$, is -0.03 and the ratio of debts to output, $d \equiv D / Y$, is -0.6 , where, output = *GDP* and surplus and lending are shown each plus, which differs from De Grauwe's sign.

First, the author clarifies three aspects in the *EMU* rule:

1. The balance of payments and deficits are each expressed as the difference between saving and investment of the total economy or the government sector. For the above case, gross (including depreciation) and net (excluding depreciation) have no problem. However, there are a few problems as a whole. One is that the lower limit is zero for gross investment while a minus value exists for net investment. When minus returns in the government sector are measured due to minus saving that is caused by huge deficits, as in the author's model, it is inevitable to take the 'net' base. Balassone F. and Franco D. (2000, 217–224) cites and compares the proposal of Modigliani et al (1998), the German model, and the UK model, with the EMU rule, where the German model is only based on gross.¹⁰⁾ Therefore, the above literature only reviews the level of saving by the level of investment. There is no step into returns using the saving in the government sector.

2. The rule for budget analysis is well examined by using corresponding ratios in the government sector: the ratio of government saving to government output, s_G , and the ratio of government net investment to government output, i_G , where $s_G = (s_G - i_G) + i_G$ based on $S_G = (S_G - I_G) + I_G$, under $s_G = S_G / Y_G$, $i_G = I_G / Y_G$ and $Y_G =$ government output. However, in the above comparison of Modigliani et al, the German, and the UK models, each denominator is not government output but *GDP* or output, as shown by $s_{\frac{G}{Y}} = (s_{\frac{G}{Y}} - i_{\frac{G}{Y}}) + i_{\frac{G}{Y}}$, where $s_{\frac{G}{Y}} = s_G \cdot \frac{Y_G}{Y}$ and $i_{\frac{G}{Y}} = i_G \cdot \frac{Y_G}{Y}$. A reason is that there is no systematic method for dividing the total economy into the government and private sectors. The author proposes this method, first connecting $s_G = (s_G - i_G) + i_G$ with $s_{\frac{G}{Y}} = (s_{\frac{G}{Y}} - i_{\frac{G}{Y}}) + i_{\frac{G}{Y}}$, by using the government share of output, Y_G / Y . Note that the author uses national disposable income, Y , instead of *GDP*, where Y / GDP is 80 to 90% by country and by year and Y is more consumption or sustainability oriented in the long run.

3. From the viewpoint of solvency, the relationship between deficits and debts must be examined, by shifting De Grauwe's external relationship between the interest rate and the current growth rate of output to endogenous values such as the cost of capital and the valuation ratio, as the author clarified in this paper.

Taking into consideration the results of the comparison of Japan with the US 1997–2005 and also the case study by the level of deficits and debts each to output, new findings are summarized as follows:

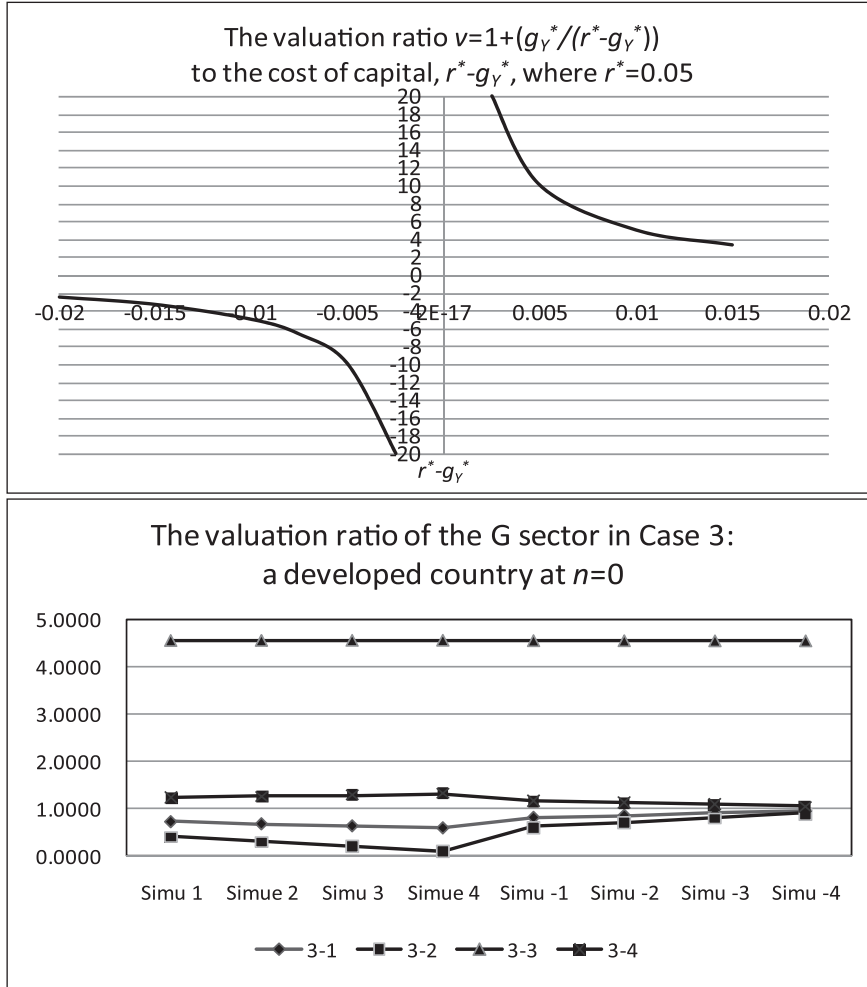
10) The constraint of each model is commonly shown by using the government 'saving' derived from $s_G = \Delta d + i_G$: (1) Proposal of Modigliani and et al; $s_G \leq -0.03$ and $s_G = -0.01$, (2) German model; $s_G = -0.02$, and (3) The UK model; $s_G = -0.02$. The author, then, justifies a minus rate of return in the government sector, by setting consumption coefficient/national taste = 1, under the neutrality of wages and consumption in the government sector.

1. The higher the balance of payments to output the higher the growth rate of output at convergence, assuming that the level of deficits and debts is given. It is indispensable for an economy to increase investment in order to decrease a huge positive balance of payments. If the balance of payments is minus, it is indispensable to increase the saving of the total economy (by using tax redistribution function) assuming that investment is given. Nevertheless, if investment increases at a rapid rate continuously over years, the economy will get into a developed stage soon, resulting in a low growth rate of output at convergence due to the existence of an upper limit of the capital-output ratio.
2. Assuming that the balance of payments and the capital-output ratio, $\Omega = K/Y$, are given, the higher positively the rate of increase/decrease in net investment the higher the growth rate of output at convergence, and vice versa. Note that the qualitative investment to total investment, $1 - \beta^*$, cannot easily rise up at above $\Omega = 3$, even if R & D, education, and human capital are input much. This is justified by the elasticity of $1 - \beta^*$ w. r. t. Ω (see Figure 1). Investment must be qualitatively examined and investment is endogenously divided into qualitative and quantitative investments.
3. The higher the ratio of net investment to output, $i = I/Y$, the higher the growth rate of output but, at the sacrifice of the higher Ω . This implies that an optimum $i = I/Y$ exists at between 0.1 and 0.2. Note that the rate of technological progress, $g_A^* = i(1 - \beta^*)$, mostly depends on $i = I/Y$. Assume that the government share of output, Y_G/Y , is 0.15 and $i = 0.2$, then an optimum $i_{G/Y} = 0.03 = 0.15 \times 0.2$ is obtained. The higher negatively the level of Δd the higher the value of Y_G/Y . In this respect, the upper limit of $\Delta d = -0.03$ in the EMU rule is theoretically justified at $\frac{Y_G}{Y} = 0.15$ and $i = 0.2$ or $i_{G/Y} = 0.03$, which is within the range of optimum.
4. If the level of government net investment to GDP is 3% as derived above (neglecting the difference between GDP and output $Y = C + S$), the EMU rule implies that government saving (to output) must be zero. What does $S_G = 0$ mean? $S_G = 0$ guarantees a minimum growth rate of output in the government sector. The government saving to GDP or output is, in a sense, a final indicator of this rule.¹¹⁾ With the change in net investment, the BOP and saving move positively or negatively in parallel. This is a fact-base yet without further

11) This situation realizes Arrow's (1970) unique rate of return in an economy, where the rate of return of the private sector equals that of the total economy. The author advocates that if government saving is more or less than zero, then the rate of return will be more or less than zero, where an idea that government should not earn money in budgeting is denied.

extending to endogenous ratios.

5. The solvency of primary balance in De Grauwe's differential at the continuous time is replaced by the endogenous cost of capital at the discrete time. The cost of capital with the



Note: Top of this figure shows the characteristics of the valuation ratio, where the vertical asymptote crosses the origin of $r^* - g_Y^* = 0$. Bottom of this figure shows that the maximum/optimum of the valuation ratio holds only at deficit = 0 and debt = 0. Case 3-1 shows the upper limit of the *EMU* rule. Case 3-2 shows $\Delta d = -0.015$ and $d = -0.3$. Each valuation ratio is much lower than 1.0, in particular when the increase/decrease rate of net investment is positive. Case 3-3 has no deficit and no debt in an economy. The valuation ratio of Case 3-3 shows highest value stably regardless of the increase/decrease rate of net investment. Case 3-4 has surplus and lending (Δd and d are positive) yet, the valuation ratio is much lower than that of Case 3-3, though it is still above 1.0.

Figure 8 The characteristics of the valuation ratio and its maximum at $debt = 0$

changes in net investment under four cases is optimized at $\Delta d = d = 0$ as well as the valuation ratio (see **Figure 8** with Eq. 10-2). The higher negatively the level of deficits and debts, the lower the valuation ratio, which falls below 1.0. The higher the level of surplus and lending, the higher the valuation ratio remains above 1.0. This implies that the smaller the deficits and debts the more healthy an economy is. The higher positively the change in net investment the more $d = D/Y$ increases. The higher negatively the change in net investment the more $d = D/Y$ decreases. All of these results are traced back to the change in technology: it is indispensable for an economy to decrease government investment unless $1 - \beta_G^* > 1 - \beta^*$ is guaranteed.

6. The comparison of the interest rate of $r_{(DEBT)}$ with the rate of return at convergence, r^* , is useful to the correction of financial policy¹²⁾: $r_{(DEBT)} - r^*$ and/or $r_{(DEBT)} / r^*$. The interest rate for the long-term is based on the market principle and holds with the least risk of national debts. The rate of return at convergence is based on the endogenous growth model and connects it with the relative share of output and the capital-output ratio in the real assets. The central bank must be rigidly neutral to arbitrary decision-making: $r_{(DEBT)} > r^*$ (often as in the US) is preferable to $r_{(DEBT)} < r^*$ (as in Japan for many years). As a guideline, the former is more responsible to the next generations than the latter.
7. Finally, $d = D/Y = -60\%$ in the EMU upper limit is externally justified using the theoretical capital-output ratio, $\Omega = K/Y$, even when the set of equations are not measured: Assume $\Omega = 2.5$ on average among countries in the world, and define $d_{D/K} \equiv -D/K$. Then, $d = -60\%$ is expressed by $d_{D/K} = 24\% = 0.6/2.5$. Define the theoretical leverage of debts to equity or national wealth: $l_{EV} \equiv d_{D/K}/(1 - d_{D/K})$. Then, the leverage is $30.16\% = 0.24/0.76$ under an assumption that debts are only used for capital. For example, if $d_{D/Y} = 2.0$ (as in Japan, using national disposable income instead of *GDP*), the leverage will be $l_{EV} = 400\% = 0.8/(1 - 0.8)$ using $\Omega = 2.5$ and $d_{D/K} = 80\% = 2.0/2.5$. If the actual capital-output is used, $\Omega_{actual} = 3.7$, the actual leverage will be; $l_{EV} = 117\% = 0.54/0.46$ under $d_{D/K} = 54\% = 2.0/3.7$. Note that $\Omega = 2.5$ is a common criterion for sustainability in the global world that controls the relationship between the relative share of capital and the rate of return by country. The higher the leverage the higher the risk of debts, where a concept of non-risk does not hold: the interest rate of debts will turn to risk-bearing, unless the private sector vividly absorbs the heavy burden of debts that depresses the total economy.

12) The relationship between real assets and financial assets will be discussed using the Marshall's $k = M/Y$ and comparing this k with the author's theoretical capital-output ratio in a separate paper.

6. Conclusions

The *EMU* fiscal rule was set empirically. The author proved that the fiscal rule was modest, as shown by endogenous findings in this paper. The upper limit of 3% deficit to output, $\Delta d = \Delta D/Y$, is replaced by zero government saving under a sustainable level of the ratio of net investment to output in an open economy, where the balance of payments and budget are simultaneously taken into consideration. The upper limit of 60% debts to output, $d = D/Y$, is converted to the theoretical leverage by connecting $d_{D/K} \equiv -D/K$ with the theoretical-output ratio. $D/Y = -60\%$ corresponds with 30% of the leverage under $\Omega = 2.0$. These results are stable and ready for challenges. Also, the more modest an economy the closer the interest rate of debts to the rate of return at convergence, as shown by the Petersburg coefficient.

The quantitative net investment to the sum of qualitative and quantitative investment measured by sector is a supreme criterion to fiscal policy, since the valuation ratio $r^*/(r^* - g_Y^*)$ is replaced by $\alpha/(\alpha - i \cdot \beta^*)$. The maximized level of the valuation ratio is only realized at $\Delta d = 0$ and $d = 0$. And, the higher the capital-output ratio the higher the β^* is. Therefore, it is inevitable for policy-makers to have the capital-output lower and this is maintained by converting physical capital to human capital.

The condition for primary balance set by De Grauwe (ibid., 225) was tested by connecting deficits with debts, primarily comparing the interest rate of debts with the rate of return of the government sector in the financial degree of solvency and, replacing the difference of the interest rate and the external growth rate of output by an endogenous cost of capital and its valuation ratio at convergence, in the endogenous degree of solvency. The results of De Grauwe's equation using Japan and the US 1997–2005 (see Figure 6) do not contradict with the results of 'dynamic efficiency' in the literature (for comparison, see Abel et al (1989)), where if the growth rate of output is lower than the interest rate, the condition for solvency of debts will be well maintained. The dynamic efficiency is tested using the Petersburg coefficient between the rate of return and the growth rate of output each at convergence or directly using the cost of capital and the valuation ratio each at convergence.

The growth rate of output at convergence of the total economy is higher if the balance of payments (*BOP*) is positively higher and vice versa, so that the saving at $BOP = 0$ is a base for any country. The growth rate of output at convergence of the government sector, on the other hand, remains unchanged if the ratio of net investment to output and the theoretical capital-out-

put ratio remain unchanged regardless of whether or not the levels of deficits and debts are extremely minus. Nevertheless, the relative share of capital and the rate of return of the government sector strongly reflect the levels of deficits and debts: the more minus deficits and debts, the more minus these ratios of the government sector. These results in the long run influence the *BOP* and variables of the private sector and the total economy, through the aggravation of the capital-output ratio.

The government share of output, where taxes equal government output, reflects these results: the more minus deficits and debts the more the government share of output (as an expression of a large government). Here is much room for choosing alternative policies among people and policy-makers. As a result, if deficits and debts are more minus, the cost of capital of the government sector is more minus and accordingly, the valuation ratio is below 1.0, where national net wealth reduces. In adverse, if surplus and lending are plus, the valuation ratio is above 1.0 and national net wealth increases.

The author assumed in this paper that the differential of the level of high-powered money was zero, similarly to De Grauwe. The author will test the above neutrality by comparing the Marshall's k with the author's capital-output ratio in a separate paper.

Table 1 Simulation of the endogenous fiscal rule, commonly to all the cases

Table 2 Simulation of basic data in the endogenous fiscal rule: Case 3

Table 3 The cost of capital and the valuation ratio in the endogenous fiscal rule

Table 4 New relationship between deficits and debts: from De Grauwe's (2005) to the endogenous fiscal rule

Figure 1 Elasticity values of beta with respect to the capital-output ratio and the ratio of net investment to output

Figure 2 De Grauwe's (2005) condition for primary solvency: Japan 1997–2005

Figure 3 De Grauwe's (2005) condition for primary solvency: the US 1997–2005

Figure 4 Simulation of the growth rate of output and the ratio of saving to output, in the balance of payments and budget: Cases 1, 2, and 3

Figure 5 Simulation of primary balance to output and deficits and debts: Case 3

Figure 6 Simulation of the cost of capital, the valuation ratio, and the Petersburg coefficient, each at convergence: Case 3

Figure 7 The neutrality of the interest rate and the government share of output by simulation in Case 3

Figure 8 The characteristics of the valuation ratio and its maximum at debt = 0

Table A1 De Grauwe's (2005, 225) condition for primary solvency: 1997–2005

Table A2 Specific simulation of deficits and debts: four cases of Japan 1997–2005

Table A3 De Grauwe's (2005, 225) condition for primary solvency: the US 1997–2005

Table A4 Specific simulation of deficits and debts: four cases of the US 1997–2005

Table A5 Simulation 3.1 & 3.2 of the *EMU* rule to deficits and debts: a developed country

Table A6 Simulation 3.3 & 3.4 of the *EMU* rule to deficits and debts: a developed country

Table A7 Simulation 3.1 & 3.2 of the EMU rule to deficits and debts: a developing country

Table A8 Simulation 3.3 & 3.4 of the EMU rule to deficits and debts: a developing country

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Table 1 Simulation of the endogenous fiscal rule, commonly to all the cases

A DEVELOPED COUNTRY	Before simu	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
g_Y^* of 1-1, 1-2, 1-3, 1-4	0.0379	0.0398	0.0429	0.0460	0.0491	0.0334	0.0302	0.0269	0.0237
1-beta [*] of 1-1, 1-2, 1-3, 1-4	0.2178	0.2131	0.2182	0.2224	0.2259	0.1995	0.1900	0.1777	0.1613
1-beta _(G) [*] of 1-3 (Note)	0.3694	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526
$\Omega^*=(K/Y)^*$ of 1-1, 1-2, 1-3, 1-4	2.0658	2.1504	2.0919	2.0562	2.0221	2.2095	2.2526	2.2979	2.3456
alpha of 1-1, 1-2, 1-3, 1-4	0.1467	0.1551	0.1792	0.1944	0.2090	0.1294	0.1111	0.0920	0.0719
r^* of 1-1, 1-2, 1-3, 1-4	0.0710	0.0721	0.0857	0.0946	0.1034	0.0585	0.0493	0.0400	0.0306
g_Y^* of 2-1, 2-2, 2-3, 2-4	0.0379	0.0371	0.0376	0.0381	0.0386	0.0361	0.0356	0.0351	0.0346
1-beta [*] of 2-1, 2-2, 2-3, 2-4	0.2178	0.2080	0.2091	0.2101	0.2110	0.2059	0.2048	0.2036	0.2024
1-beta _(G) [*] of 2-3 (Note)	0.3694	0.3776	0.3951	0.4080	0.4177	0.3144	0.2499	0.1197	(0.2736)
$\Omega^*=(K/Y)^*$ of 2-1, 2-2, 2-3, 2-4	2.0659	2.1621	2.1558	2.1496	2.1433	2.1748	2.1813	2.1877	2.1942
alpha of 2-1, 2-2, 2-3, 2-4	0.1467	0.1494	0.1521	0.1547	0.1574	0.1440	0.1413	0.1386	0.1358
r^* of 2-1, 2-2, 2-3, 2-4	0.0710	0.0691	0.0706	0.0720	0.0734	0.0662	0.0648	0.0633	0.0619
g_Y^* of 3-1, 3-2, 3-3, 3-4	0.0265	0.0285	0.0312	0.0339	0.0365	0.0231	0.0204	0.0176	0.0149
1-beta [*] of 3-1, 3-2, 3-3, 3-4	0.2262	0.2201	0.2199	0.2198	0.2196	0.2204	0.2205	0.2207	0.2208
1-beta _(G) [*] of 3-3 (Note)	0.2190	0.2167	0.2166	0.2165	0.2165	0.2169	0.2170	0.2171	0.2172
$\Omega^*=(K/Y)^*$ of 3-1, 3-2, 3-3, 3-4	2.9188	2.9599	2.9020	2.8467	2.7938	3.0847	3.1519	3.2227	3.2974
alpha of 3-1, 3-2, 3-3, 3-4	0.1467	0.1647	0.1817	0.1981	0.2137	0.1279	0.1082	0.0874	0.0654
r^* of 3-1, 3-2, 3-3, 3-4	0.0503	0.0556	0.0626	0.0696	0.0765	0.0415	0.0343	0.0271	0.0198

A DEVELOPING COUNTRY	Before simu	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
g_Y^* of 1-1, 1-2, 1-3, 1-4	0.0629	0.0646	0.0696	0.0745	0.0793	0.0544	0.0492	0.0439	0.0385
1-beta [*] of 1-1, 1-2, 1-3, 1-4	0.3398	0.3195	0.3223	0.3245	0.3263	0.3116	0.3058	0.2982	0.2877
1-beta _(G) [*] of 1-3 (Note)	0.3589	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419
$\Omega^*=(K/Y)^*$ of 1-1, 1-2, 1-3, 1-4	1.4000	1.5046	1.4734	1.4441	1.4164	1.5730	1.6107	1.6511	1.6945
alpha of 1-1, 1-2, 1-3, 1-4	0.0983	0.1215	0.1427	0.1628	0.1818	0.0749	0.0494	0.0221	(0.0071)
r^* of 1-1, 1-2, 1-3, 1-4	0.0702	0.0807	0.0969	0.1127	0.1283	0.0476	0.0307	0.0134	(0.0042)
g_Y^* of 2-1, 2-2, 2-3, 2-4	0.0629	0.0602	0.0609	0.0616	0.0623	0.0588	0.0581	0.0574	0.0567
1-beta [*] of 2-1, 2-2, 2-3, 2-4	0.3398	0.3165	0.3170	0.3175	0.3180	0.3155	0.3149	0.3143	0.3137
1-beta _(G) [*] of 2-3 (Note)	0.3589	0.3643	0.3800	0.3915	0.4001	0.3075	0.2493	0.1317	(0.2237)
$\Omega^*=(K/Y)^*$ of 2-1, 2-2, 2-3, 2-4	1.4000	1.5330	1.5256	1.5156	1.5030	1.5424	1.5499	1.5604	1.5740
alpha of 2-1, 2-2, 2-3, 2-4	0.0983	0.1021	0.1068	0.1131	0.1209	0.0957	0.0909	0.0842	0.0758
r^* of 2-1, 2-2, 2-3, 2-4	0.0702	0.0666	0.0700	0.0746	0.0804	0.0621	0.0586	0.0540	0.0481
g_Y^* of 3-1, 3-2, 3-3, 3-4	0.0697	0.0716	0.0777	0.0837	0.0896	0.0589	0.0524	0.0458	0.0390
1-beta [*] of 3-1, 3-2, 3-3, 3-4	0.5300	0.4925	0.4907	0.4889	0.4872	0.4962	0.4981	0.5000	0.5019
1-beta _(G) [*] of 3-3 (Note)	0.4677	0.4533	0.4521	0.4509	0.4497	0.4557	0.4569	0.4581	0.4593
$\Omega^*=(K/Y)^*$ of 3-1, 3-2, 3-3, 3-4	0.8995	1.0166	0.9979	0.9803	0.9639	1.0581	1.0812	1.1062	1.1333
alpha of 3-1, 3-2, 3-3, 3-4	(0.0144)	0.0134	0.0386	0.0623	0.0846	(0.0423)	(0.0731)	(0.1062)	(0.1419)
r^* of 3-1, 3-2, 3-3, 3-4	(0.0160)	0.0132	0.0387	0.0635	0.0877	(0.0399)	(0.0676)	(0.0960)	(0.1252)

Note: Case 1 changes ‘net investment’ of the total economy, Case 2 that of the government sector and, Case 3 changes those of the total economy and the government sector. $\Delta D/Y = -0.03$ and $D/Y = -0.6$ in Sub-case 1, $\Delta D/Y = -0.015$ and $D/Y = -0.3$ in Sub-case 2, $\Delta D/Y = 0.0$ and $D/Y = 0.0$ in Sub-case 3, and $\Delta D/Y = 0.03$ and $D/Y = 0.1$ in Sub-case 4. In this table, results are the same, regardless of the combination of the changes in net investments of the total economy and the G sector. The qualitative net investment to total net investment of the government sector, $1-\beta_{(G)}^*$, however, differs by case but, slightly. This implies that fiscal policy does not influence technological improvement in the short-run. In the long-run, fiscal policy influences technological progress via the aggravation of the capital-output ratio of the government sector.

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Table 2 Simulation of basic data in the endogenous fiscal rule: Case 3

	Before simu	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
A DEVELOPED COUNTRY									Cases of 3--1 to 3--4
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.1	(0.0441)	(0.0429)	(0.0415)	(0.0402)	(0.0389)	(0.0459)	(0.0475)	(0.0491)	(0.0508)
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.2	(0.0221)	(0.0215)	(0.0208)	(0.0201)	(0.0194)	(0.0229)	(0.0237)	(0.0246)	(0.0254)
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.4	0.0324	0.0322	0.0319	0.0317	0.0315	0.0326	0.0329	0.0331	0.0334
$\Delta d=\Delta D/Y$ of 3-1	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$\Delta d=\Delta D/Y$ of 3-2	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$\Delta d=\Delta D/Y$ of 3-3	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d=\Delta D/Y$ of 3-4	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
$d=D/Y$ of 3-1	(0.6000)	(0.5898)	(0.5797)	(0.5700)	(0.5606)	(0.6111)	(0.6223)	(0.6341)	(0.6463)
$d=D/Y$ of 3-2	(0.3000)	(0.2949)	(0.2898)	(0.2850)	(0.2803)	(0.3055)	(0.3112)	(0.3170)	(0.3231)
$d=D/Y$ of 3-3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d=D/Y$ of 3-4	0.1000	0.0987	0.0975	0.0963	0.0952	0.1014	0.1027	0.1042	0.1057
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-1	4.0533	4.0785	4.0530	4.0279	4.0033	4.1310	4.1580	4.1855	4.2136
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-2	3.7060	3.7260	3.6996	3.6738	3.6486	3.7803	3.8083	3.8369	3.8661
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-3	3.4135	3.4295	3.4030	3.3770	3.3515	3.4844	3.5128	3.5418	3.5715
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-4	2.9481	2.9588	2.9327	2.9072	2.8823	3.0129	3.0410	3.0697	3.0992
$\alpha(G)$ of 3-1	(0.1365)	(0.1286)	(0.1209)	(0.1133)	(0.1059)	(0.1444)	(0.1525)	(0.1608)	(0.1692)
$\alpha(G)$ of 3-2	(0.0391)	(0.0311)	(0.0232)	(0.0155)	(0.0079)	(0.0473)	(0.0556)	(0.0641)	(0.0728)
$\alpha(G)$ of 3-3	0.0429	0.0510	0.0589	0.0666	0.0742	0.0347	0.0263	0.0177	0.0090
$\alpha(G)$ of 3-4	0.1734	0.1812	0.1889	0.1964	0.2038	0.1653	0.1571	0.1486	0.1400
$r(G)^*$ of 3-1	(0.0337)	(0.0315)	(0.0298)	(0.0281)	(0.0265)	(0.0350)	(0.0367)	(0.0384)	(0.0402)
$r(G)^*$ of 3-2	(0.0106)	(0.0083)	(0.0063)	(0.0042)	(0.0022)	(0.0125)	(0.0146)	(0.0167)	(0.0188)
$r(G)^*$ of 3-3	0.0126	0.0149	0.0173	0.0197	0.0221	0.0100	0.0075	0.0050	0.0025
$r(G)^*$ of 3-4	0.0588	0.0613	0.0644	0.0676	0.0707	0.0549	0.0517	0.0484	0.0452
A DEVELOPING COUNTRY									
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.1	(0.0139)	(0.0133)	(0.0103)	(0.0076)	(0.0050)	(0.0200)	(0.0239)	(0.0281)	(0.0327)
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.2	(0.0070)	(0.0212)	(0.0196)	(0.0182)	(0.0168)	(0.0248)	(0.0268)	(0.0290)	(0.0314)
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d-(r_{(DEBT)} \cdot B_Y^*)(-d)$ of 3.4	0.0273	0.0272	0.0267	0.0262	0.0257	0.0284	0.0290	0.0297	0.0304
$\Delta d=\Delta D/Y$ of 3-1	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$\Delta d=\Delta D/Y$ of 3-2	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$\Delta d=\Delta D/Y$ of 3-3	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d=\Delta D/Y$ of 3-4	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
$d=D/Y$ of 3-1	(0.6000)	(0.5834)	(0.5659)	(0.5495)	(0.5340)	(0.6220)	(0.6435)	(0.6668)	(0.6919)
$d=D/Y$ of 3-2	(0.3000)	(0.3067)	(0.2980)	(0.2898)	(0.2822)	(0.3260)	(0.3368)	(0.3484)	(0.3610)
$d=D/Y$ of 3-3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$d=D/Y$ of 3-4	0.1000	0.0980	0.0958	0.0938	0.0919	0.1027	0.1054	0.1082	0.1113
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-1	1.3240	1.3921	1.3891	1.3862	1.3836	1.3987	1.4022	1.4060	1.4100
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-2	1.1850	1.2436	1.2383	1.2332	1.2283	1.2550	1.2612	1.2676	1.2743
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-3	1.0724	1.1238	1.1170	1.1106	1.1044	1.1382	1.1459	1.1540	1.1625
$\Omega(G)^*=(K/Y) \cdot (G)^*$ of 3-4	0.9011	0.9422	0.9341	0.9264	0.9190	0.9595	0.9688	0.9786	0.9889
$\alpha(G)$ of 3-1	(0.1636)	(0.1544)	(0.1464)	(0.1385)	(0.1307)	(0.1712)	(0.1799)	(0.1888)	(0.1979)
$\alpha(G)$ of 3-2	(0.0414)	(0.0313)	(0.0219)	(0.0128)	(0.0039)	(0.0510)	(0.0612)	(0.0718)	(0.0827)
$\alpha(G)$ of 3-3	0.0576	0.0681	0.0781	0.0879	0.0974	0.0469	0.0358	0.0243	0.0124
$\alpha(G)$ of 3-4	0.2081	0.2187	0.2291	0.2392	0.2489	0.1965	0.1848	0.1726	0.1598
$r(G)^*$ of 3-1	(0.1235)	(0.1109)	(0.1054)	(0.0999)	(0.0945)	(0.1224)	(0.1283)	(0.1343)	(0.1404)
$r(G)^*$ of 3-2	(0.0349)	(0.0252)	(0.0177)	(0.0104)	(0.0031)	(0.0406)	(0.0485)	(0.0566)	(0.0649)
$r(G)^*$ of 3-3	0.0537	0.0606	0.0700	0.0792	0.0882	0.0412	0.0312	0.0210	0.0106
$r(G)^*$ of 3-4	0.2310	0.2321	0.2453	0.2582	0.2709	0.2048	0.1908	0.1764	0.1616

Note: Case 3 shows the results closer to the real world when the changes in net investments of the total economy and the government sector occur at the same time. This table shows three current values used for the solvency of debts in De Grauwe, P. (2005) and also three theoretical basic ratios at convergence of the capital-output ratio, the relative share of capital, and the rate of return, each of the government sector, where $\alpha_G = \Omega_G \cdot r_G$. The government share of output, Y_G/Y , is used for connecting the government sector's value with the total economy's.

Table 3 The cost of capital and the valuation ratio in the endogenous fiscal rule

A DEVELOPED COUNTRY	Before simu	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
$g_Y^*(G)$ of 3-1	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$g_Y^*(G)$ of 3-2	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$g_Y^*(G)$ of 3-3	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$g_Y^*(G)$ of 3-4	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
The cost of capital of the government sector This cost changes due to the changes in $r(G)$ and Petersburg coefficient.									
$r(G)-g_Y^*(G)$ of 3-1	(0.0448)	(0.0447)	(0.0452)	(0.0457)	(0.0462)	(0.0437)	(0.0432)	(0.0427)	(0.0422)
$r(G)-g_Y^*(G)$ of 3-2	(0.0210)	(0.0207)	(0.0207)	(0.0207)	(0.0207)	(0.0208)	(0.0208)	(0.0208)	(0.0208)
$r(G)-g_Y^*(G)$ of 3-3	0.0028	0.0032	0.0037	0.0043	0.0048	0.0022	0.0016	0.0011	0.0005
$r(G)-g_Y^*(G)$ of 3-4	0.0503	0.0511	0.0527	0.0542	0.0557	0.0481	0.0465	0.0449	0.0433
The valuation ratio= $V(G)/K(G)$ This ratio is shown by the rate of returns divided by the cost of capital at convergence.									
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-1	0.7807	0.7396	0.7002	0.6618	0.6243	0.8217	0.8644	0.9084	0.9535
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-2	0.5328	0.4388	0.3462	0.2538	0.1617	0.6249	0.7183	0.8119	0.9058
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-3	4.5665	4.6137	4.6158	4.6179	4.6200	4.6097	4.6077	4.6057	4.6038
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-4	1.1953	1.2276	1.2572	1.2849	1.3111	1.1623	1.1261	1.0872	1.0453
Petersburg coefficient This coefficient is shown by 'alpha(G)/(i(G) beta(G)*)' and connects $g_Y^*(G)$ with $r(G)$.									
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-1	(3.5608)	(2.8407)	(2.3356)	(1.9567)	(1.6619)	(4.6086)	(6.3763)	(9.9116)	(20.5172)
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-2	(1.1402)	(0.7820)	(0.5295)	(0.3401)	(0.1928)	(1.6658)	(2.5496)	(4.3171)	(9.6198)
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-3	1.2804	1.2767	1.2766	1.2764	1.2762	1.2770	1.2772	1.2773	1.2775
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-4	6.1216	5.3942	4.8887	4.5095	4.2144	7.1627	8.9307	12.4663	23.0721
The government sector's share of output/income This share more increases when budget is more surplus.									
$Y(G)/Y$ of 3-1	0.1549	0.1525	0.1501	0.1479	0.1457	0.1576	0.1604	0.1633	0.1663
$Y(G)/Y$ of 3-2	0.1750	0.1736	0.1721	0.1706	0.1693	0.1767	0.1783	0.1801	0.1819
$Y(G)/Y$ of 3-3	0.1849	0.1825	0.1801	0.1779	0.1757	0.1876	0.1904	0.1933	0.1963
$Y(G)/Y$ of 3-4	0.2149	0.2125	0.2101	0.2079	0.2057	0.2176	0.2204	0.2233	0.2263
A DEVELOPING COUNTRY	Before simu	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
$g_Y^*(G)$ of 3-1	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
$g_Y^*(G)$ of 3-2	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
$g_Y^*(G)$ of 3-3	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
$g_Y^*(G)$ of 3-4	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
The cost of capital of the government sector This cost changes due to the changes in $r(G)$ and Petersburg coefficient.									
$r(G)-g_Y^*(G)$ of 3-1	(0.1521)	(0.1441)	(0.1437)	(0.1433)	(0.1430)	(0.1448)	(0.1452)	(0.1457)	(0.1461)
$r(G)-g_Y^*(G)$ of 3-2	(0.0635)	(0.0583)	(0.0560)	(0.0538)	(0.0517)	(0.0630)	(0.0655)	(0.0680)	(0.0706)
$r(G)-g_Y^*(G)$ of 3-3	0.0251	0.0275	0.0316	0.0357	0.0397	0.0188	0.0143	0.0096	0.0049
$r(G)-g_Y^*(G)$ of 3-4	0.2024	0.1990	0.2070	0.2147	0.2223	0.1824	0.1738	0.1650	0.1559
The valuation ratio= $V(G)/K(G)$ This ratio is shown by the rate of returns divided by the cost of capital at convergence.									
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-1	0.8120	0.7702	0.7333	0.6968	0.6607	0.8451	0.8832	0.9217	0.9607
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-2	0.5496	0.4320	0.3159	0.1924	0.0610	0.6441	0.7410	0.8324	0.9186
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-3	2.1379	2.2058	2.2117	2.2177	2.2236	2.1942	2.1884	2.1827	2.1770
$v(G)=r(G)/(r(G)-g_Y^*(G))$ of 3-4	1.1413	1.1664	1.1852	1.2024	1.2183	1.1230	1.0976	1.0691	1.0369
Petersburg coefficient This coefficient is shown by 'alpha(G)/(i(G) beta(G)*)' and connects $g_Y^*(G)$ with $r(G)$.									
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-1	(4.3194)	(3.3507)	(2.7490)	(2.2979)	(1.9471)	(5.4573)	(7.5641)	(11.7780)	(24.4198)
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-2	(1.2203)	(0.7607)	(0.4619)	(0.2383)	(0.0649)	(1.8099)	(2.8613)	(4.9662)	(11.2851)
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-3	1.8788	1.8293	1.8253	1.8213	1.8173	1.8374	1.8414	1.8455	1.8496
$\alpha(G)/(i(G)\beta(G)^*)$ of 3-4	8.0770	7.0093	6.3996	5.9404	5.5816	9.1320	11.2470	15.4690	28.1190
The government sector's share of output/income This share more increases when budget is more surplus.									
$Y(G)/Y$ of 3-1	0.1278	0.1256	0.1232	0.1209	0.1187	0.1311	0.1341	0.1374	0.1409
$Y(G)/Y$ of 3-2	0.1428	0.1406	0.1382	0.1359	0.1337	0.1461	0.1491	0.1524	0.1559
$Y(G)/Y$ of 3-3	0.1578	0.1556	0.1532	0.1509	0.1487	0.1611	0.1641	0.1674	0.1709
$Y(G)/Y$ of 3-4	0.1878	0.1856	0.1832	0.1809	0.1787	0.1911	0.1941	0.1974	0.2009

Note: The cost of capital and the valuation ratio show an essence of the endogenous fiscal rule. These optimum values exist when both deficits and debts are zero. The higher deficits and debts are the higher the valuation ratio below 1.0 and, the higher surplus and lending are the higher the valuation ratio above 1.0, where the horizontal asymptote is 1.0, by setting the cost of capital at the X axis.

Table 4 New relationship between deficits and debts: from De Grauwe's (2005) to the endogenous fiscal rule

A DEVELOPED COUNTRY	Before simu	Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.1	0.0141	0.0129	0.0115	0.0102	0.0089	0.0159	0.0175	0.0191	0.0208
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.2	0.0071	0.0065	0.0058	0.0051	0.0044	0.0079	0.0087	0.0096	0.0104
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.4	(0.0024)	(0.0022)	(0.0019)	(0.0017)	(0.0015)	(0.0026)	(0.0029)	(0.0031)	(0.0034)
$(r^*-g_Y^*)(-d)$ of 3.1	0.0261	0.0254	0.0251	0.0248	0.0245	0.0261	0.0265	0.0268	0.0272
$(r^*-g_Y^*)(-d)$ of 3.2	0.0061	0.0059	0.0057	0.0056	0.0054	0.0062	0.0064	0.0065	0.0067
$(r^*-g_Y^*)(-d)$ of 3.3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r^*-g_Y^*)(-d)$ of 3.4	0.0049	0.0049	0.0050	0.0050	0.0051	0.0048	0.0047	0.0046	0.0046
difference of 3.1	(0.0120)	(0.0125)	(0.0136)	(0.0147)	(0.0157)	(0.0102)	(0.0090)	(0.0077)	(0.0064)
difference of 3.2	0.0010	0.0006	0.0000	(0.0005)	(0.0010)	0.0017	0.0024	0.0030	0.0037
difference of 3.3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
difference of 3.4	(0.0073)	(0.0071)	(0.0069)	(0.0067)	(0.0066)	(0.0074)	(0.0076)	(0.0078)	(0.0080)
A DEVELOPING COUNTRY									
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.1	(0.0161)	(0.0167)	(0.0197)	(0.0224)	(0.0250)	(0.0100)	(0.0061)	(0.0019)	0.0027
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.2	(0.0080)	0.0062	0.0046	0.0032	0.0018	0.0098	0.0118	0.0140	0.0164
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$(r_{(DEBT)}^*g_Y^*)(-d)$ of 3.4	0.0027	0.0028	0.0033	0.0038	0.0043	0.0016	0.0010	0.0003	(0.0004)
$(r^*-g_Y^*)(-d)$ of 3.1	0.0913	0.0840	0.0813	0.0788	0.0764	0.0901	0.0935	0.0971	0.1011
$(r^*-g_Y^*)(-d)$ of 3.2	0.0190	0.0179	0.0167	0.0156	0.0146	0.0205	0.0221	0.0237	0.0255
$(r^*-g_Y^*)(-d)$ of 3.3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$(r^*-g_Y^*)(-d)$ of 3.4	0.0202	0.0195	0.0198	0.0201	0.0204	0.0187	0.0183	0.0178	0.0173
difference of 3.1	(0.1074)	(0.1008)	(0.1010)	(0.1012)	(0.1013)	(0.1001)	(0.0996)	(0.0991)	(0.0984)
difference of 3.2	(0.0271)	(0.0117)	(0.0121)	(0.0124)	(0.0128)	(0.0108)	(0.0103)	(0.0097)	(0.0091)
difference of 3.3	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	0.0000	0.0000	0.0000	0.0000
difference of 3.4	(0.0176)	(0.0167)	(0.0165)	(0.0163)	(0.0161)	(0.0171)	(0.0173)	(0.0175)	(0.0178)
A DEVELOPED COUNTRY									
$(r_{(DEBT)}^*g_Y^*)$ of 3-1, 3-2, 3-3, 3-4	0.0050	0.0031	(0.0000)	(0.0031)	(0.0062)	0.0095	0.0127	0.0160	0.0192
$(r^*-g_Y^*)$ of 3-1, 3-2, 3-3, 3-4	0.0217	0.0243	0.0276	0.0309	0.0341	0.0177	0.0143	0.0110	0.0076
$(r_{(DEBT)}-r^*)$ of 3-1, 3-2, 3-3, 3-5	(0.0166)	(0.0212)	(0.0276)	(0.0340)	(0.0404)	(0.0082)	(0.0016)	0.0050	0.0117
A DEVELOPING COUNTRY									
$(r_{(DEBT)}^*g_Y^*)$ of 3-1, 3-2, 3-3, 3-4	(0.0268)	(0.0287)	(0.0348)	(0.0408)	(0.0467)	(0.0160)	(0.0095)	(0.0029)	0.0039
$(r^*-g_Y^*)$ of 3-1, 3-2, 3-3, 3-4	(0.0857)	(0.0584)	(0.0390)	(0.0202)	(0.0019)	(0.0989)	(0.1200)	(0.1418)	(0.1642)
$(r_{(DEBT)}-r^*)$ of 3-1, 3-2, 3-3, 3-5	0.0589	0.0297	0.0042	(0.0206)	(0.0448)	0.0828	0.1105	0.1389	0.1681

Note: De Grauwe (2005, 225) forms a necessary condition of solvency of debts in the continuous time, by making deficit equal to the product of debts and the difference between the external interest rate, $r_{(DEBT)}$, and a given growth rate of GDP ; $r_{DEBT} - g_Y$. The necessary condition only holds when deficits and debts are both zero in the discrete time. Kamiryo (2007, 63–64) forms an endogenous cost of capital at convergence in the discrete time, $r^* - g_Y^*$, which corresponds with the above difference of De Grauwe's. The differences between $r_{(DEBT)}$ and r^* and between $g_Y - g_Y^*$ are the keys to bury the two frameworks.

Table A1 De Grauwe's (2005, 225) condition for primary solvency: 1997–2005

Japan	1997	1998	1999	2000	2001	2002	2003	2004	2005
Case 1 XXX Japan									
Measured g_Y^* and given π and $r_{(DEBT)}$:					$g_Y^* \neq \pi \neq r_{(DEBT)}$		Case 1 XXX		
g_Y^* (see the equation)	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$\pi = \Delta D/D$	0.0803	0.1103	0.1220	0.0962	0.0861	0.0957	0.0806	0.0715	0.0552
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0121	0.0139	0.0151	0.0159	0.0156	0.0153	0.0144	0.0180	0.0194
$d = D/Y$	-0.6046	-0.6965	-0.7952	-0.8813	-0.9769	-1.0901	-1.2037	-1.2841	-1.3405
(1) $\Delta d = \Delta D/Y$	-0.0486	-0.0768	-0.0970	-0.0848	-0.0841	-0.1043	-0.0970	-0.0919	-0.0740
(2) $t_{AX} = g_{SPEND(G)}$	-0.0486	-0.0768	-0.0970	-0.0848	-0.0841	-0.1043	-0.0970	-0.0919	-0.0740
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
difference between (2) and (3)	-0.0056	-0.0006	0.0002	0.0013	0.0020	0.0048	0.0073	0.0125	0.0111
difference between (1) and (3)	-0.0430	-0.0763	-0.0972	-0.0860	-0.0860	-0.1091	-0.1043	-0.1043	-0.0851
Case 2 OOX Japan									
Adjusting $\pi = \Delta D/D$ by g_Y^* and given $r_{(DEBT)}$:					$g_Y^* = \pi \neq r_{(DEBT)}$		Case 2 OOX		
g_Y^* (see the equation)	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$\pi = \Delta D/D$	0.0292	0.0199	0.0188	0.0166	0.0140	0.0097	0.0060	0.0044	0.0062
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0121	0.0126	0.0123	0.0118	0.0108	0.0096	0.0084	0.0098	0.0101
$d = D/Y$	-0.6046	-0.6323	-0.6459	-0.6579	-0.6760	-0.6889	-0.7036	-0.6999	-0.6947
(1) $\Delta d = \Delta D/Y$	-0.0177	-0.0126	-0.0121	-0.0109	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043
(2) $t_{AX} = g_{SPEND(G)}$	-0.0177	-0.0126	-0.0121	-0.0109	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
difference between (2) and (3)	-0.0121	-0.0121	-0.0123	-0.0119	-0.0108	-0.0097	-0.0085	-0.0099	-0.0101
difference between (1) and (3)	-0.0121	-0.0121	-0.0123	-0.0119	-0.0108	-0.0097	-0.0085	-0.0099	-0.0101
Case 3 XOO Japan									
Adjusting $\pi = \Delta D/D$ by given $r_{(DEBT)}$:					$g_Y^* \neq \pi = r_{(DEBT)}$		Case 3 XOO		
g_Y^* (see the equation)	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$\pi = \Delta D/D$	0.0200	0.0190	0.0191	0.0180	0.0160	0.0141	0.0121	0.0140	0.0136
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0121	0.0126	0.0123	0.0119	0.0108	0.0097	0.0086	0.0100	0.0104
$d = D/Y$	-0.6046	-0.6317	-0.6455	-0.6585	-0.6779	-0.6939	-0.7131	-0.7163	-0.7163
(1) $\Delta d = \Delta D/Y$	-0.0121	-0.0120	-0.0123	-0.0119	-0.0109	-0.0098	-0.0086	-0.0101	-0.0098
(2) $t_{AX} = g_{SPEND(G)}$	-0.0121	-0.0120	-0.0123	-0.0119	-0.0109	-0.0098	-0.0086	-0.0101	-0.0098
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
difference between (2) and (3)	-0.0065	-0.0115	-0.0125	-0.0128	-0.0122	-0.0128	-0.0129	-0.0170	-0.0157
difference between (1) and (3)	-0.0065	-0.0115	-0.0125	-0.0128	-0.0122	-0.0128	-0.0129	-0.0170	-0.0157
Case 4 OOO Japan									
Adjusting both $\pi = \Delta D/D$ and $r_{(DEBT)}$ by g_Y^* :					$g_Y^* = \pi = r_{(DEBT)}$		Case 4 OOO		
g_Y^* (see the equation)	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$\pi = \Delta D/D$	0.0292	0.0199	0.0188	0.0166	0.0140	0.0097	0.0060	0.0044	0.0062
interest rate to debt, $r_{(DEBT)}$	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0177	0.0132	0.0121	0.0109	0.0095	0.0066	0.0042	0.0030	0.0043
$d = D/Y$	-0.6046	-0.6323	-0.6459	-0.6579	-0.6760	-0.6889	-0.7036	-0.6999	-0.6947
(1) $\Delta d = \Delta D/Y$	-0.0177	-0.0126	-0.0121	-0.0109	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043
(2) $t_{AX} = g_{SPEND(G)}$	-0.0177	-0.0126	-0.0121	-0.0109	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (2) and (3)	-0.0177	-0.0126	-0.0121	-0.0110	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043
difference between (1) and (3)	-0.0177	-0.0126	-0.0121	-0.0110	-0.0095	-0.0067	-0.0042	-0.0031	-0.0043

Note: Deficit ΔD , debt D , and interest paid $R_{(DEBT)}$, each are shown as minus, to distinguish surplus, lending, and interest received, each shown as plus.

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table A2 Specific simulation of deficits and debts: four cases of Japan 1997–2005

Japan	1997	1998	1999	2000	2001	2002	2003	2004	2005
Case 2-2 OOX Japan	Set debt=290000 in 1997 instead of debt=250000 as shown in Case 2.								
g_Y^* (see the equation)	$g_Y^* \neq \pi \neq r_{(DEBT)}$	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043
$\pi = \Delta D / D$	0.0292	0.0203	0.0175	0.0150	0.0123	0.0083	0.0050	0.0036	0.0052
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)}) / Y$	0.0140	0.0147	0.0142	0.0137	0.0125	0.0111	0.0097	0.0113	0.0116
$d = D / Y$	-0.7013	-0.7337	-0.7485	-0.7613	-0.7808	-0.7945	-0.8107	-0.8059	-0.7990
(1) $\Delta d = \Delta D / Y$	-0.0205	-0.0149	-0.0131	-0.0114	-0.0096	-0.0066	-0.0041	-0.0029	-0.0041
(2) $t_{AX} - \xi_{SPEND(G)}$	-0.0205	-0.0149	-0.0131	-0.0114	-0.0096	-0.0066	-0.0041	-0.0029	-0.0041
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0064	-0.0006	0.0002	0.0011	0.0016	0.0035	0.0049	0.0078	0.0066
difference between (2) and (3)	-0.0140	-0.0143	-0.0133	-0.0125	-0.0112	-0.0101	-0.0090	-0.0107	-0.0107
difference between (1) and (3)	-0.0140	-0.0143	-0.0133	-0.0125	-0.0112	-0.0101	-0.0090	-0.0107	-0.0107
Case 1-2 XXX Primary, Japan	Primary is shown by deficit less debt interest.								
g_Y^* (see the equation)	$g_Y^* \neq \pi \neq r_{(DEBT)}$	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043
$\pi = \Delta D / D$	0.1003	0.1303	0.1410	0.1142	0.1021	0.1097	0.0926	0.0855	0.0697
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)}) / Y$	0.0121	0.0139	0.0151	0.0159	0.0156	0.0153	0.0144	0.0180	0.0194
$d = D / Y$	-0.6046	-0.6965	-0.7952	-0.8813	-0.9769	-1.0901	-1.2037	-1.2841	-1.3405
(1) $\Delta d = \Delta D / Y$	-0.0606	-0.0908	-0.1121	-0.1006	-0.0997	-0.1196	-0.1114	-0.1099	-0.0934
(2) $t_{AX} - \xi_{SPEND(G)}$	-0.0606	-0.0908	-0.1121	-0.1006	-0.0997	-0.1196	-0.1114	-0.1099	-0.0934
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0056	-0.0006	0.0002	0.0013	0.0020	0.0048	0.0073	0.0125	0.0111
difference between (2) and (3)	-0.0551	-0.0902	-0.1124	-0.1019	-0.1017	-0.1244	-0.1188	-0.1223	-0.1045
difference between (1) and (3)	-0.0551	-0.0902	-0.1124	-0.1019	-0.1017	-0.1244	-0.1188	-0.1223	-0.1045
Case 1-3 XXX Primary, Japan	Assume taxes are the same as those in non-primary deficit.								
g_Y^* (see the equation)	$g_Y^* \neq \pi \neq r_{(DEBT)}$	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043
$\pi = \Delta D / D$	0.1003	0.1303	0.1410	0.1142	0.1021	0.1097	0.0926	0.0855	0.0697
interest rate to debt, $r_{(DEBT)}$	0.0200	0.0200	0.0190	0.0180	0.0160	0.0140	0.0120	0.0140	0.0145
$r_{(DEBT)} = (-R_{(DEBT)}) / Y$	0.0121	0.0139	0.0151	0.0159	0.0156	0.0153	0.0144	0.0180	0.0194
$d = D / Y$	-0.6046	-0.6965	-0.7952	-0.8813	-0.9769	-1.0901	-1.2037	-1.2841	-1.3405
(1) $\Delta d = \Delta D / Y$	-0.0606	-0.0908	-0.1121	-0.1006	-0.0997	-0.1196	-0.1114	-0.1099	-0.0934
(2) $t_{AX} - \xi_{SPEND(G)}$	-0.0606	-0.0908	-0.1121	-0.1006	-0.0997	-0.1196	-0.1114	-0.1099	-0.0934
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	-0.0092	-0.0008	0.0003	0.0014	0.0020	0.0044	0.0061	0.0097	0.0083
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0056	-0.0006	0.0002	0.0013	0.0020	0.0048	0.0073	0.0125	0.0111
difference between (2) and (3)	-0.0551	-0.0902	-0.1124	-0.1019	-0.1017	-0.1244	-0.1188	-0.1223	-0.1045
difference between (1) and (3)	-0.0551	-0.0902	-0.1124	-0.1019	-0.1017	-0.1244	-0.1188	-0.1223	-0.1045
Case 1-4 OOO Japan	$\pi = 0$ and $g_Y^* = r_{(DEBT)}$: $(t_{AX} - \xi_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d) = 0$ holds only under no deficit.								
g_Y^* (see the equation)	0.0292	0.0208	0.0187	0.0166	0.0140	0.0096	0.0059	0.0043	0.0062
$\pi = \Delta D / D$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
interest rate to debt, $r_{(DEBT)}$	0.0292	0.0200	0.0190	0.0180	0.0142	0.0097	0.0059	0.0043	0.0062
$r_{(DEBT)} = (-R_{(DEBT)}) / Y$	0.0176	0.0124	0.0118	0.0112	0.0090	0.0061	0.0038	0.0028	0.0039
$d = D / Y$	-0.6046	-0.6197	-0.6211	-0.6222	-0.6303	-0.6361	-0.6458	-0.6396	-0.6309
(1) $\Delta d = \Delta D / Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(2) $t_{AX} - \xi_{SPEND(G)}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0000	-0.0008	0.0003	0.0014	0.0002	0.0001	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	0.0000	-0.0005	0.0002	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000
difference between (2) and (3)	0.0000	0.0005	-0.0002	-0.0009	-0.0002	0.0000	0.0000	0.0000	0.0000
difference between (1) and (3)	0.0000	0.0005	-0.0002	-0.0009	-0.0002	0.0000	0.0000	0.0000	0.0000

Note: Deficit ΔD , debt D , and interest paid $R_{(DEBT)}$, each are shown as minus, to distinguish surplus, lending, and interest received, each shown as plus.

Table A3 De Grauwe's (2005, 225) condition for primary solvency: the US 1997–2005

the US	1997	1998	1999	2000	2001	2002	2003	2004	2005
Case 1 XXX the US									
Measured g_Y^* and given π and $r_{(DEBT)}$:					$g_Y^* \neq \pi \neq r_{(DEBT)}$		Case 1 XXX		
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0006	-0.0143	-0.0429	-0.0749	-0.0279	0.0651	0.1007	0.0923	0.0684
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0334	0.0266	0.0257	0.0240	0.0188	0.0177	0.0161	0.0177	0.0180
$d = D/Y$	-0.5254	-0.5049	-0.4547	-0.3974	-0.3739	-0.3833	-0.4007	-0.4149	-0.4188
(1) $\Delta d = \Delta D/Y$	-0.0003	0.0072	0.0195	0.0298	0.0104	-0.0250	-0.0404	-0.0383	-0.0286
(2) $t_{AX} \cdot g_{SPEND(G)}$	-0.0003	0.0072	0.0195	0.0298	0.0104	-0.0250	-0.0404	-0.0383	-0.0286
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*) \cdot (-d)$	-0.0157	-0.0126	-0.0116	-0.0107	-0.0084	-0.0069	-0.0027	-0.0027	-0.0020
difference between (2) and (3)	0.0154	0.0198	0.0311	0.0405	0.0188	-0.0180	-0.0377	-0.0356	-0.0266
difference between (1) and (3)	0.0154	0.0198	0.0311	0.0405	0.0188	-0.0180	-0.0377	-0.0356	-0.0266
Case 2 OOX the US									
Adjusting $\pi = \Delta D/D$ by g_Y^* and given $r_{(DEBT)}$:					$g_Y^* = \pi \neq r_{(DEBT)}$		Case 2 OOX		
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0337	0.0270	0.0304	0.0331	0.0272	0.0279	0.0335	0.0354	0.0381
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0334	0.0277	0.0288	0.0299	0.0247	0.0224	0.0190	0.0196	0.0193
$d = D/Y$	-0.5254	-0.5262	-0.5098	-0.4953	-0.4924	-0.4854	-0.4722	-0.4601	-0.4498
(1) $\Delta d = \Delta D/Y$	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171
(2) $t_{AX} \cdot g_{SPEND(G)}$	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*) \cdot (-d)$	-0.0157	-0.0131	-0.0130	-0.0134	-0.0110	-0.0088	-0.0032	-0.0030	-0.0021
difference between (2) and (3)	-0.0020	-0.0010	-0.0025	-0.0030	-0.0024	-0.0047	-0.0127	-0.0133	-0.0150
difference between (1) and (3)	-0.0020	-0.0010	-0.0025	-0.0030	-0.0024	-0.0047	-0.0127	-0.0133	-0.0150
Case 3 XO the US									
Adjusting $\pi = \Delta D/D$ by given $r_{(DEBT)}$:					$g_Y^* \neq \pi = r_{(DEBT)}$		Case 3 XO		
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0635	0.0526	0.0564	0.0602	0.0498	0.0454	0.0399	0.0425	0.0428
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0334	0.0284	0.0304	0.0324	0.0275	0.0253	0.0216	0.0226	0.0223
$d = D/Y$	-0.5254	-0.5405	-0.5379	-0.5378	-0.5473	-0.5495	-0.5381	-0.5282	-0.5190
(1) $\Delta d = \Delta D/Y$	-0.0334	-0.0284	-0.0303	-0.0324	-0.0273	-0.0250	-0.0215	-0.0225	-0.0222
(2) $t_{AX} \cdot g_{SPEND(G)}$	-0.0334	-0.0284	-0.0303	-0.0324	-0.0273	-0.0250	-0.0215	-0.0225	-0.0222
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*) \cdot (-d)$	-0.0157	-0.0135	-0.0138	-0.0145	-0.0122	-0.0099	-0.0036	-0.0034	-0.0025
difference between (2) and (3)	-0.0177	-0.0149	-0.0166	-0.0179	-0.0150	-0.0150	-0.0179	-0.0190	-0.0198
difference between (1) and (3)	-0.0177	-0.0149	-0.0166	-0.0179	-0.0150	-0.0150	-0.0179	-0.0190	-0.0198
Case 4 OOO the US									
Adjusting both $\pi = \Delta D/D$ and $r_{(DEBT)}$ by g_Y^* :					$g_Y^* = \pi = r_{(DEBT)}$		Case 4 OOO		
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0337	0.0270	0.0304	0.0331	0.0272	0.0279	0.0335	0.0354	0.0381
interest rate to debt, $r_{(DEBT)}$	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0177	0.0145	0.0158	0.0165	0.0137	0.0136	0.0158	0.0166	0.0172
$d = D/Y$	-0.5254	-0.5262	-0.5098	-0.4953	-0.4924	-0.4854	-0.4722	-0.4601	-0.4498
(1) $\Delta d = \Delta D/Y$	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171
(2) $t_{AX} \cdot g_{SPEND(G)}$	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*) \cdot (-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (2) and (3)	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171
difference between (1) and (3)	-0.0177	-0.0142	-0.0155	-0.0164	-0.0134	-0.0135	-0.0158	-0.0163	-0.0171

Note: Deficit ΔD , debt D , and interest paid $R_{(DEBT)}$, each are shown as minus, to distinguish surplus, lending, and interest received, each shown as plus.

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table A4 Specific simulation of deficits and debts: four cases of the US 1997–2005

the US	1997	1998	1999	2000	2001	2002	2003	2004	2005
Case 2-2 OOX the US									
$g_Y^* = \pi \neq r_{(DEBT)}$	Set debt=4500 in 1997 instead of debt=3867 as shown in Case 2.								
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0337	0.0275	0.0303	0.0330	0.0272	0.0280	0.0330	0.0359	0.0380
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0388	0.0322	0.0335	0.0348	0.0288	0.0261	0.0221	0.0229	0.0225
$d = D/Y$	-0.6115	-0.6128	-0.5936	-0.5767	-0.5733	-0.5652	-0.5495	-0.5356	-0.5237
(1) $\Delta d = \Delta D/Y$	-0.0206	-0.0169	-0.0180	-0.0190	-0.0156	-0.0158	-0.0181	-0.0192	-0.0199
(2) $t_{AX} - g_{SPEND(G)}$	-0.0206	-0.0169	-0.0180	-0.0190	-0.0156	-0.0158	-0.0181	-0.0192	-0.0199
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0182	-0.0153	-0.0152	-0.0155	-0.0128	-0.0102	-0.0037	-0.0035	-0.0025
difference between (2) and (3)	-0.0024	-0.0016	-0.0028	-0.0035	-0.0028	-0.0056	-0.0144	-0.0157	-0.0174
difference between (1) and (3)	-0.0024	-0.0016	-0.0028	-0.0035	-0.0028	-0.0056	-0.0144	-0.0157	-0.0174
Case 1-2 XXX Primary, US									
$g_Y^* \neq \pi \neq r_{(DEBT)}$	Primary is shown by deficit less debt interest.								
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0641	0.0387	0.0181	0.0015	0.0293	0.1011	0.1445	0.1605	0.1462
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0334	0.0273	0.0287	0.0305	0.0251	0.0209	0.0155	0.0139	0.0119
$d = D/Y$	-0.5254	-0.5193	-0.5072	-0.5063	-0.5000	-0.4542	-0.3867	-0.3251	-0.2771
(1) $\Delta d = \Delta D/Y$	-0.0337	-0.0201	-0.0092	-0.0008	-0.0147	-0.0459	-0.0559	-0.0522	-0.0405
(2) $t_{AX} - g_{SPEND(G)}$	-0.0337	-0.0201	-0.0092	-0.0008	-0.0147	-0.0459	-0.0559	-0.0522	-0.0405
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0157	-0.0130	-0.0130	-0.0136	-0.0112	-0.0082	-0.0026	-0.0021	-0.0013
difference between (2) and (3)	-0.0180	-0.0071	0.0038	0.0129	-0.0035	-0.0377	-0.0533	-0.0501	-0.0392
difference between (1) and (3)	-0.0180	-0.0071	0.0038	0.0129	-0.0035	-0.0377	-0.0533	-0.0501	-0.0392
Case 1-3 XXX Primary, US									
$g_Y^* \neq \pi \neq r_{(DEBT)}$	Assume taxes are the same as those in non-primary deficit.								
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0641	0.0387	0.0181	0.0015	0.0293	0.1011	0.1445	0.1605	0.1462
interest rate to debt, $r_{(DEBT)}$	0.0635	0.0526	0.0565	0.0603	0.0502	0.0461	0.0402	0.0427	0.0429
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0334	0.0273	0.0287	0.0305	0.0251	0.0209	0.0155	0.0139	0.0119
$d = D/Y$	-0.5254	-0.5193	-0.5072	-0.5063	-0.5000	-0.4542	-0.3867	-0.3251	-0.2771
(1) $\Delta d = \Delta D/Y$	-0.0337	-0.0201	-0.0092	-0.0008	-0.0147	-0.0459	-0.0559	-0.0522	-0.0405
(2) $t_{AX} - g_{SPEND(G)}$	-0.0337	-0.0201	-0.0092	-0.0008	-0.0147	-0.0459	-0.0559	-0.0522	-0.0405
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0298	0.0250	0.0256	0.0270	0.0224	0.0181	0.0067	0.0065	0.0047
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	-0.0157	-0.0130	-0.0130	-0.0136	-0.0112	-0.0082	-0.0026	-0.0021	-0.0013
difference between (2) and (3)	-0.0180	-0.0071	0.0038	0.0129	-0.0035	-0.0377	-0.0533	-0.0501	-0.0392
difference between (1) and (3)	-0.0180	-0.0071	0.0038	0.0129	-0.0035	-0.0377	-0.0533	-0.0501	-0.0392
Case 1-4 O00 the US									
$\pi = 0$ and $g_Y^* = r_{(DEBT)}$	$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d) = 0$ holds only under no deficit.								
g_Y^* (see the equation)	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$\pi = \Delta D/D$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
interest rate to debt, $r_{(DEBT)}$	0.0337	0.0276	0.0309	0.0333	0.0278	0.0280	0.0335	0.0362	0.0382
$r_{(DEBT)} = (-R_{(DEBT)})/Y$	0.0177	0.0141	0.0149	0.0151	0.0122	0.0117	0.0132	0.0134	0.0133
$d = D/Y$	-0.5254	-0.5121	-0.4809	-0.4518	-0.4369	-0.4187	-0.3937	-0.3700	-0.3480
(1) $\Delta d = \Delta D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(2) $t_{AX} - g_{SPEND(G)}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (1) and (2)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(r_{(DEBT)} - g_Y^*)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
(3) $(r_{(DEBT)} - g_Y^*)(-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (2) and (3)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
difference between (1) and (3)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: Deficit ΔD , debt D , and interest paid $R_{(DEBT)}$, each are shown as minus, to distinguish surplus, lending, and interest received, each shown as plus.

Table A5 Simulation 3.1 & 3.2 of the EMU rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 3-1		Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1000$				$n = gL = 0.0000$		$(\rho/r) = 1.13151$	
change rate of private net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0265	0.0285	0.0312	0.0339	0.0365	0.0231	0.0204	0.0176	0.0149
output=income, $Y=C+S$	11091	11352	11620	11888	12156	10815	10547	10278	10009
balance of payment, BOP	(727)	(607)	(487)	(367)	(248)	(846)	(966)	(1086)	(1205)
net investment, I	1109	1229	1349	1468	1588	989	870	750	630
deficit, ΔD (For surplus, plus)	(333)	(341)	(349)	(357)	(365)	(324)	(316)	(308)	(300)
debt, D	(6655)	(6662)	(6671)	(6679)	(6687)	(6646)	(6638)	(6630)	(6622)
G net investment, $I(G)$	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	(245)	(235)	(225)	(216)	(206)	(254)	(264)	(273)	(283)
BOP/Y	(0.0655)	(0.0535)	(0.0419)	(0.0309)	(0.0204)	(0.0782)	(0.0916)	(0.1056)	(0.1204)
$\pi = \Delta D/D$	0.0500	0.0511	0.0523	0.0534	0.0545	0.0488	0.0477	0.0465	0.0453
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5869)	(0.5740)	(0.5618)	(0.5501)	(0.6145)	(0.6294)	(0.6451)	(0.6616)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y) \cdot (-d)$	(0.0398)	(0.0384)	(0.0367)	(0.0351)	(0.0335)	(0.0422)	(0.0442)	(0.0463)	(0.0485)
$c = C/Y$	0.9655	0.9452	0.9259	0.9074	0.8897	0.9868	1.0091	1.0327	1.0575
capital-output ratio, $\Omega = K/Y$	2.9188	2.9599	2.9020	2.8467	2.7938	3.0847	3.1519	3.2227	3.2974
1-beta* (see the equation)	0.2262	0.2201	0.2199	0.2198	0.2196	0.2204	0.2205	0.2207	0.2208
$i_{(G)} = I(G)/Y(G)$	0.0512	0.0610	0.0706	0.0801	0.0894	0.0413	0.0312	0.0210	0.0106
$\Omega(G) = K(G)/Y(G)$	4.0746	4.1042	4.0830	4.0621	4.0416	4.1477	4.1700	4.1927	4.2157
1-beta $_{(G)}$ * (see the equation)	0.2190	0.2167	0.2166	0.2165	0.2165	0.2169	0.2170	0.2171	0.2172
$g_{Y(G)}^*$	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$\alpha = \Pi/Y$	0.1467	0.1647	0.1817	0.1981	0.2137	0.1279	0.1082	0.0874	0.0654
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.1425)	(0.1357)	(0.1292)	(0.1228)	(0.1165)	(0.1490)	(0.1559)	(0.1628)	(0.1698)
$r = \Pi/K$	0.0503	0.0556	0.0626	0.0696	0.0765	0.0415	0.0343	0.0271	0.0198
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.0350)	(0.0331)	(0.0316)	(0.0302)	(0.0288)	(0.0359)	(0.0374)	(0.0388)	(0.0403)
$Y(G)/Y$	0.1549	0.1525	0.1501	0.1479	0.1457	0.1576	0.1604	0.1633	0.1663

A DEVELOPED COUNTRY 3-2		Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1000$				$n = gL = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0265	0.0285	0.0312	0.0339	0.0365	0.0231	0.0204	0.0176	0.0149
output=income, $Y=C+S$	11091	11352	11620	11888	12156	10815	10547	10278	10009
balance of payment, BOP	(727)	(607)	(487)	(367)	(248)	(846)	(966)	(1086)	(1205)
net investment, I	1109	1229	1349	1468	1588	989	870	750	630
deficit, ΔD (For surplus, plus)	(166)	(170)	(174)	(178)	(182)	(162)	(158)	(154)	(150)
debt, D	(3327)	(3331)	(3335)	(3339)	(3343)	(3323)	(3319)	(3315)	(3311)
G net investment, $I(G)$	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	(78)	(65)	(51)	(38)	(24)	(92)	(105)	(119)	(133)
BOP/Y	(0.0655)	(0.0535)	(0.0419)	(0.0309)	(0.0204)	(0.0782)	(0.0916)	(0.1056)	(0.1204)
$\pi = \Delta D/D$	0.0500	0.0511	0.0523	0.0534	0.0545	0.0488	0.0477	0.0465	0.0453
$\Delta d = \Delta D/Y$	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$d = D/Y$	(0.3000)	(0.2934)	(0.2870)	(0.2809)	(0.2750)	(0.3073)	(0.3147)	(0.3225)	(0.3308)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	(0.0000)	(0.0000)	0.0000	(0.0000)	(0.0000)	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y) \cdot (-d)$	(0.0199)	(0.0192)	(0.0184)	(0.0175)	(0.0168)	(0.0211)	(0.0221)	(0.0231)	(0.0243)
$c = C/Y$	0.9655	0.9452	0.9259	0.9074	0.8897	0.9868	1.0091	1.0327	1.0575
capital-output ratio, $\Omega = K/Y$	2.9188	2.9599	2.9020	2.8467	2.7938	3.0847	3.1519	3.2227	3.2974
1-beta* (see the equation)	0.2262	0.2201	0.2199	0.2198	0.2196	0.2204	0.2205	0.2207	0.2208
$i_{(G)} = I(G)/Y(G)$	0.0467	0.0555	0.0642	0.0727	0.0811	0.0377	0.0285	0.0192	0.0097
$\Omega(G) = K(G)/Y(G)$	3.7148	3.7367	3.7121	3.6880	3.6644	3.7872	3.8133	3.8399	3.8670
1-beta $_{(G)}$ * (see the equation)	0.2190	0.2167	0.2166	0.2165	0.2165	0.2169	0.2170	0.2171	0.2172
$g_{Y(G)}^*$	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$\alpha = \Pi/Y$	0.1467	0.1647	0.1817	0.1981	0.2137	0.1279	0.1082	0.0874	0.0654
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.0416)	(0.0340)	(0.0266)	(0.0194)	(0.0123)	(0.0492)	(0.0570)	(0.0649)	(0.0730)
$r = \Pi/K$	0.0503	0.0556	0.0626	0.0696	0.0765	0.0415	0.0343	0.0271	0.0198
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.0112)	(0.0091)	(0.0072)	(0.0053)	(0.0033)	(0.0130)	(0.0149)	(0.0169)	(0.0189)
$Y(G)/Y$	0.1750	0.1736	0.1721	0.1706	0.1693	0.1767	0.1783	0.1801	0.1819

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table A6 Simulation 3.3 & 3.4 of the EMU rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 3-3		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$		$i = I/Y = 0.1000$				$n = g_L = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I, as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, I(G), as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y (see the equation)	0.0265	0.0285	0.0312	0.0339	0.0365	0.0231	0.0204	0.0176	0.0149
output=income, $Y=C+S$	11091	11352	11620	11888	12156	10815	10547	10278	10009
balance of payment, BOP	(727)	(607)	(487)	(367)	(248)	(846)	(966)	(1086)	(1205)
net investment, I	1109	1229	1349	1468	1588	989	870	750	630
deficit, ΔD (For surplus, plus)	(0)	0	0	0	0	0	0	0	0
debt, D	0	0	0	0	0	0	0	0	0
G net investment, I(G)	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	88	106	123	141	158	70	53	35	18
BOP/Y	(0.0655)	(0.0535)	(0.0419)	(0.0309)	(0.0204)	(0.0782)	(0.0916)	(0.1056)	(0.1204)
$\pi = \Delta D/D$	#DIV/0!	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d = \Delta D/Y$	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d = D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y) \cdot (-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$c = C/Y$	0.9655	0.9452	0.9259	0.9074	0.8897	0.9868	1.0091	1.0327	1.0575
capital-output ratio, $\Omega = K/Y$	2.9188	2.9599	2.9020	2.8467	2.7938	3.0847	3.1519	3.2227	3.2974
1-beta (see the equation)	0.2262	0.2201	0.2199	0.2198	0.2196	0.2204	0.2205	0.2207	0.2208
$i_{(G)} = I(G)/Y(G)$	0.0429	0.0510	0.0589	0.0666	0.0742	0.0347	0.0263	0.0177	0.0090
$\Omega(G) = K(G)/Y(G)$	3.4135	3.4295	3.4030	3.3770	3.3515	3.4844	3.5128	3.5418	3.5715
1-beta _(G) (see the equation)	0.2190	0.2167	0.2166	0.2165	0.2165	0.2169	0.2170	0.2171	0.2172
$g_{Y(G)}$	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$\alpha = \Pi/Y$	0.1467	0.1647	0.1817	0.1981	0.2137	0.1279	0.1082	0.0874	0.0654
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.0429	0.0510	0.0589	0.0666	0.0742	0.0347	0.0263	0.0177	0.0090
$r = \Pi/K$	0.0503	0.0556	0.0626	0.0696	0.0765	0.0415	0.0343	0.0271	0.0198
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0126	0.0149	0.0173	0.0197	0.0221	0.0100	0.0075	0.0050	0.0025
$Y(G)/Y$	0.1849	0.1825	0.1801	0.1779	0.1757	0.1876	0.1904	0.1933	0.1963
A DEVELOPED COUNTRY 3-4		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$		$i = I/Y = 0.1000$				$n = g_L = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I, as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, I(G), as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y (see the equation)	0.0265	0.0285	0.0312	0.0339	0.0365	0.0231	0.0204	0.0176	0.0149
output=income, $Y=C+S$	11091	11352	11620	11888	12156	10815	10547	10278	10009
balance of payment, BOP	(727)	(607)	(487)	(367)	(248)	(846)	(966)	(1086)	(1205)
net investment, I	1109	1229	1349	1468	1588	989	870	750	630
deficit, ΔD (For surplus, plus)	333	341	349	357	365	324	316	308	300
debt, D	1109	1117	1125	1133	1141	1101	1093	1085	1077
G net investment, I(G)	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	421	446	472	497	523	395	369	344	318
BOP/Y	(0.0655)	(0.0535)	(0.0419)	(0.0309)	(0.0204)	(0.0782)	(0.0916)	(0.1056)	(0.1204)
$\pi = \Delta D/D$	0.3000	0.3049	0.3099	0.3148	0.3196	0.2947	0.2895	0.2843	0.2789
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
$d = D/Y$	0.1000	0.0984	0.0968	0.0953	0.0939	0.1018	0.1036	0.1055	0.1076
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	(0.0000)	(0.0000)	0.0000	(0.0000)	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y) \cdot (-d)$	0.0316	0.0314	0.0311	0.0309	0.0306	0.0320	0.0323	0.0327	0.0330
$c = C/Y$	0.9655	0.9452	0.9259	0.9074	0.8897	0.9868	1.0091	1.0327	1.0575
capital-output ratio, $\Omega = K/Y$	2.9188	2.9599	2.9020	2.8467	2.7938	3.0847	3.1519	3.2227	3.2974
1-beta (see the equation)	0.2262	0.2201	0.2199	0.2198	0.2196	0.2204	0.2205	0.2207	0.2208
$i_{(G)} = I(G)/Y(G)$	0.0369	0.0438	0.0505	0.0570	0.0633	0.0299	0.0227	0.0153	0.0078
$\Omega(G) = K(G)/Y(G)$	2.9369	2.9454	2.9172	2.8896	2.8628	3.0041	3.0346	3.0659	3.0981
1-beta _(G) (see the equation)	0.2190	0.2167	0.2166	0.2165	0.2165	0.2169	0.2170	0.2171	0.2172
$g_{Y(G)}$	0.0098	0.0116	0.0135	0.0154	0.0173	0.0078	0.0059	0.0039	0.0020
$\alpha = \Pi/Y$	0.1467	0.1647	0.1817	0.1981	0.2137	0.1279	0.1082	0.0874	0.0654
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.1765	0.1849	0.1932	0.2013	0.2092	0.1678	0.1589	0.1497	0.1403
$r = \Pi/K$	0.0503	0.0556	0.0626	0.0696	0.0765	0.0415	0.0343	0.0271	0.0198
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0601	0.0628	0.0662	0.0697	0.0731	0.0558	0.0523	0.0488	0.0453
$Y(G)/Y$	0.2149	0.2125	0.2101	0.2079	0.2057	0.2176	0.2204	0.2233	0.2263

Table A7 Simulation 3.1 & 3.2 of the EMU rule to deficits and debts: a developing country

A DEVELOPING COUNTRY 3-1		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1335$				$n = gL = 0.0000$	$(\rho/r) = 0.80000$		
change rate of private net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0697	0.0716	0.0777	0.0837	0.0896	0.0589	0.0524	0.0458	0.0390
output=income, $Y=C+S$	13196	13593	14037	14480	14923	12705	12260	11813	11364
balance of payment, BOP	727	915	1103	1291	1479	538	350	162	(26)
net investment, I	1761	1949	2137	2325	2514	1573	1385	1197	1009
deficit, ΔD (For surplus, plus)	(396)	(408)	(421)	(434)	(448)	(381)	(368)	(354)	(341)
debt, D	(7918)	(7930)	(7943)	(7956)	(7969)	(7903)	(7890)	(7876)	(7863)
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24
G saving=returns, $S(G)=\Delta D+I(G)$	(276)	(264)	(253)	(242)	(232)	(285)	(296)	(306)	(317)
BOP/Y	0.0551	0.0673	0.0786	0.0891	0.0991	0.0424	0.0286	0.0137	(0.0023)
$\pi = \Delta D/D$	0.0500	0.0514	0.0530	0.0546	0.0562	0.0482	0.0466	0.0450	0.0434
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5834)	(0.5659)	(0.5495)	(0.5340)	(0.6220)	(0.6435)	(0.6668)	(0.6919)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)} - (d))$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)} - (d))$	(0.0139)	(0.0133)	(0.0103)	(0.0076)	(0.0050)	(0.0200)	(0.0239)	(0.0281)	(0.0327)
$c = C/Y$	0.8115	0.7893	0.7692	0.7503	0.7325	0.8338	0.8585	0.8850	0.9135
capital-output ratio, $\Omega = K/Y$	0.8995	1.0166	0.9979	0.9803	0.9639	1.0581	1.0812	1.1062	1.1333
1-beta* (see the equation)	0.5300	0.4925	0.4907	0.4889	0.4872	0.4962	0.4981	0.5000	0.5019
$i_{(G)} = I(G)/Y(G)$	0.0711	0.0843	0.0972	0.1097	0.1220	0.0576	0.0438	0.0296	0.0150
$\Omega(G) = K(G)/Y(G)$	1.3240	1.3921	1.3891	1.3862	1.3836	1.3987	1.4022	1.4060	1.4100
1-beta $_{(G)}$ * (see the equation)	0.4677	0.4533	0.4521	0.4509	0.4497	0.4557	0.4569	0.4581	0.4593
$g_{Y(G)}^*$	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
$\alpha = \Pi/Y$ (0.0144)	0.0134	0.0386	0.0623	0.0846	(0.0423)	(0.0731)	(0.1062)	(0.1419)	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ (0.1636)	(0.1544)	(0.1464)	(0.1385)	(0.1307)	(0.1712)	(0.1799)	(0.1888)	(0.1979)	
$r = \Pi/K$ (0.0160)	0.0132	0.0387	0.0635	0.0877	(0.0399)	(0.0676)	(0.0960)	(0.1252)	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ (0.1235)	(0.1109)	(0.1054)	(0.0999)	(0.0945)	(0.1224)	(0.1283)	(0.1343)	(0.1404)	
$Y(G)/Y$ (0.1278)	0.1256	0.1232	0.1209	0.1187	0.1311	0.1341	0.1374	0.1409	

A DEVELOPING COUNTRY 3-2		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1335$				$n = gL = 0.01203$	$(\rho/r) = 0.80000$		
change rate of private net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0697	0.0716	0.0777	0.0837	0.0897	0.0589	0.0524	0.0458	0.0390
output=income, $Y=C+S$	13196	13592	14034	14475	14914	12704	12258	11811	11362
balance of payment, BOP	727	915	1103	1291	1479	538	350	162	(26)
net investment, I	1761	1949	2137	2325	2514	1573	1385	1197	1009
deficit, ΔD (For surplus, plus)	(198)	(408)	(421)	(434)	(447)	(381)	(368)	(354)	(341)
debt, D	(3959)	(4169)	(4182)	(4195)	(4208)	(4142)	(4129)	(4115)	(4102)
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24
G saving=returns, $S(G)=\Delta D+I(G)$	(78)	(264)	(253)	(242)	(231)	(285)	(296)	(306)	(317)
BOP/Y	0.0551	0.0673	0.0786	0.0892	0.0992	0.0424	0.0286	0.0137	(0.0023)
$\pi = \Delta D/D$	0.0500	0.0978	0.1007	0.1035	0.1063	0.0920	0.0891	0.0861	0.0831
$\Delta d = \Delta D/Y$	(0.0150)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.3000)	(0.3067)	(0.2980)	(0.2898)	(0.2822)	(0.3260)	(0.3368)	(0.3484)	(0.3610)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)} - (d))$	(0.0000)	0.0000	(0.0000)	0.0000	0.0000	0.0000	0.0000	(0.0000)	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)} - (d))$	(0.0070)	(0.0212)	(0.0196)	(0.0182)	(0.0168)	(0.0248)	(0.0268)	(0.0290)	(0.0314)
$c = C/Y$	0.8115	0.7893	0.7691	0.7502	0.7323	0.8338	0.8585	0.8849	0.9135
capital-output ratio, $\Omega = K/Y$	0.8995	1.0167	0.9981	0.9807	0.9644	1.0581	1.0813	1.1063	1.1335
1-beta* (see the equation)	0.5300	0.4925	0.4906	0.4888	0.4870	0.4962	0.4981	0.5000	0.5019
$i_{(G)} = I(G)/Y(G)$	0.0637	0.0753	0.0866	0.0976	0.1083	0.0517	0.0394	0.0267	0.0135
$\Omega(G) = K(G)/Y(G)$	1.1850	1.2436	1.2383	1.2332	1.2283	1.2550	1.2612	1.2676	1.2743
1-beta $_{(G)}$ * (see the equation)	0.4677	0.4533	0.4521	0.4509	0.4497	0.4557	0.4569	0.4581	0.4593
$g_{Y(G)}^*$	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057
$\alpha = \Pi/Y$ (0.0144)	0.0134	0.0386	0.0623	0.0846	(0.0423)	(0.0731)	(0.1062)	(0.1419)	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ (0.0414)	(0.0313)	(0.0219)	(0.0128)	(0.0039)	(0.0510)	(0.0612)	(0.0718)	(0.0827)	
$r = \Pi/K$ (0.0160)	0.0132	0.0387	0.0635	0.0877	(0.0399)	(0.0676)	(0.0960)	(0.1252)	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ (0.0349)	(0.0252)	(0.0177)	(0.0104)	(0.0031)	(0.0406)	(0.0485)	(0.0566)	(0.0649)	
$Y(G)/Y$ 0.1428	0.1406	0.1382	0.1359	0.1337	0.1461	0.1491	0.1524	0.1559	

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table A8 Simulation 3.3 & 3.4 of the EMU rule to deficits and debts: a developing country

A DEVELOPING COUNTRY		3--3	Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$			$i = I/Y = 0.1335$				$n = g_L = 0.01203$	$(\rho/r) = 0.80000$		
change rate of net investment, I , as a base			0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base			0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0697	0.0716	0.0777	0.0837	0.0897	0.0589	0.0524	0.0458	0.0390	
output=income, $Y=C+S$	13196	13592	14034	14475	14914	12704	12258	11811	11362	
balance of payment, BOP	727	915	1103	1291	1479	538	350	162	162	(26)
net investment, I	1761	1949	2137	2325	2514	1573	1385	1197	1009	
deficit, ΔD (For surplus, plus)	0	0	0	0	0	0	0	0	0	
debt, D	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24	
G saving=returns, $S(G)=\Delta D+I(G)$	120	144	168	192	216	96	72	48	24	
BOP/ Y	0.0551	0.0673	0.0786	0.0892	0.0992	0.0424	0.0286	0.0137	(0.0023)	
$\pi = \Delta D/D$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$\Delta d = \Delta D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$d = D/Y$	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y^*)(-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	(0.0000)	
$c = C/Y$	0.8115	0.7893	0.7691	0.7502	0.7323	0.8338	0.8585	0.8849	0.9135	
capital-output ratio, $\Omega = K/Y$	0.8995	1.0167	0.9981	0.9807	0.9644	1.0581	1.0813	1.1063	1.1335	
1-beta* (see the equation)	0.5300	0.4925	0.4906	0.4888	0.4870	0.4962	0.4981	0.5000	0.5019	
$i_{(G)} = I(G)/Y(G)$	0.0576	0.0681	0.0781	0.0879	0.0974	0.0469	0.0358	0.0243	0.0124	
$\Omega(G) = K(G)/Y(G)$	1.0724	1.1238	1.1170	1.1106	1.1044	1.1382	1.1459	1.1540	1.1625	
1-beta $_{(G)}$ * (see the equation)	0.4677	0.4533	0.4521	0.4509	0.4497	0.4557	0.4569	0.4581	0.4593	
$g_{Y(G)}^*$	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057	
$\alpha = \Pi/Y$ (0.0144)	0.0134	0.0386	0.0623	0.0846	0.0846	(0.0423)	(0.0731)	(0.1062)	(0.1419)	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ 0.0576	0.0681	0.0781	0.0879	0.0974	0.0974	0.0469	0.0358	0.0243	0.0124	
$r = \Pi/K$ (0.0160)	0.0132	0.0387	0.0635	0.0877	0.0877	(0.0399)	(0.0676)	(0.0960)	(0.1252)	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ 0.0537	0.0606	0.0700	0.0792	0.0882	0.0882	0.0412	0.0312	0.0210	0.0106	
$Y(G)/Y$ 0.1578	0.1556	0.1532	0.1509	0.1487	0.1487	0.1611	0.1641	0.1674	0.1709	
A DEVELOPING COUNTRY		3--4	Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$			$i = I/Y = 0.1335$				$n = g_L = 0.01203$	$(\rho/r) = 0.80000$		
change rate of net investment, I , as a base			0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base			0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0697	0.0716	0.0777	0.0837	0.0897	0.0589	0.0524	0.0458	0.0390	
output=income, $Y=C+S$	13196	13592	14034	14475	14914	12704	12258	11811	11362	
balance of payment, BOP	727	915	1103	1291	1479	538	350	162	162	(26)
net investment, I	1761	1949	2137	2325	2514	1573	1385	1197	1009	
deficit, ΔD (For surplus, plus)	396	408	421	434	447	381	368	354	341	
debt, D	1320	1331	1345	1358	1371	1305	1291	1278	1265	
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24	
G saving=returns, $S(G)=\Delta D+I(G)$	516	552	589	626	663	477	440	402	365	
BOP/ Y	0.0551	0.0673	0.0786	0.0892	0.0992	0.0424	0.0286	0.0137	(0.0023)	
$\pi = \Delta D/D$	0.0000	0.3062	0.3131	0.3198	0.3263	0.2921	0.2848	0.2772	0.2695	
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	
$d = D/Y$	0.1000	0.0980	0.0958	0.0938	0.0919	0.1027	0.1054	0.1082	0.1113	
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	(0.0000)	(0.0000)	0.0000	(0.0000)	0.0000	0.0000	0.0000	
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y^*)(-d)$	0.0273	0.0272	0.0267	0.0262	0.0257	0.0284	0.0290	0.0297	0.0304	
$c = C/Y$	0.8115	0.7893	0.7691	0.7502	0.7323	0.8338	0.8585	0.8849	0.9135	
capital-output ratio, $\Omega = K/Y$	0.8995	1.0167	0.9981	0.9807	0.9644	1.0581	1.0813	1.1063	1.1335	
1-beta* (see the equation)	0.5300	0.4925	0.4906	0.4888	0.4870	0.4962	0.4981	0.5000	0.5019	
$i_{(G)} = I(G)/Y(G)$	0.0484	0.0571	0.0653	0.0733	0.0810	0.0395	0.0303	0.0206	0.0105	
$\Omega(G) = K(G)/Y(G)$	0.9011	0.9422	0.9341	0.9264	0.9190	0.9595	0.9688	0.9786	0.9889	
1-beta $_{(G)}$ * (see the equation)	0.4677	0.4533	0.4521	0.4509	0.4497	0.4557	0.4569	0.4581	0.4593	
$g_{Y(G)}^*$	0.0286	0.0331	0.0383	0.0435	0.0485	0.0224	0.0170	0.0114	0.0057	
$\alpha = \Pi/Y$ (0.0144)	0.0134	0.0386	0.0623	0.0846	0.0846	(0.0423)	(0.0731)	(0.1062)	(0.1419)	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ 0.2081	0.2187	0.2291	0.2392	0.2489	0.2489	0.1965	0.1848	0.1726	0.1598	
$r = \Pi/K$ (0.0160)	0.0132	0.0387	0.0635	0.0877	0.0877	(0.0399)	(0.0676)	(0.0960)	(0.1252)	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ 0.2310	0.2321	0.2453	0.2582	0.2709	0.2709	0.2048	0.1908	0.1764	0.1616	
$Y(G)/Y$ 0.1878	0.1856	0.1832	0.1809	0.1787	0.1787	0.1911	0.1941	0.1974	0.2009	

Appendix: Tables AA1 to AA8 show the results of simulations in the degree of financial solvency and those in the endogenous degree of solvency: under ‘the BOP’ or ‘ ΔD & D ’

Table AA1 Simulation 1.1 & 1.2 of the EMU rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 1-1		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1000$				$n = gL = 0.01203$ $(\rho/r) = 1.13151$			
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0379	0.0398	0.0429	0.0460	0.0491	0.0334	0.0302	0.0269	0.0237
output=income, $Y = C + S$	11091	11223	11589	11844	12099	10822	10565	10309	10052
balance of payment, BOP	(727)	(727)	(505)	(394)	(283)	(837)	(948)	(1059)	(1170)
net investment, I	1109	1220	1331	1442	1553	998	887	776	665
deficit, ΔD (For surplus, plus)	(333)	(337)	(348)	(355)	(363)	(325)	(317)	(309)	(302)
debt, D	(6655)	(6659)	(6670)	(6677)	(6685)	(6647)	(6639)	(6631)	(6623)
G net investment, $I(G)$	88	88	88	88	88	88	88	88	88
G saving=returns, $S(G) = \Delta D + I(G)$	(245)	(249)	(260)	(267)	(275)	(237)	(229)	(221)	(214)
BOP/Y	(0.0655)	(0.0647)	(0.0435)	(0.0332)	(0.0234)	(0.0774)	(0.0898)	(0.1028)	(0.1164)
$\pi = \Delta D/D$	0.0500	0.0506	0.0521	0.0532	0.0543	0.0488	0.0477	0.0466	0.0455
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5933)	(0.5755)	(0.5638)	(0.5525)	(0.6142)	(0.6284)	(0.6433)	(0.6589)
$\Delta D/Y - (I_{AX} - g_{SPEND(G)})$	0.0000	0.0000	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	(0.0000)
$(I_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d)$	(0.0330)	(0.0319)	(0.0300)	(0.0282)	(0.0266)	(0.0358)	(0.0380)	(0.0403)	(0.0427)
$c = C/Y$	0.9655	0.9560	0.9287	0.9115	0.8950	0.9851	1.0058	1.0274	1.0502
capital-output ratio, $\Omega = K/Y$	2.0658	2.1504	2.0919	2.0562	2.0221	2.2095	2.2526	2.2979	2.3456
1-beta* (see the equation)	0.2178	0.2131	0.2182	0.2224	0.2259	0.1995	0.1900	0.1777	0.1613
$i_{(G)} = I(G)/Y(G)$	0.0512	0.0514	0.0517	0.0520	0.0522	0.0510	0.0508	0.0506	0.0503
$\Omega(G) = K(G)/Y(G)$	1.0017	1.0563	1.0631	1.0680	1.0728	1.0488	1.0442	1.0395	1.0349
1-beta $_{(G)}$ * (see the equation)	0.3694	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526
$g_{Y(G)}^*$	0.0322	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315
$\alpha = \Pi/Y$	0.1467	0.1551	0.1792	0.1944	0.2090	0.1294	0.1111	0.0920	0.0719
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.1425)	(0.1452)	(0.1526)	(0.1578)	(0.1631)	(0.1372)	(0.1322)	(0.1272)	(0.1222)
$r = \Pi/K$	0.0710	0.0721	0.0857	0.0946	0.1034	0.0585	0.0493	0.0400	0.0306
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.1422)	(0.1375)	(0.1436)	(0.1478)	(0.1520)	(0.1308)	(0.1266)	(0.1223)	(0.1181)
$Y(G)/Y$	0.1549	0.1526	0.1468	0.1430	0.1394	0.1594	0.1640	0.1688	0.1739
A DEVELOPED COUNTRY 1-2		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1000$				$n = gL = 0.01203$ $(\rho/r) = 1.13151$			
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0379	0.0398	0.0429	0.0460	0.0491	0.0334	0.0302	0.0269	0.0237
output=income, $Y = C + S$	11091	11223	11589	11844	12099	10822	10565	10309	10052
balance of payment, BOP	(727)	(727)	(505)	(394)	(283)	(837)	(948)	(1059)	(1170)
net investment, I	1109	1220	1331	1442	1553	998	887	776	665
deficit, ΔD (For surplus, plus)	(166)	(168)	(174)	(178)	(181)	(162)	(158)	(155)	(151)
debt, D	(3327)	(3329)	(3335)	(3339)	(3342)	(3323)	(3319)	(3316)	(3312)
G net investment, $I(G)$	88	88	88	88	88	88	88	88	88
G saving=returns, $S(G) = \Delta D + I(G)$	(78)	(80)	(86)	(90)	(93)	(74)	(70)	(67)	(63)
BOP/Y	(0.0655)	(0.0647)	(0.0435)	(0.0332)	(0.0234)	(0.0774)	(0.0898)	(0.1028)	(0.1164)
$\pi = \Delta D/D$	0.0500	0.0506	0.0521	0.0532	0.0543	0.0488	0.0477	0.0466	0.0455
$\Delta d = \Delta D/Y$	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$d = D/Y$	(0.3000)	(0.2967)	(0.2878)	(0.2819)	(0.2763)	(0.3071)	(0.3142)	(0.3216)	(0.3295)
$\Delta D/Y - (I_{AX} - g_{SPEND(G)})$	(0.0000)	(0.0000)	0.0000	0.0000	(0.0000)	0.0000	0.0000	(0.0000)	(0.0000)
$(I_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d)$	(0.0165)	(0.0159)	(0.0150)	(0.0141)	(0.0133)	(0.0179)	(0.0190)	(0.0201)	(0.0213)
$c = C/Y$	0.9655	0.9560	0.9287	0.9115	0.8950	0.9851	1.0058	1.0274	1.0502
capital-output ratio, $\Omega = K/Y$	2.0658	2.1504	2.0919	2.0562	2.0221	2.2095	2.2526	2.2979	2.3456
1-beta* (see the equation)	0.2178	0.2131	0.2182	0.2224	0.2259	0.1995	0.1900	0.1777	0.1613
$i_{(G)} = I(G)/Y(G)$	0.0467	0.0468	0.0469	0.0470	0.0471	0.0466	0.0465	0.0464	0.0463
$\Omega(G) = K(G)/Y(G)$	0.9132	0.9617	0.9646	0.9666	0.9686	0.9586	0.9566	0.9547	0.9527
1-beta $_{(G)}$ * (see the equation)	0.3694	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526
$g_{Y(G)}^*$	0.0322	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315
$\alpha = \Pi/Y$	0.1467	0.1551	0.1792	0.1944	0.2090	0.1294	0.1111	0.0920	0.0719
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.0416)	(0.0427)	(0.0458)	(0.0479)	(0.0501)	(0.0394)	(0.0373)	(0.0352)	(0.0331)
$r = \Pi/K$	0.0710	0.0721	0.0857	0.0946	0.1034	0.0585	0.0493	0.0400	0.0306
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.0455)	(0.0444)	(0.0475)	(0.0496)	(0.0517)	(0.0411)	(0.0390)	(0.0368)	(0.0347)
$Y(G)/Y$	0.1699	0.1676	0.1618	0.1580	0.1544	0.1744	0.1790	0.1838	0.1889

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table AA2 Simulation 1.3 & 1.4 of the EMU rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 1-3					Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$					$i = I/Y = 0.1000$				$n = gL = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I , as a base					0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base					0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0379	0.0398	0.0429	0.0460	0.0491	0.0334	0.0302	0.0269	0.0237			
output=income, $Y=C+S$	11091	11223	11589	11844	12099	10822	10565	10309	10052			
balance of payment, BOP	(727)	(727)	(505)	(394)	(283)	(837)	(948)	(1059)	(1170)			
net investment, I	1109	1220	1331	1442	1553	998	887	776	665			
deficit, ΔD (For surplus, plus)	2	0	0	0	0	0	0	0	0			
debt, D	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)			
G net investment, $I(G)$	88	88	88	88	88	88	88	88	88			
G saving=returns, $S(G)=\Delta D+I(G)$	88	88	88	88	88	88	88	88	88			
BOP/ Y	(0.0655)	(0.0647)	(0.0435)	(0.0332)	(0.0234)	(0.0774)	(0.0898)	(0.1028)	(0.1164)			
$\pi = \Delta D/D$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000			
$\Delta d = \Delta D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000			
$d = D/Y$	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)			
$\Delta D/Y - (t_{AX} - gSPEND(G))$	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000			
$(t_{AX} - gSPEND(G)) - (r_{(DEBT)} - g_Y^*)(-d)$	(0.0000)	(0.0000)	0.0000	0.0000	0.0000	(0.0000)	(0.0000)	(0.0000)	(0.0000)			
$c = C/Y$	0.9655	0.9560	0.9287	0.9115	0.8950	0.9851	1.0058	1.0274	1.0502			
capital-output ratio, $\Omega = K/Y$	2.0658	2.1504	2.0919	2.0562	2.0221	2.2095	2.2526	2.2979	2.3456			
$1 - \beta_{G^*}$ (see the equation)	0.2178	0.2131	0.2182	0.2224	0.2259	0.1995	0.1900	0.1777	0.1613			
$i_{(G)} = I(G)/Y(G)$	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429			
$\Omega(G) = K(G)/Y(G)$	0.8392	0.8827	0.8827	0.8828	0.8828	0.8827	0.8827	0.8826	0.8826			
$1 - \beta_{G(G)^*}$ (see the equation)	0.3694	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526			
$g_{Y(G)}^*$	0.0322	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315			
$\alpha = \Pi/Y$	0.1467	0.1551	0.1792	0.1944	0.2090	0.1294	0.1111	0.0920	0.0719			
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429			
$r = \Pi/K$	0.0710	0.0721	0.0857	0.0946	0.1034	0.0585	0.0493	0.0400	0.0306			
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0511	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486	0.0486			
$Y(G)/Y$	0.1849	0.1826	0.1768	0.1730	0.1694	0.1894	0.1940	0.1988	0.2039			
A DEVELOPED COUNTRY 1-4					Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$					$i = I/Y = 0.1000$				$n = gL = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I , as a base					0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base					0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0379	0.0398	0.0429	0.0460	0.0491	0.0334	0.0302	0.0269	0.0237			
output=income, $Y=C+S$	11091	11223	11589	11844	12099	10822	10565	10309	10052			
balance of payment, BOP	(727)	(727)	(505)	(394)	(283)	(837)	(948)	(1059)	(1170)			
net investment, I	1109	1220	1331	1442	1553	998	887	776	665			
deficit, ΔD (For surplus, plus)	333	337	348	355	363	325	317	309	302			
debt, D	1109	1113	1124	1132	1139	1101	1093	1086	1078			
G net investment, $I(G)$	88	88	88	88	88	88	88	88	88			
G saving=returns, $S(G)=\Delta D+I(G)$	421	425	436	443	451	413	405	397	390			
BOP/ Y	(0.0655)	(0.0647)	(0.0435)	(0.0332)	(0.0234)	(0.0774)	(0.0898)	(0.1028)	(0.1164)			
$\pi = \Delta D/D$	0.3000	0.3025	0.3093	0.3140	0.3186	0.2949	0.2899	0.2849	0.2798			
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300			
$d = D/Y$	0.1000	0.0992	0.0970	0.0955	0.0942	0.1017	0.1035	0.1053	0.1072			
$\Delta D/Y - (t_{AX} - gSPEND(G))$	0.0000	(0.0000)	0.0000	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000			
$(t_{AX} - gSPEND(G)) - (r_{(DEBT)} - g_Y^*)(-d)$	0.0305	0.0303	0.0300	0.0297	0.0294	0.0310	0.0313	0.0317	0.0321			
$c = C/Y$	0.9655	0.9560	0.9287	0.9115	0.8950	0.9851	1.0058	1.0274	1.0502			
capital-output ratio, $\Omega = K/Y$	2.0658	2.1504	2.0919	2.0562	2.0221	2.2095	2.2526	2.2979	2.3456			
$1 - \beta_{G^*}$ (see the equation)	0.2178	0.2131	0.2182	0.2224	0.2259	0.1995	0.1900	0.1777	0.1613			
$i_{(G)} = I(G)/Y(G)$	0.0369	0.0369	0.0367	0.0366	0.0365	0.0371	0.0372	0.0373	0.0374			
$\Omega(G) = K(G)/Y(G)$	0.7220	0.7582	0.7547	0.7523	0.7499	0.7620	0.7644	0.7669	0.7694			
$1 - \beta_{G(G)^*}$ (see the equation)	0.3694	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526	0.3526			
$g_{Y(G)}^*$	0.0322	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315	0.0315			
$\alpha = \Pi/Y$	0.1467	0.1551	0.1792	0.1944	0.2090	0.1294	0.1111	0.0920	0.0719			
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.1765	0.1780	0.1818	0.1844	0.1870	0.1738	0.1711	0.1684	0.1657			
$r = \Pi/K$	0.0710	0.0721	0.0857	0.0946	0.1034	0.0585	0.0493	0.0400	0.0306			
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.2445	0.2348	0.2409	0.2451	0.2493	0.2281	0.2239	0.2196	0.2154			
$Y(G)/Y$	0.2149	0.2126	0.2068	0.2030	0.1994	0.2194	0.2240	0.2288	0.2339			

Table AA3 Simulation 2.1 & 2.2 of the *EMU* rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 2-1		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1000$				$n = gL = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I , as a base		0	0	0	0	0	0	0	0
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0379	0.0371	0.0376	0.0381	0.0386	0.0361	0.0356	0.0351	0.0346
output=income, $Y=C+S$	11091	11118	11159	11200	11240	11037	10997	10956	10915
balance of payment, BOP	(727)	(709)	(691)	(674)	(656)	(744)	(762)	(779)	(797)
net investment, I	1109	1127	1144	1162	1179	1091	1074	1056	1039
deficit, ΔD (For surplus, plus)	(333)	(334)	(335)	(336)	(337)	(331)	(330)	(329)	(327)
debt, D	(6655)	(6655)	(6657)	(6658)	(6659)	(6653)	(6652)	(6651)	(6649)
G net investment, $I(G)$	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	(245)	(228)	(212)	(195)	(179)	(261)	(277)	(293)	(310)
BOP/Y	(0.0655)	(0.0638)	(0.0620)	(0.0602)	(0.0584)	(0.0674)	(0.0693)	(0.0711)	(0.0730)
$\pi = \Delta D/D$	0.0500	0.0501	0.0503	0.0505	0.0506	0.0498	0.0496	0.0494	0.0492
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5986)	(0.5965)	(0.5945)	(0.5924)	(0.6028)	(0.6049)	(0.6070)	(0.6092)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	(0.0000)	(0.0000)	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y)(-d)$	(0.0330)	(0.0335)	(0.0331)	(0.0328)	(0.0325)	(0.0341)	(0.0344)	(0.0347)	(0.0351)
$c = C/Y$	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315
capital-output ratio, $\Omega = K/Y$	2.0659	2.1621	2.1558	2.1496	2.1433	2.1748	2.1813	2.1877	2.1942
1-beta* (see the equation)	0.2178	0.2080	0.2091	0.2101	0.2110	0.2059	0.2048	0.2036	0.2024
$i_{(G)} = I(G)/Y(G)$	0.0512	0.0606	0.0697	0.0786	0.0872	0.0416	0.0317	0.0215	0.0109
$\Omega(G) = K(G)/Y(G)$	1.0017	1.0484	1.0434	1.0386	1.0340	1.0590	1.0646	1.0704	1.0765
1-beta $_{(G)}$ * (see the equation)	0.3694	0.3776	0.3951	0.4080	0.4177	0.3144	0.2499	0.1197	(0.2736)
$g_{Y(G)}^*$	0.0322	0.0360	0.0404	0.0448	0.0491	0.0269	0.0223	0.0176	0.0129
$\alpha = \Pi/Y$	0.1467	0.1494	0.1521	0.1547	0.1574	0.1440	0.1413	0.1386	0.1358
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.1425)	(0.1308)	(0.1197)	(0.1089)	(0.0984)	(0.1541)	(0.1663)	(0.1789)	(0.1919)
$r = \Pi/K$	0.0710	0.0691	0.0706	0.0720	0.0734	0.0662	0.0648	0.0633	0.0619
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.1422)	(0.1248)	(0.1147)	(0.1048)	(0.0951)	(0.1455)	(0.1562)	(0.1671)	(0.1782)
$Y(G)/Y$	0.1549	0.1567	0.1584	0.1600	0.1617	0.1533	0.1515	0.1497	0.1479

A DEVELOPED COUNTRY 2-2		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1000$				$n = gL = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I , as a base		0	0	0	0	0	0	0	0
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0379	0.0371	0.0376	0.0381	0.0386	0.0361	0.0356	0.0351	0.0346
output=income, $Y=C+S$	11091	11118	11159	11199	11240	11037	10996	10956	10915
balance of payment, BOP	(727)	(709)	(691)	(674)	(656)	(744)	(762)	(779)	(797)
net investment, I	1109	1127	1144	1162	1179	1091	1074	1056	1039
deficit, ΔD (For surplus, plus)	(166)	(167)	(167)	(168)	(169)	(166)	(165)	(164)	(164)
debt, D	(3327)	(3328)	(3328)	(3329)	(3330)	(3327)	(3326)	(3325)	(3325)
G net investment, $I(G)$	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	(78)	(61)	(44)	(27)	(10)	(95)	(112)	(129)	(146)
BOP/Y	(0.0655)	(0.0638)	(0.0620)	(0.0602)	(0.0584)	(0.0674)	(0.0693)	(0.0711)	(0.0730)
$\pi = \Delta D/D$	0.0500	0.0501	0.0503	0.0505	0.0506	0.0498	0.0496	0.0494	0.0492
$\Delta d = \Delta D/Y$	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$d = D/Y$	(0.3000)	(0.2993)	(0.2983)	(0.2972)	(0.2962)	(0.3014)	(0.3025)	(0.3035)	(0.3046)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	(0.0000)	0.0000	0.0000	0.0000	(0.0000)	(0.0000)	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r(DEBT) - g_Y)(-d)$	(0.0165)	(0.0167)	(0.0166)	(0.0164)	(0.0163)	(0.0170)	(0.0172)	(0.0174)	(0.0175)
$c = C/Y$	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315
capital-output ratio, $\Omega = K/Y$	2.0659	2.1621	2.1558	2.1496	2.1434	2.1748	2.1813	2.1878	2.1943
1-beta* (see the equation)	0.2178	0.2080	0.2091	0.2100	0.2110	0.2059	0.2048	0.2036	0.2024
$i_{(G)} = I(G)/Y(G)$	0.0467	0.0553	0.0637	0.0718	0.0798	0.0379	0.0288	0.0195	0.0099
$\Omega(G) = K(G)/Y(G)$	0.9132	0.9568	0.9532	0.9498	0.9465	0.9645	0.9686	0.9729	0.9774
1-beta $_{(G)}$ * (see the equation)	0.3694	0.3776	0.3951	0.4079	0.4176	0.3144	0.2499	0.1197	(0.2735)
$g_{Y(G)}^*$	0.0322	0.0360	0.0404	0.0448	0.0491	0.0269	0.0223	0.0176	0.0129
$\alpha = \Pi/Y$	0.1467	0.1494	0.1521	0.1547	0.1574	0.1440	0.1413	0.1386	0.1358
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	(0.0416)	(0.0320)	(0.0228)	(0.0139)	(0.0051)	(0.0512)	(0.0612)	(0.0715)	(0.0822)
$r = \Pi/K$	0.0710	0.0691	0.0706	0.0720	0.0734	0.0662	0.0648	0.0633	0.0619
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	(0.0455)	(0.0335)	(0.0240)	(0.0146)	(0.0054)	(0.0531)	(0.0632)	(0.0735)	(0.0841)
$Y(G)/Y$	0.1699	0.1717	0.1734	0.1750	0.1766	0.1683	0.1665	0.1647	0.1629

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table AA4 Simulation 2.3 & 2.4 of the EMU rule to deficits and debts: a developed country

A DEVELOPED COUNTRY 2--3		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$		$i = I/Y = 0.1000$				$n = g_L = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I, as a base		0	0	0	0	0	0	0	0
change rate of G net investment, I(G), as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y (see the equation)	0.0379	0.0371	0.0376	0.0381	0.0386	0.0361	0.0356	0.0351	0.0346
output=income, $Y=C+S$	11091	11118	11159	11199	11240	11037	10996	10956	10915
balance of payment, BOP	(727)	(709)	(691)	(674)	(656)	(744)	(762)	(779)	(797)
net investment, I	1109	1127	1144	1162	1179	1091	1074	1056	1039
deficit, ΔD (For surplus, plus)	(0)	0	0	0	0	0	0	0	0
debt, D	0	0	0	0	0	0	0	0	0
G net investment, I(G)	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	88	106	123	141	158	70	53	35	18
BOP/Y	(0.0655)	(0.0638)	(0.0620)	(0.0602)	(0.0584)	(0.0674)	(0.0693)	(0.0711)	(0.0730)
$\pi = \Delta D/D$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta d = \Delta D/Y$	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d = D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y)(-d)$	(0.0000)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$c = C/Y$	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315
capital-output ratio, $\Omega = K/Y$	2.0659	2.1621	2.1558	2.1496	2.1434	2.1748	2.1813	2.1878	2.1943
1-beta (see the equation)	0.2178	0.2080	0.2091	0.2100	0.2110	0.2059	0.2048	0.2036	0.2024
$i_{(G)} = I(G)/Y(G)$	0.0429	0.0509	0.0586	0.0662	0.0735	0.0348	0.0265	0.0179	0.0091
$\Omega(G) = K(G)/Y(G)$	0.8392	0.8800	0.8773	0.8748	0.8724	0.8856	0.8886	0.8917	0.8950
1-beta _(G) (see the equation)	0.3694	0.3776	0.3951	0.4079	0.4176	0.3144	0.2499	0.1197	(0.2735)
$g_{Y(G)}$	0.0322	0.0360	0.0404	0.0448	0.0491	0.0269	0.0223	0.0176	0.0129
$\alpha = \Pi/Y$	0.1467	0.1494	0.1521	0.1547	0.1574	0.1440	0.1413	0.1386	0.1358
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.0429	0.0509	0.0586	0.0662	0.0735	0.0348	0.0265	0.0179	0.0091
$r = \Pi/K$	0.0710	0.0691	0.0706	0.0720	0.0734	0.0662	0.0648	0.0633	0.0619
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0511	0.0578	0.0668	0.0756	0.0843	0.0393	0.0298	0.0200	0.0101
$Y(G)/Y$	0.1849	0.1867	0.1884	0.1900	0.1916	0.1833	0.1815	0.1797	0.1779
A DEVELOPED COUNTRY 2--4		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$		$i = I/Y = 0.1000$				$n = g_L = 0.01203$		$(\rho/r) = 1.13151$	
change rate of net investment, I, as a base		0	0	0	0	0	0	0	0
change rate of G net investment, I(G), as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y (see the equation)	0.0379	0.0371	0.0376	0.0381	0.0386	0.0361	0.0356	0.0351	0.0346
output=income, $Y=C+S$	11091	11118	11159	11199	11240	11037	10996	10956	10915
balance of payment, BOP	(727)	(709)	(691)	(674)	(656)	(744)	(762)	(779)	(797)
net investment, I	1109	1127	1144	1162	1179	1091	1074	1056	1039
deficit, ΔD (For surplus, plus)	333	334	335	336	337	331	330	329	327
debt, D	1109	1110	1111	1112	1114	1107	1106	1105	1104
G net investment, I(G)	88	106	123	141	158	70	53	35	18
G saving=returns, $S(G)=\Delta D+I(G)$	421	439	458	477	496	402	383	364	345
BOP/Y	(0.0655)	(0.0638)	(0.0620)	(0.0602)	(0.0584)	(0.0674)	(0.0693)	(0.0711)	(0.0730)
$\pi = \Delta D/D$	0.3000	0.3005	0.3013	0.3020	0.3028	0.2990	0.2982	0.2974	0.2967
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
$d = D/Y$	0.1000	0.0998	0.0996	0.0993	0.0991	0.1003	0.1006	0.1009	0.1011
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y)(-d)$	0.0305	0.0306	0.0305	0.0305	0.0304	0.0307	0.0307	0.0308	0.0308
$c = C/Y$	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315	1.1315
capital-output ratio, $\Omega = K/Y$	2.0659	2.1621	2.1558	2.1496	2.1434	2.1748	2.1813	2.1878	2.1943
1-beta (see the equation)	0.2178	0.2080	0.2091	0.2100	0.2110	0.2059	0.2048	0.2036	0.2024
$i_{(G)} = I(G)/Y(G)$	0.0369	0.0438	0.0506	0.0571	0.0636	0.0299	0.0227	0.0153	0.0078
$\Omega(G) = K(G)/Y(G)$	0.7220	0.7581	0.7568	0.7555	0.7543	0.7610	0.7626	0.7642	0.7659
1-beta _(G) (see the equation)	0.3694	0.3776	0.3951	0.4079	0.4176	0.3144	0.2499	0.1197	(0.2735)
$g_{Y(G)}$	0.0322	0.0360	0.0404	0.0448	0.0491	0.0269	0.0223	0.0176	0.0129
$\alpha = \Pi/Y$	0.1467	0.1494	0.1521	0.1547	0.1574	0.1440	0.1413	0.1386	0.1358
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.1765	0.1823	0.1879	0.1935	0.1989	0.1706	0.1645	0.1584	0.1520
$r = \Pi/K$	0.0710	0.0691	0.0706	0.0720	0.0734	0.0662	0.0648	0.0633	0.0619
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.2445	0.2404	0.2483	0.2561	0.2637	0.2242	0.2158	0.2072	0.1985
$Y(G)/Y$	0.2149	0.2167	0.2184	0.2200	0.2216	0.2133	0.2115	0.2097	0.2079

Table AA5 Simulation 1.1 & 1.2 of the EMU rule to deficits and debts: a developing country

A DEVELOPING COUNTRY 1-1		Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1335$				$n = gL = 0.01203$	$(\rho/r) = 0.90000$		
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0629	0.0646	0.0696	0.0745	0.0793	0.0544	0.0492	0.0439	0.0385
output=income, $Y=C+S$	13196	13566	13973	14379	14784	12753	12345	11936	11527
balance of payment, BOP	727	903	1079	1255	1431	550	374	198	22
net investment, I	1761	1937	2113	2289	2466	1585	1409	1233	1057
deficit, ΔD (For surplus, plus)	(396)	(407)	(419)	(431)	(444)	(383)	(370)	(358)	(346)
debt, D	(7918)	(7929)	(7941)	(7953)	(7965)	(7904)	(7892)	(7880)	(7868)
G net investment, $I(G)$	120	120	120	120	120	120	120	120	120
G saving=returns, $S(G)=\Delta D+I(G)$	(276)	(287)	(299)	(311)	(324)	(263)	(250)	(238)	(226)
BOP/Y	0.0551	0.0665	0.0772	0.0873	0.0968	0.0432	0.0303	0.0166	0.0019
$\pi = \Delta D/D$	0.0500	0.0513	0.0528	0.0542	0.0557	0.0484	0.0469	0.0454	0.0440
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5844)	(0.5683)	(0.5531)	(0.5388)	(0.6198)	(0.6393)	(0.6602)	(0.6826)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	(0.0000)	0.0000	0.0000	(0.0000)	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d)$	(0.0180)	(0.0173)	(0.0148)	(0.0125)	(0.0104)	(0.0229)	(0.0260)	(0.0293)	(0.0330)
$c = C/Y$	0.8115	0.7907	0.7716	0.7535	0.7364	0.8326	0.8556	0.8801	0.9064
capital-output ratio, $\Omega = K/Y$	1.4000	1.5046	1.4734	1.4441	1.4164	1.5730	1.6107	1.6511	1.6945
1-beta* (see the equation)	0.3398	0.3195	0.3223	0.3245	0.3263	0.3116	0.3058	0.2982	0.2877
$i_{(G)} = I(G)/Y(G)$	0.0593	0.0597	0.0600	0.0604	0.0608	0.0590	0.0586	0.0583	0.0579
$\Omega(G) = K(G)/Y(G)$	1.1034	1.1703	1.1774	1.1846	1.1919	1.1562	1.1493	1.1425	1.1357
1-beta $_{(G)}$ * (see the equation)	0.3589	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419
$g_{Y(G)}^*$	0.0344	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336
$\alpha = I/Y$	0.0983	0.1215	0.1427	0.1628	0.1818	0.0749	0.0494	0.0221	(0.0071)
$\alpha_{(G)} = I_{(G)}/Y_{(G)}$	(0.1363)	(0.1427)	(0.1497)	(0.1567)	(0.1638)	(0.1290)	(0.1223)	(0.1156)	(0.1090)
$r = I/K$	0.0702	0.0807	0.0969	0.1127	0.1283	0.0476	0.0307	0.0134	(0.0042)
$r_{(G)} = I_{(G)}/K_{(G)}$	(0.1235)	(0.1219)	(0.1271)	(0.1323)	(0.1374)	(0.1116)	(0.1064)	(0.1012)	(0.0959)
$Y(G)/Y$	0.1534	0.1482	0.1431	0.1382	0.1336	0.1596	0.1659	0.1726	0.1798

A DEVELOPING COUNTRY 1-2		Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1335$				$n = gL = 0.01203$	$(\rho/r) = 0.90000$		
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0629	0.0646	0.0696	0.0745	0.0793	0.0544	0.0492	0.0439	0.0385
output=income, $Y=C+S$	13196	13566	13972	14376	14781	12752	12344	11936	11526
balance of payment, BOP	727	903	1079	1255	1431	550	374	198	22
net investment, I	1761	1937	2113	2289	2466	1585	1409	1233	1057
deficit, ΔD (For surplus, plus)	(198)	(203)	(210)	(216)	(222)	(191)	(185)	(179)	(173)
debt, D	(3959)	(3964)	(3970)	(3977)	(3983)	(3952)	(3946)	(3940)	(3934)
G net investment, $I(G)$	120	120	120	120	120	120	120	120	120
G saving=returns, $S(G)=\Delta D+I(G)$	(78)	(83)	(90)	(96)	(102)	(71)	(65)	(59)	(53)
BOP/Y	0.0551	0.0665	0.0772	0.0873	0.0968	0.0432	0.0303	0.0166	0.0019
$\pi = \Delta D/D$	0.0500	0.0513	0.0528	0.0542	0.0557	0.0484	0.0469	0.0454	0.0439
$\Delta d = \Delta D/Y$	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$d = D/Y$	(0.3000)	(0.2922)	(0.2842)	(0.2766)	(0.2694)	(0.3099)	(0.3197)	(0.3301)	(0.3413)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	(0.0000)	(0.0000)	0.0000	0.0000	0.0000	(0.0000)	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*)(-d)$	(0.0090)	(0.0087)	(0.0074)	(0.0063)	(0.0052)	(0.0114)	(0.0130)	(0.0147)	(0.0165)
$c = C/Y$	0.8115	0.7907	0.7715	0.7535	0.7364	0.8326	0.8555	0.8801	0.9064
capital-output ratio, $\Omega = K/Y$	1.4000	1.5046	1.4736	1.4443	1.4167	1.5730	1.6107	1.6511	1.6945
1-beta* (see the equation)	0.3398	0.3195	0.3222	0.3244	0.3262	0.3116	0.3058	0.2982	0.2877
$i_{(G)} = I(G)/Y(G)$	0.0540	0.0542	0.0543	0.0545	0.0546	0.0539	0.0537	0.0536	0.0535
$\Omega(G) = K(G)/Y(G)$	1.0051	1.0627	1.0657	1.0686	1.0715	1.0569	1.0540	1.0511	1.0483
1-beta $_{(G)}$ * (see the equation)	0.3589	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419
$g_{Y(G)}^*$	0.0344	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336
$\alpha = I/Y$	0.0983	0.1215	0.1427	0.1628	0.1818	0.0749	0.0494	0.0221	(0.0071)
$\alpha_{(G)} = I_{(G)}/Y_{(G)}$	(0.0351)	(0.0377)	(0.0406)	(0.0434)	(0.0463)	(0.0320)	(0.0292)	(0.0264)	(0.0236)
$r = I/K$	0.0702	0.0807	0.0969	0.1127	0.1283	0.0476	0.0307	0.0134	(0.0042)
$r_{(G)} = I_{(G)}/K_{(G)}$	(0.0349)	(0.0355)	(0.0381)	(0.0406)	(0.0432)	(0.0303)	(0.0277)	(0.0251)	(0.0225)
$Y(G)/Y$	0.1684	0.1632	0.1581	0.1532	0.1486	0.1746	0.1809	0.1876	0.1948

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table AA6 Simulation 1.3 & 1.4 of the EMU rule to deficits and debts: a developing country

A DEVELOPING COUNTRY	1-3	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$				$i = I/Y =$	0.1335	$n = gL =$	0.01203	$(\rho/r) =$	0.90000
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0629	0.0646	0.0696	0.0745	0.0793	0.0544	0.0492	0.0439	0.0385
output=income, $Y=C+S$	13196	13566	13972	14377	14781	12752	12344	11936	11526
balance of payment, BOP	727	903	1079	1255	1431	550	374	198	22
net investment, I	1761	1937	2113	2289	2466	1585	1409	1233	1057
deficit, ΔD (For surplus, plus)	0	0	0	0	0	0	0	0	0
debt, D	0	0	0	0	0	0	0	0	0
G net investment, $I(G)$	120	120	120	120	120	120	120	120	120
G saving=returns, $S(G)=\Delta D+I(G)$	120	120	120	120	120	120	120	120	120
BOP/ Y	0.0551	0.0665	0.0772	0.0873	0.0968	0.0432	0.0303	0.0166	0.0019
$\pi = \Delta D/D$	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
$\Delta d = \Delta D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$d = D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*) (-d)$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$c = C/Y$	0.8115	0.7907	0.7715	0.7535	0.7364	0.8326	0.8555	0.8801	0.9064
capital-output ratio, $\Omega = K/Y$	1.4000	1.5046	1.4735	1.4443	1.4167	1.5730	1.6107	1.6511	1.6945
$1 - \beta_{G^*}$ (see the equation)	0.3398	0.3195	0.3222	0.3244	0.3262	0.3116	0.3058	0.2982	0.2877
$i_{(G)} = I(G)/Y(G)$	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496
$\Omega(G) = K(G)/Y(G)$	0.9229	0.9733	0.9733	0.9733	0.9733	0.9733	0.9733	0.9733	0.9733
$1 - \beta_{G(G)^*}$ (see the equation)	0.3589	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419
$g_{Y(G)}^*$	0.0344	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336
$\alpha = \Pi/Y$	0.0983	0.1215	0.1427	0.1628	0.1818	0.0749	0.0494	0.0221	(0.0071)
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496	0.0496
$r = \Pi/K$	0.0702	0.0807	0.0969	0.1127	0.1283	0.0476	0.0307	0.0134	(0.0042)
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0537	0.0510	0.0510	0.0510	0.0510	0.0510	0.0510	0.0510	0.0510
$Y(G)/Y$	0.1834	0.1782	0.1731	0.1682	0.1636	0.1896	0.1959	0.2026	0.2098

A DEVELOPING COUNTRY	1-4	Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$				$i = I/Y =$	0.1335	$n = gL =$	0.01203	$(\rho/r) =$	0.90000
change rate of net investment, I , as a base		0.1	0.2	0.3	0.4	-0.1	-0.2	-0.3	-0.4
change rate of G net investment, $I(G)$, as a base		0	0	0	0	0	0	0	0
g_Y^* (see the equation)	0.0629	0.0646	0.0696	0.0745	0.0793	0.0544	0.0492	0.0439	0.0385
output=income, $Y=C+S$	13196	13566	13972	14377	14781	12752	12344	11936	11526
balance of payment, BOP	727	903	1079	1255	1431	550	374	198	22
net investment, I	1761	1937	2113	2289	2466	1585	1409	1233	1057
deficit, ΔD (For surplus, plus)	396	407	419	431	443	383	370	358	346
debt, D	1320	1331	1343	1355	1367	1306	1294	1282	1270
G net investment, $I(G)$	120	120	120	120	120	120	120	120	120
G saving=returns, $S(G)=\Delta D+I(G)$	516	527	539	551	563	503	490	478	466
BOP/ Y	0.0551	0.0665	0.0772	0.0873	0.0968	0.0432	0.0303	0.0166	0.0019
$\pi = \Delta D/D$	0.3000	0.3058	0.3121	0.3183	0.3243	0.2929	0.2862	0.2793	0.2724
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300
$d = D/Y$	0.1000	0.0981	0.0961	0.0943	0.0925	0.1024	0.1048	0.1074	0.1101
$\Delta D/Y - (t_{AX} - g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT)} - g_Y^*) (-d)$	0.0280	0.0279	0.0274	0.0270	0.0266	0.0288	0.0293	0.0299	0.0305
$c = C/Y$	0.8115	0.7907	0.7715	0.7535	0.7364	0.8326	0.8555	0.8801	0.9064
capital-output ratio, $\Omega = K/Y$	1.4000	1.5046	1.4735	1.4443	1.4167	1.5730	1.6107	1.6511	1.6945
$1 - \beta_{G^*}$ (see the equation)	0.3398	0.3195	0.3222	0.3244	0.3262	0.3116	0.3058	0.2982	0.2877
$i_{(G)} = I(G)/Y(G)$	0.0426	0.0425	0.0423	0.0421	0.0419	0.0428	0.0430	0.0432	0.0434
$\Omega(G) = K(G)/Y(G)$	0.7931	0.8331	0.8295	0.8260	0.8225	0.8403	0.8440	0.8478	0.8515
$1 - \beta_{G(G)^*}$ (see the equation)	0.3589	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419	0.3419
$g_{Y(G)}^*$	0.0344	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336	0.0336
$\alpha = \Pi/Y$	0.0983	0.1215	0.1427	0.1628	0.1818	0.0749	0.0494	0.0221	(0.0071)
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.1832	0.1865	0.1900	0.1935	0.1969	0.1795	0.1759	0.1722	0.1685
$r = \Pi/K$	0.0702	0.0807	0.0969	0.1127	0.1283	0.0476	0.0307	0.0134	(0.0042)
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.2310	0.2239	0.2291	0.2343	0.2394	0.2136	0.2084	0.2031	0.1979
$Y(G)/Y$	0.2134	0.2082	0.2031	0.1982	0.1936	0.2196	0.2259	0.2326	0.2398

Table AA7 Simulation 2.1 & 2.2 of the EMU rule to deficits and debts: a developing country

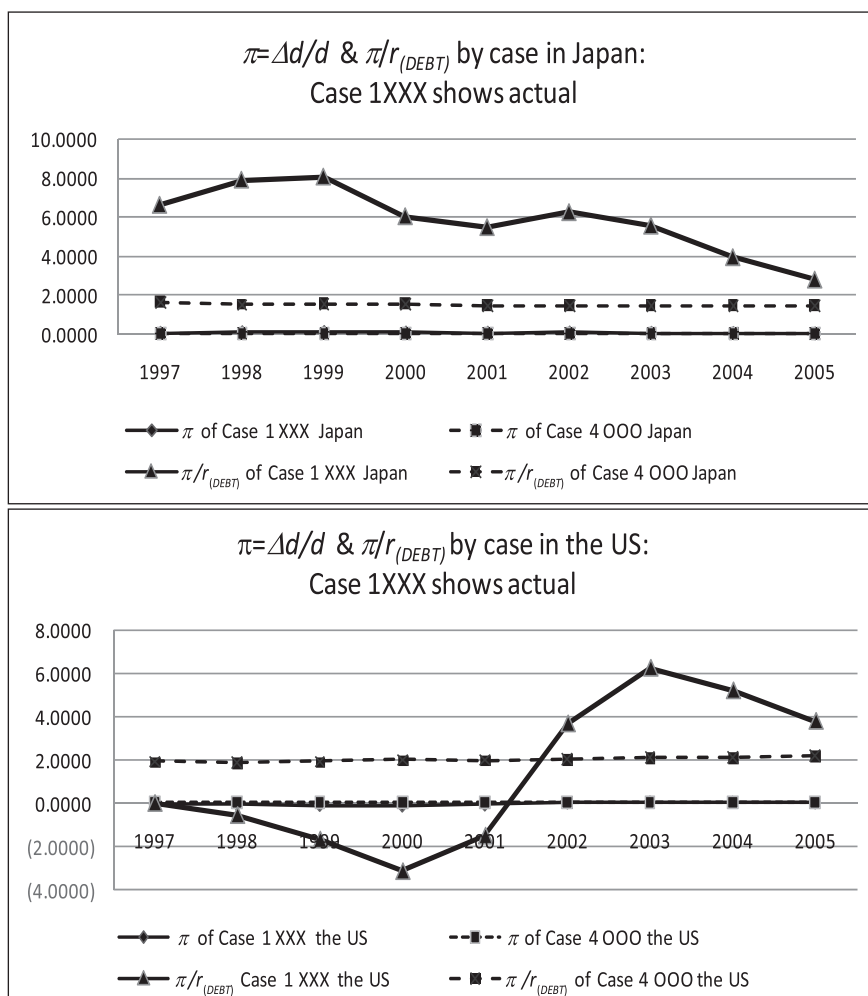
A DEVELOPING COUNTRY 2-1		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(1) Under the EMU rule of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.6$		$i = I/Y = 0.1335$				$n = gL = 0.01203$		$(\rho/r) = 0.90000$	
change rate of net investment, I , as a base		0	0	0	0	0	0	0	0
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0629	0.0602	0.0609	0.0616	0.0623	0.0588	0.0581	0.0574	0.0567
output=income, $Y=C+S$	13196	13215	13295	13399	13527	13104	13025	12922	12795
balance of payment, BOP	727	751	799	871	967	703	655	583	487
net investment, I	1761	1785	1809	1833	1857	1737	1713	1689	1665
deficit, ΔD (For surplus, plus)	(396)	(396)	(399)	(402)	(406)	(393)	(391)	(388)	(384)
debt, D	(7918)	(7918)	(7921)	(7924)	(7928)	(7915)	(7913)	(7909)	(7906)
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24
G saving=returns, $S(G)=\Delta D+I(G)$	(276)	(252)	(231)	(210)	(190)	(297)	(319)	(340)	(360)
BOP/Y	0.0551	0.0568	0.0601	0.0650	0.0714	0.0536	0.0502	0.0451	0.0380
$\pi = \Delta D/D$	0.0500	0.0501	0.0504	0.0507	0.0512	0.0497	0.0494	0.0490	0.0486
$\Delta d = \Delta D/Y$	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)	(0.0300)
$d = D/Y$	(0.6000)	(0.5992)	(0.5957)	(0.5914)	(0.5860)	(0.6040)	(0.6075)	(0.6121)	(0.6179)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)}(-d))$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)}(-d))$	(0.0180)	(0.0196)	(0.0193)	(0.0189)	(0.0186)	(0.0204)	(0.0207)	(0.0211)	(0.0214)
$c = C/Y$	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000
capital-output ratio, $\Omega = K/Y$	1.4000	1.5330	1.5256	1.5156	1.5030	1.5424	1.5499	1.5604	1.5740
1-beta* (see the equation)	0.3398	0.3165	0.3170	0.3175	0.3180	0.3155	0.3149	0.3143	0.3137
$i_{(G)} = I(G)/Y(G)$	0.0593	0.0700	0.0804	0.0905	0.1003	0.0483	0.0368	0.0250	0.0127
$\Omega(G) = K(G)/Y(G)$	1.1034	1.1558	1.1492	1.1432	1.1378	1.1710	1.1787	1.1863	1.1938
1-beta $_{(G)}$ * (see the equation)	0.3589	0.3643	0.3800	0.3915	0.4001	0.3075	0.2493	0.1317	(0.2237)
$g_{Y(G)}^*$	0.0344	0.0385	0.0434	0.0482	0.0529	0.0285	0.0234	0.0183	0.0130
$\alpha = \Pi/Y$	0.0983	0.1021	0.1068	0.1131	0.1209	0.0957	0.0909	0.0842	0.0758
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ (0.1363)	(0.1227)	(0.1105)	(0.0990)	(0.0882)	(0.1494)	(0.1630)	(0.1766)	(0.1903)	
$r = \Pi/K$	0.0702	0.0666	0.0700	0.0746	0.0804	0.0621	0.0586	0.0540	0.0481
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ (0.1235)	(0.1062)	(0.0961)	(0.0866)	(0.0775)	(0.1276)	(0.1383)	(0.1489)	(0.1594)	
$Y(G)/Y$	0.1534	0.1556	0.1572	0.1584	0.1592	0.1518	0.1502	0.1488	0.1478

A DEVELOPING COUNTRY 2-2		Simu 1	Simu 2	Simu 3	Simu 4	Simu -1	Simu -2	Simu -3	Simu -4
(2) For the base of $\Delta d = \Delta D/Y = 0.015$ and $d = D/Y = 0.30$		$i = I/Y = 0.1335$				$n = gL = 0.01203$		$(\rho/r) = 0.90000$	
change rate of net investment, I , as a base		0	0	0	0	0	0	0	0
change rate of G net investment, $I(G)$, as a base		0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0629	0.0602	0.0609	0.0616	0.0623	0.0588	0.0581	0.0574	0.0567
output=income, $Y=C+S$	13196	13215	13294	13398	13525	13104	13025	12921	12794
balance of payment, BOP	727	751	799	871	967	703	655	583	487
net investment, I	1761	1785	1809	1833	1857	1737	1713	1689	1665
deficit, ΔD (For surplus, plus)	(198)	(198)	(199)	(201)	(203)	(197)	(195)	(194)	(192)
debt, D	(3959)	(3959)	(3960)	(3962)	(3964)	(3957)	(3956)	(3955)	(3953)
G net investment, $I(G)$	120	144	168	192	216	96	72	48	24
G saving=returns, $S(G)=\Delta D+I(G)$	(78)	(54)	(31)	(9)	(13)	(101)	(123)	(146)	(168)
BOP/Y	0.0551	0.0568	0.0601	0.0650	0.0715	0.0536	0.0503	0.0451	0.0380
$\pi = \Delta D/D$	0.0500	0.0501	0.0504	0.0507	0.0512	0.0497	0.0494	0.0490	0.0485
$\Delta d = \Delta D/Y$	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)	(0.0150)
$d = D/Y$	(0.3000)	(0.2996)	(0.2979)	(0.2957)	(0.2931)	(0.3020)	(0.3037)	(0.3061)	(0.3090)
$\Delta D/Y - (t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)}(-d))$	0.0000	(0.0000)	0.0000	(0.0000)	0.0000	(0.0000)	(0.0000)	0.0000	(0.0000)
$(t_{AX} - g_{SPEND(G)}) - (r_{(DEBT - g_Y^*)}(-d))$	(0.0090)	(0.0098)	(0.0096)	(0.0095)	(0.0093)	(0.0102)	(0.0104)	(0.0105)	(0.0107)
$c = C/Y$	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000
capital-output ratio, $\Omega = K/Y$	1.4000	1.5331	1.5257	1.5157	1.5032	1.5424	1.5499	1.5605	1.5742
1-beta* (see the equation)	0.3398	0.3165	0.3170	0.3175	0.3180	0.3155	0.3149	0.3143	0.3137
$i_{(G)} = I(G)/Y(G)$	0.0540	0.0639	0.0734	0.0827	0.0917	0.0439	0.0335	0.0227	0.0115
$\Omega(G) = K(G)/Y(G)$	1.0051	1.0542	1.0491	1.0443	1.0398	1.0657	1.0716	1.0777	1.0838
1-beta $_{(G)}$ * (see the equation)	0.3589	0.3643	0.3800	0.3915	0.4001	0.3075	0.2493	0.1317	(0.2237)
$g_{Y(G)}^*$	0.0344	0.0385	0.0434	0.0482	0.0529	0.0285	0.0234	0.0183	0.0130
$\alpha = \Pi/Y$	0.0983	0.1021	0.1068	0.1131	0.1209	0.0957	0.0909	0.0842	0.0758
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$ (0.0351)	(0.0240)	(0.0137)	(0.0039)	0.0056	(0.0460)	(0.0573)	(0.0689)	(0.0806)	
$r = \Pi/K$	0.0702	0.0666	0.0700	0.0746	0.0804	0.0621	0.0586	0.0540	0.0481
$r_{(G)} = \Pi_{(G)}/K_{(G)}$ (0.0349)	(0.0228)	(0.0131)	(0.0037)	0.0054	(0.0432)	(0.0535)	(0.0639)	(0.0744)	
$Y(G)/Y$	0.1684	0.1706	0.1722	0.1734	0.1742	0.1668	0.1652	0.1638	0.1628

How to Simulate Budgeting towards Endogenous Rules of Deficits and Debts?

Table AA8 Simulation 2.3 & 2.4 of the EMU rule to deficits and debts: a developing country

A DEVELOPING COUNTRY		2-3	Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(3) For the base of $\Delta d = \Delta D/Y = 0.0$ and $d = D/Y = 0.0$			$i = I/Y = 0.1335$				$n = gL = 0.01203$		$(\rho/r) = 0.90000$	
change rate of net investment, I, as a base			0	0	0	0	0	0	0	0
change rate of G net investment, I(G), as a base			0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0629	0.0602	0.0609	0.0616	0.0623	0.0588	0.0581	0.0574	0.0567	
output=income, $Y=C+S$	13196	13215	13295	13398	13525	13104	13025	12921	12794	
balance of payment, BOP	727	751	799	871	967	703	655	583	487	
net investment, I	1761	1785	1809	1833	1857	1737	1713	1689	1665	
deficit, ΔD (For surplus, plus)	0	0	0	0	0	0	0	0	0	
debt, D	0	0	0	0	0	0	0	0	0	
G net investment, I(G)	120	144	168	192	216	96	72	48	24	
G saving=returns, $S(G)=\Delta D+I(G)$	120	144	168	192	216	96	72	48	24	
BOP/Y	0.0551	0.0568	0.0601	0.0650	0.0715	0.0536	0.0503	0.0451	0.0380	
$\pi = \Delta D/D$	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	
$\Delta d = \Delta D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$d = D/Y$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$\Delta D/Y - (t_{AX} \cdot g_{SPEND(G)})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$(t_{AX} \cdot g_{SPEND(G)}) - (r_{(DEBT \cdot g_Y^*)} \cdot (-d))$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
$c = C/Y$	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	
capital-output ratio, $\Omega = K/Y$	1.4000	1.5331	1.5257	1.5157	1.5032	1.5424	1.5499	1.5605	1.5742	
1-beta* (see the equation)	0.3398	0.3165	0.3170	0.3175	0.3180	0.3155	0.3149	0.3143	0.3137	
$i_{(G)} = I(G)/Y(G)$	0.0496	0.0587	0.0675	0.0761	0.0844	0.0403	0.0307	0.0208	0.0106	
$\Omega(G) = K(G)/Y(G)$	0.9229	0.9691	0.9650	0.9611	0.9574	0.9777	0.9824	0.9873	0.9924	
1-beta $_{(G)}^*$ (see the equation)	0.3589	0.3643	0.3800	0.3915	0.4001	0.3075	0.2493	0.1317	(0.2237)	
$g_{Y(G)}^*$	0.0344	0.0385	0.0434	0.0482	0.0529	0.0285	0.0234	0.0183	0.0130	
$\alpha = \Pi/Y$	0.0983	0.1021	0.1068	0.1131	0.1209	0.0957	0.0909	0.0842	0.0758	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.0496	0.0587	0.0675	0.0761	0.0844	0.0403	0.0307	0.0208	0.0106	
$r = \Pi/K$	0.0702	0.0666	0.0700	0.0746	0.0804	0.0621	0.0586	0.0540	0.0481	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.0537	0.0606	0.0700	0.0792	0.0882	0.0412	0.0312	0.0210	0.0106	
$Y(G)/Y$	0.1834	0.1856	0.1872	0.1884	0.1892	0.1818	0.1802	0.1788	0.1778	
A DEVELOPING COUNTRY		2-4	Simu 1	Simue 2	Simu 3	Simue 4	Simu -1	Simu -2	Simu -3	Simu -4
(4) For the base of $\Delta d = \Delta D/Y = 0.03$ and $d = D/Y = 0.1$			$i = I/Y = 0.1335$				$n = gL = 0.01203$		$(\rho/r) = 0.90000$	
change rate of net investment, I, as a base			0	0	0	0	0	0	0	0
change rate of G net investment, I(G), as a base			0.2	0.4	0.6	0.8	-0.2	-0.4	-0.6	-0.8
g_Y^* (see the equation)	0.0629	0.0602	0.0609	0.0616	0.0623	0.0588	0.0581	0.0574	0.0567	
output=income, $Y=C+S$	13196	13215	13295	13398	13525	13104	13025	12921	12794	
balance of payment, BOP	727	751	799	871	967	703	655	583	487	
net investment, I	1761	1785	1809	1833	1857	1737	1713	1689	1665	
deficit, ΔD (For surplus, plus)	396	396	399	402	406	393	391	388	384	
debt, D	1320	1320	1323	1326	1329	1317	1314	1311	1308	
G net investment, I(G)	120	144	168	192	216	96	72	48	24	
G saving=returns, $S(G)=\Delta D+I(G)$	516	540	567	594	622	489	463	436	408	
BOP/Y	0.0551	0.0568	0.0601	0.0650	0.0715	0.0536	0.0503	0.0451	0.0380	
$\pi = \Delta D/D$	0.3000	0.3003	0.3016	0.3032	0.3052	0.2985	0.2973	0.2956	0.2935	
$\Delta d = \Delta D/Y$	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	0.0300	
$d = D/Y$	0.1000	0.0999	0.0995	0.0989	0.0983	0.1005	0.1009	0.1015	0.1022	
$\Delta D/Y - (t_{AX} \cdot g_{SPEND(G)})$	0.0000	(0.0000)	0.0000	0.0000	0.0000	(0.0000)	0.0000	0.0000	0.0000	
$(t_{AX} \cdot g_{SPEND(G)}) - (r_{(DEBT \cdot g_Y^*)} \cdot (-d))$	0.0280	0.0283	0.0282	0.0281	0.0281	0.0284	0.0285	0.0285	0.0286	
$c = C/Y$	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	0.9000	
capital-output ratio, $\Omega = K/Y$	1.4000	1.5331	1.5257	1.5157	1.5032	1.5424	1.5499	1.5605	1.5742	
1-beta* (see the equation)	0.3398	0.3165	0.3170	0.3175	0.3180	0.3155	0.3149	0.3143	0.3137	
$i_{(G)} = I(G)/Y(G)$	0.0426	0.0505	0.0582	0.0656	0.0729	0.0346	0.0263	0.0178	0.0090	
$\Omega(G) = K(G)/Y(G)$	0.7931	0.8342	0.8317	0.8291	0.8263	0.8393	0.8422	0.8455	0.8491	
1-beta $_{(G)}^*$ (see the equation)	0.3589	0.3643	0.3800	0.3915	0.4001	0.3075	0.2493	0.1317	(0.2237)	
$g_{Y(G)}^*$	0.0344	0.0385	0.0434	0.0482	0.0529	0.0285	0.0234	0.0183	0.0130	
$\alpha = \Pi/Y$	0.0983	0.1021	0.1068	0.1131	0.1209	0.0957	0.0909	0.0842	0.0758	
$\alpha_{(G)} = \Pi_{(G)}/Y_{(G)}$	0.1832	0.1896	0.1963	0.2030	0.2098	0.1762	0.1690	0.1614	0.1534	
$r = \Pi/K$	0.0702	0.0666	0.0700	0.0746	0.0804	0.0621	0.0586	0.0540	0.0481	
$r_{(G)} = \Pi_{(G)}/K_{(G)}$	0.2310	0.2273	0.2360	0.2449	0.2538	0.2100	0.2007	0.1910	0.1807	
$Y(G)/Y$	0.2134	0.2156	0.2172	0.2184	0.2192	0.2118	0.2102	0.2088	0.2078	



Note: Case 1XXX shows the actual situation by year, where $g_y^* \neq \Delta d/d \neq r_{(DEBT)}$.

Case 4000 shows the actual situation by year, where $g_y^* = \Delta d/d = r_{(DEBT)}$.

This Figure shows an overall image to Tables 1 to 4, soon below, that deepen the implications of Du Grauwe's (2005) equations.

Figure AA1 $\pi = \Delta d/d$ as a sort of the internal rate of return: Japan and the US 1997–2005