

# Endogenous Size of Government and the Growth Rates in the Endogenous-Equilibrium by Country: Using 65 Country Data-Sets, 1990–2009

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## Abstract

The paper theoretically and empirically deepens the mechanics and background of the endogenous model and system and focuses how definitely endogenous taxes to output as the size of government determines the rate of technological progress and the related growth rates. The base is the structure of balance of payments at the real-assets. For evidences, actual data in IMF statistics and endogenous data in equilibrium are compared by aspect. The real-assets facts are against common sense of policy-makers, yet are proved and justified using 65 country data by sector, 1990–2009. If the government size is controllable by policy-makers, moderate growth rates are within hands under dynamic balances at the real assets. (JEL: E62; E01).

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References

## 1. Introduction

The purpose of this paper is to focus the endogenous size of government measured by the ratio of endogenous taxes to output in the author's endogenous model and system and to clarify its influences on the endogenous rate of technological progress and related growth rates in the endogenous-equilibrium. The endogenous-equilibrium is measured by the speed years for convergence by country, year, and sector (government and private and total economy as the weighted average). The endogenous model and system realize a simultaneous unity of theory (model) and practice (system) by year. The endogenous system is unique in that all the data are consistently derived from 25 real and financial data in *International Financial Statistics yearbook*, IMF. To understand the background of the size of government, the author, first of all, theoretically and empirically needs to clarify the mechanics of the endogenous model and system (data-sets). The endogenous model and system are most precisely compared with the neoclassical growth model yet, it is not enough. Each model uses

different data, endogenous versus actual and others available. Without clarifying the differences between endogenous data and actual data, differences between the two models are not fully clarified.

The author, at the beginning, clarifies essentials of ‘endogenous’ and accordingly, deepens background of the endogenous model and system. The author, in the text, fully clarifies definite differences between the endogenous model and system and the neoclassical and Keynesian models, at each pertinent context and, with citations to show more clearly.

The endogenous model and system are based on a ‘discrete’ Cobb-Douglas production function, where the author reveals ‘seven’ endogenous parameters that absorb all policies and policy-changes at the real assets during one year. Lucas’ critique (1976) is thus avoided. All strategies and tactics using non-rival human capital, education, and R & D, and learning by doing, are wholly taken in ‘seven’ endogenous parameters.<sup>1)</sup>

The endogenous size of government strictly constitutes a core of the endogenous model and system. The government size influences on the level of endogenous equilibrium, starting with the endogenous structure of the balance of payments. The government size first determines the endogenous level of net investment after capital consumption by sector. The dynamic balance of net investment between government and private sectors becomes the most important target of policies at the real assets. A net investment by sector is primarily indispensable to the growth rates of the total economy in equilibrium, because the qualitative net investment coefficient,  $\beta^*$ , as another determinant, varies

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1) ‘Seven’ endogenous parameters are: the relative share of capital  $\alpha$ ; the growth rate of population  $n$ ; the ratio of net investment to output  $i = I/Y$ ; the qualitative net investment coefficient  $\beta^*$ ; the diminishing returns to capital (DRC) coefficient  $\delta_0$ ; the capital-output ratio  $\Omega$ ; and, the ratio of government net investment to government output  $i_G = I_G/Y_G$  or  $i_G = I_G/Y$  (for condensed explanations, see Eq. 11 to Eq. 16 at section 3.3.2).

slowly, compared with net investment, by sector and by year. Directly, the endogenous-equilibrium measured by the speed years determines sustainable growth and returns, led by the size of government.

Basic character-differences between the endogenous model and system and the neoclassical and Keynesian models are: 1) the purely endogenous rate of technology (simultaneously measuring capital and its rate of return) versus the external rate of technological progress; 2) precise measure of the relative share of capital<sup>2)</sup> *versus* 'vaguely or using actual capital estimate of the relative share of capital; 3) no assumption (e.g., perfect competition is measured by marginal productivities of capital and labor) *versus* nine assumptions as in Meade James (1962). though some assumptions may be overlapped; 4) policy-oriented theory and endogenous data-sets *versus* record-oriented theory and actual data-sets; 5) the real assets *versus* the financial assets; 6) the market neutral to the real assets *versus* the market principle; 7) endogenous data by sector *versus* actual data, neglecting government performances; 8) no panel data required for analyses *versus* panel data necessitated for actual analyses; 9) causes and results specified at the real assets *versus* market intuitive results with unknown causes; 10) precisely measured (no estimated) *versus* forecasted and estimated; 11) no ad-hoc, no probability, and no expected *versus* repeating, probability, and expected. The endogenous model and system start with Solow's exogenous model and has the same root as the neoclassical growth model. It is 'not against' but strengthens the characters of the neoclassical growth model, integrating the whole system as a one unity of theory and practice in the discrete case, but producing different results by year similarly to the results of actual data. Keynesian model naturally and directly (from actual data) looked for

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2) The relative share of capital is constant by year in recursive programming at the endogenous model, while it varies at KEWT data-sets by year. Both are measured by sector, consistently over years.

stylized facts, as shown by Kaldor (1961, 1978).

The above differences are ultimately expressed by a fact that capital stock and its rate of return are simultaneously measured, changing the individual micro utility to a macro utility as national taste/preferences and culture, where all the parameters and variables are simultaneously measured, by country, year, sector, and over years. The current economic models use actual statistics and other data while the endogenous model and system measure endogenous data and improve the results through real-assets policies by year. Results of both (actual and endogenous) sides naturally differ. Each actual data, however, are within a certain range of the corresponding endogenous data when endogenous data stay within a moderate range of the endogenous-equilibrium. There is much room for both sides to cooperate each other.

Section 2 shows brief review of the literature without using equations. Section 3 is divided into two: The first half clarifies key mechanics essential to the empirical analysis of this paper, with Eq. 1 to Eq. 10. The latter half covers fundamental background of the key mechanics, using Eq. 11 to Eq. 16. The processes to formulate each equation are abbreviated for the sake of simplicity (for ‘the first appearances,’ see References). In section 4, as the centre of this paper, the author tests and proves the relationship between the government size and key growth rates, with the level of the speed years in the endogenous-equilibrium, comparing 65 countries and with resultant implications. Keywords here are actual and endogenous data analyses and the speed years by country.

## **2. Brief review of the literature, compared with the endogenous model and system**

The author first affirms that the Cobb-Douglas production function is

essential to growth models, since constant returns to scale<sup>3)</sup> as its attribute have a growth model simply formulated. The origin is shown by Solow, R. M. (1956), where the rate of technological progress is externally given. I interpret Solow's model such that the continuous Cobb-Douglas production function is essentially exogenous in terms of technological progress, even if human capital and others are consistently inserted into. The continuous Cobb-Douglas production function uses the capital-labor ratio as a base instead of the capital-output ratio. In Solow's exogenous model, the capital coefficient or the capital-output ratio,  $K/Y$ , becomes constant at the steady state (after correction, 1969, 94; before, 1956, 86). The endogenous model changes his external rate of technological progress to the endogenous rate of technological progress so that the author here does discuss his external technology. Apart from his external technology, the author finds a problem inherent in the continuous Cobb-Douglas production function.<sup>4)</sup> This comes back to the relationship between model and data. The author defines 'the Cobb-Douglas type' as the continuous Cobb-Douglas production function that independently and selectively uses actual data and also external data, under a given rate of technological progress. Then, the Cobb-Douglas type hardly estimates the relative share of capital or labor. The reason returns to a fact that a system for national accounts (the SNA, 1993; United Nations) neglects returns at the government sector since wages finally belong to households. The Cobb-Douglas type commonly has a contradiction in

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3) The author finds a few endogenous technological progress articles in the literature, one of which uses a CES function. This function uses elasticity so that it cannot be thoroughly endogenous.

4) A defect of Solow's model is represented by a vague estimation of the relative share, commonly to neoclassical growth models. When  $Y$  = output = income = expenditure is measured using national accounts data, the defect is mitigated. Solow, therefore, had anxiously discusses this problem repeatedly, as the author touches citing below.

that the steady state holds under an assumption of perfect competition, with a vague relative share of capital under the SNA. The existence of assumptions is a source of producing contradictions.

In detail; the author advocates that the capital-output ratio is most difficult to treat since it has its range and cannot disperse or converge in the transitional path, differently from per capita labor or the capital-labor ratio that increases in parallel with per capita output. In the case of Cobb-Douglas type, the capital-labor ratio and the capital-output ratio each converge to the steady state very slowly in the transitional path. This fact contradicts the textbook illustration of  $k_1 < k^* > k_2$  on the x axis. It is not possible for  $x^y$  to express it by a known function so that  $x^y$  is approximately solved by using Taylor's expansion each by each; first, second, and then, third degree and so on. This is a serious problem of the Cobb-Douglas type. The endogenous model challenges for this problem and measures endogenous equations. The endogenous equations measures the capital-labor ratio, the capital-output ratio, the relative share of capital, and technological progress, stock and flow, consistently with the speed years at convergence by country, as shown at section 3.3.2.

The endogenous model and system, however, never deny respectable accumulations of growth models up to date. The author finds most precise summary of assumptions common to Keynesian and neoclassical models in Meade, J. E. (1962, 1–7; for nine assumptions with these implications), as each by each explained at section 3.3.2. Is it possible for any model to delete nine assumptions? Nine assumptions are all disappear only in the case of the endogenous model and system, so far as the author has investigated after 21 Century. In fact, when endogenous is perfectly consistent among all the parameters and variables in a model, all the assumptions in the model are formulated by equations and their empirical evidences.

Endogenous growth models have appeared in the literature after the mid

1980s. The textbook commonly raises Romer, P. M. (1986; for increasing returns..) and, Lucas, R. E. Jr. (1988; for On the mechanics..). Barro, R. J., and Sala-i-Martin, X. (1995, 182–197) connected Lucas (1988; the above) with Uzawa, H. (1965; Optimal technical change..). Then, Romer, P. M. (1990; Endogenous technological progress..) uses both physical and human capital in the Cobb-Douglas type, to approach endogenous steady state. According to the author's viewpoint, human capital, R & D, education, knowledge, and learning by doing, are all non-rival and numerically difficult to distribute to various effective years. If a non-rival model could delete assumptions, then, it might contribute to a strict endogenous model. The author has failed in adopting these non-rival elements to the endogenous model and system. As a result, the discrete Cobb-Douglas production function exists; reveals seven endogenous parameters, absorbs the changes in policies by fiscal year and, satisfy the critique of Lucas, R. E. Jr. (1976, 19–46). In the endogenous model and system, the above non-rival items belong to 'strategies and tactics' wholly expressed through seven endogenous parameters in the discrete Cobb-Douglas production function.

The difference between Keynesian models and the endogenous model and system is whether the Cobb-Douglas type is used or not. Keynesians use a variety of equations, whose engine was Kaldor's (1961, 177–222) stylized facts. Jones, C. I. and Romer P. M. (2010, 224–245) re-examines the stylized facts, which correspond with values at a steady state or balanced growth. Is it possible for Keynesian models to obtain an endogenous model, without using the Cobb-Douglas type? Common sense may say no answer, since equations mostly remain partial and are not integrated consistently as a whole system.

The author, nevertheless, pays attention to Thirlwall, A. P. (2002, 427–435). His model,  $Y = AK$ , corresponds with  $y = A \cdot k$  as one of equations in the endogenous model and system, as shown at section 3.3.2. Keynesian models



find some endogenous results in the endogenous model and system. Thirlwall, A. P. (2002, 40–51) clarifies the nature of economic growth and stresses the importance of manufacturing industry as the engine of growth. The author has no objection to his point of view, yet still protests his treatment that the government sector has no return, similar to neoclassical models. It is the current SNA that wages are attributed to households and, returns are private sector. The current SNA aims at final distribution but, causes serious problems. Both Keynesians and Neo-classists shoulder a heavy burden from actual data. Typically, the marginal productivities of capital and labor,  $MPK = r$  and  $MPL = w$ , are assumed in the price-equilibrium and accordingly, perfect competition must be assumed. These results are common to both schools.

Lastly, the author reviews two articles, from the viewpoint of endogenous growth theory and the use of data-sets: Craft, N. (1996) and Oulton, N. (1997). The author discussed current articles for various intentions (see Reference). First, Craft, N. (1996, 30–47) refers to representative literature, uses several country actual statistics data, 1950–92, and investigates several countries' elements at 'accounting for growth' (i.e.,  $L/Y$ ,  $\dot{K}/K$ , growth due to capital, total factor productivity ( $TFP$ ) growth, and  $\dot{Y}/Y$ ) by 1950–73, 1973–92, 1966–90, and also, cites 17 country industry-financed R & D percentage of  $GDP$ , 1967, 1991. His question is: what are the policy implications of post-neoclassical endogenous growth theory? His conclusions are: 1) physical investment is not an engine to steady growth or a target should be broader capital; 2) R & D does not produce dramatic effects; 3) a high priority to identifying and remedying markets does not promote skills acquisition and foreign direct investment designed to reduce the ideas gap; and 4) enhancing returns decreases broader investment and physical investment. These findings are related to the investment at the real assets. In conclusion, Craft, N. (ibid., 44) states:

First, it should be recognized that raising growth rate is difficult and that a return to the growth rates of the Golden Age is highly unlikely... Second, raising the rate of broad capital formation without changing TFP growth will have only a levels effect and not a growth-rate effect on income... Third, in the long run growth comes from improvements in productivity resulting from better technology which involves the use and effective assimilation of new ideas.

Craft uses actual data yet, his policy implications and conclusions are consistent with the results of the endogenous system to some extent. This is because actual data are each within a certain range of the corresponding endogenous data. Craft interprets the results using both rival and non-rival and also policies and strategies. The cause-result relationship becomes vast and vague, compared with that of the endogenous system. The endogenous system, contrarily, distinguishes the rate of technological progress into two, flow and stock: the rate of technological progress,  $g_A^*$ , as flow and the TFP growth rate ( $g_{TFP}^*$ ) as stock. When actual statistics data are used,  $g_A^* \neq g_{TFP}^*$  occurs at the author's close-to-disequilibrium. When the data-sets of KEWT are used,  $g_A^* = g_{TFP}^*$  holds in a moderate equilibrium. This is a reason why Craft is prudent in stating facts.

Second, for the use of data-sets; Oulton, N. (1997, 99–121) discusses total factor productivity growth, *TFP* growth, and the role of externalities, referring to representative articles and using Penn World Table (PWT) version 5.6a, for 53 countries of OECD, Latin America, East Asia, Africa, and other area, 1965–1990. *TFP* growth and its components are output, output per worker, labor, physical capital per worker, human capital per worker, and TFP, where ‘human capital’ from Barro-Lee educational dataset (1996 version) is added. The author reviews Oulton’s article here from the viewpoint of a serious limit of actual or estimated data-sets by country. PWT 5.6a is the last data-sets in that the capital-labor ratio published instead of capital stock had some contradictions for estimation and PWT stopped its publication, according to the author’s

communications with PWT.<sup>5)</sup> It implies that data-sets prevailing today cannot publish capital stock except for each country's national accounts data such as those of Cabinet Office, Japan, and the Bureau of Economic Analysis, the US.<sup>6)</sup> Also it implies that macroeconomics and national accounts have to analyze the current situations without consistent capital stock and its rate of return or that econometrics improves tremendously up to date yet is incomplete from the viewpoint of the whole version of grasping economies.

Oulton, N. (*ibid.*, 99, 106) concludes that the role of externalities is relatively small under competing technologies and that even if externalities are relatively small the government or the European Commission should still seek to correct them. The author interprets the externalities more broadly as follows: The externalities often lead to an unbalanced equilibrium between the government and private sector. Policy-makers must aim at a balanced situation by mitigating the difference of technological progress between the government and private sector. The literature does not distinguish government with private technological progress, while the endogenous system measures the rate of

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5) The author visited PWT, University of Penn, on Dec 11, 2005 to confirm this fact. The author is grateful to Dr. Alan Heston and staff and admires their brave decision-making. When data are estimated using actual data and related econometrics, the consistency does not work over years. A unique solution was found at the endogenous system, where the consistency by data, year, country, sector, and over years, are maintained without later corrections. See, Summers, R. and Heston, A. (1988, 1–26) and, De Long, B. J. and Summers, L. H. (1991, 445–502).

6) Thirteen counties today publish actual capital stock at national accounts but not consecutively. A contradiction appears between capital stock and its rate of return by country. When the market data in the long run is used for capital estimation, the contradiction is well mitigated due to the author's neutrality of the financial assets to the real assets. For OECD capital estimation using market data, see Schreyer, P. (2004, 163–184). The author investigated the relationships between endogenous capital stock with actual capital stock by country (for twice, see References, using Japan and the US, 1960–2005 and 1960–2010).

technological progress by sector. The endogenous system declares that all the data are endogenous and without externalities. It is essential for policy-makers to control the difference of technological progress between government and private, although in the short run technological unbalance between government and private may be needed.

In short, for policy-oriented, another system in parallel to the SNA is required. Today, there is no policy-oriented system except for the endogenous system. The author advocates that the existence of actual and endogenous data is most welcome since by cooperation both systems work much and vividly contribute to policy-making by leaders and people.

### **3. Methods to set up connectors between endogenous and actual data**

#### ***3.1 Two connectors to compare endogenous data with actual data***

As a preparatory work for the purpose of this paper, the author first explains two relationships lying between two sets of three growth equations and then, two sets of connectors between endogenous and actual data. First, the endogenous model and system have two equation sets of relationships common to the neoclassical model that uses the Cobb-Douglas production function, apart from endogenous or external technology. These two equation relationships hold, regardless of whether the production function is discrete or continuous and also, regardless of whether the production function holds in the endogenous-equilibrium or in the price-equilibrium preserved in the literature.

Let the author explain the above two sets of three growth equations. Three equations are: the rate of technological progress,<sup>7)</sup>  $g_A^*$ , the growth rate of per capita output,  $g_y^*$ , and the growth rate of output,  $g_Y^*$ , each in endogenous

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7) The rate of technological progress as flow is inevitably distinguished with that as stock:  $g_A^* = g_{A(flow)}^*$  versus  $g_{TFP}^* = g_{A(stock)}^*$ . The Cobb-Douglas type cannot distinguish.

equilibrium. The three relationships are shown by:

$$g_y^* = g_A^* / (1 - \alpha) \text{ and } g_Y^* = g_y^* + n(1 + g_y^*).$$

$$\text{Or, generally, } g_y = g_A / (1 - \alpha) \text{ and } g_Y = g_y + n(1 + g_y). \quad (1)$$

Set the difference between two items, A and B. The difference is shown by A–B. Then, the growth rate of per capita output less the rate of technological progress is simply shown by:

$$g_y - g_A = \alpha \cdot g_A, \quad (2)$$

since  $g_A - g_A(1 - \alpha) = g_A / (1 - \alpha) - g_A$  holds. It implies that either  $g_y$  or  $g_A$  is only needed, with the relative share of capital,  $\alpha$ , when  $g_y$  is compared with  $g_A$ . Also, when the relative share of capital,  $\alpha$ , is constant, the use of Eq. 1 is convenient.

The growth rate of output less the growth rate of per capita output is similarly shown by:

$$g_Y - g_y = n(1 + g_y). \quad (3)$$

When a low value of  $n \cdot g_y$  is neglected,  $g_Y - g_y = n$  roughly holds. It implies that either  $g_Y$  or  $g_y$  is needed, as known in the literature.

The above Eq. 2 works when actual and endogenous data are compared, by neglecting the difference between the endogenous relative share of capital and the actual relative share of capital. The author advocates that the actual relative share of capital has not been precisely estimated in statistics, since the rate of technological progress remains externally given. This is a fact and that in the literature capital and the rate of return have not been simultaneously estimated over years.

More explanation, the endogenous relative share of capital is defined as  $\alpha = \Pi/Y$ , where endogenous returns,  $\Pi$ , is measured. The actual relative share of capital is estimated by  $\alpha_{GDP} = OS/GDP$ , where operating surplus,  $OS$ , is used, assuming that government returns are zero and under the final distribution of

income. Contrarily, national disposable income,  $Y$ , in the endogenous model and system accurately measures the equal relationship between income, expenditures, and output, as designed by Meade, James. E. (1962) and also by Meade, J. E., and Stone, R. (1969). According to the endogenous system (i.e., KEWT 5.11 by sector, 1990–2009),  $Y/GDP$  shows 0.9 to 0.85 at most countries among 65 countries, as shown below.

Next, two connectors lying between endogenous and actual data are  $g_Y / g_{GDP}$  and  $Y_G/Y$ . The two connectors are not only useful to connect endogenous and actual data but also essential to settle the endogenous model and system towards moderate range of endogenous equilibrium, using the speed years.

Actual and endogenous data are primarily related to the ratio of  $g_Y$  to  $g_{GDP}$ :  
 $g_Y / g_{GDP}$ . (4)

In the case of growth rates, there exist three classifications; ‘actual > endogenous,’ ‘actual = endogenous,’ and ‘actual < endogenous.’ A condition of ‘actual = endogenous’ is ideal, where the actual growth rate realizes the theoretical growth in the endogenous-equilibrium. **Table 1** shows six cases of

**Table 1 Relationship between actual GDP, endogenous Y, and each growth rate**

Y/GDP	<b>Case 1</b>	<b>0.8</b>				<b>Case 2</b>	<b>0.85</b>			
GDP (actual)	300	315	330	345	360	300	315	330	345	360
$g_{GDP}$		0.0500	0.1000	0.1500	0.2000		0.0500	0.1000	0.1500	0.2000
Y (endog)	240	252	264	276	288	255	267.75	280.5	293.25	306
$g_Y$		0.0500	0.1000	0.1500	0.2000		0.0500	0.1000	0.1500	0.2000
$g_{GDP}-g_Y$		0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000
Y/GDP	<b>0.800</b>	<b>0.800</b>	<b>0.800</b>	<b>0.800</b>	<b>0.800</b>		<b>0.850</b>	<b>0.850</b>	<b>0.850</b>	<b>0.850</b>
$g_Y/g_{GDP}$		1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000
Y/GDP	<b>Case 3</b>	<b>0.9</b>				<b>Case 4</b>	<b>0.95</b>			
GDP (actual)	600	608	615	630	650	600	305	310	315	325
$g_{GDP}$		0.0133	0.0250	0.0500	0.0833		(0.4917)	(0.4833)	(0.4750)	(0.4583)
Y (endog)	540	547.2	553.5	567	585	570	289.75	294.5	299.25	308.75
$g_Y$		0.0133	0.0250	0.0500	0.0833		(0.4917)	(0.4833)	(0.4750)	(0.4583)
$g_{GDP}-g_Y$		(0.0000)	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000	0.0000
Y/GDP	<b>0.900</b>	<b>0.900</b>	<b>0.900</b>	<b>0.900</b>	<b>0.900</b>	<b>0.950</b>	<b>0.950</b>	<b>0.950</b>	<b>0.950</b>	<b>0.950</b>
$g_Y/g_{GDP}$		1.000	1.000	1.000	1.000		1.000	1.000	1.000	1.000
Y/GDP	<b>Case 5</b>	<b>0.8</b>				<b>Case 6</b>	<b>0.85</b>			
GDP (actual)	300	315	330	345	360	300	315	330	345	360
$g_{GDP}$		0.0500	0.1000	0.1500	0.2000		0.0500	0.1000	0.1500	0.2000
Y (endog)	240	246	252	258	264	255	280.5	306	331.5	357
$g_Y$		0.0250	0.0500	0.0750	0.1000		0.1000	0.2000	0.3000	0.4000
$g_{GDP}-g_Y$		0.0250	0.0500	0.0750	0.1000		(0.0500)	(0.1000)	(0.1500)	(0.2000)
Y/GDP	<b>0.800</b>	<b>0.781</b>	<b>0.764</b>	<b>0.748</b>	<b>0.733</b>	<b>0.850</b>	<b>0.890</b>	<b>0.927</b>	<b>0.961</b>	<b>0.992</b>
$g_Y/g_{GDP}$		0.500	0.500	0.500	0.500		2.000	2.000	2.000	2.000
Note:										
1. Robust in equilibrium: Actual=endogeneous, the value of $g_Y/g_{GDP}$ decreases as shown in Case 5.										
2. Neutral in equilibrium: Actual=endogeneous, the value of $g_Y/g_{GDP}$ remains a constant value of 1.0, as shown in Cases 1, 2, 3, and 4.										
3. Weak in equilibrium: Actual=endogeneous, the value of $g_Y/g_{GDP}$ increases as shown in Case 6.										

relative growth rates,  $g_Y / g_{GDP}$ , by level of  $Y / GDP$ ; 0.8, 0.85, 0.9, and 0.95.

According to the above experimental relationship between  $GDP$ ,  $Y$ ,  $g_Y$  and  $g_{GDP}$ , the higher the ‘actual > endogenous,’ the lower the  $g_Y / g_{GDP}$  is. Conversely, the higher the ‘actual < endogenous,’ the higher the  $g_Y / g_{GDP}$  is. These imply that when actual data are robust in terms of equilibrium, the value of  $g_Y / g_{GDP}$  could be lower; e.g., from 0.9 to 0.84 and, that when actual data are weak in terms of equilibrium, the value of  $g_Y / g_{GDP}$  have to be higher; e.g., from 0.87 to 0.91. In other words, the level of  $g_Y / g_{GDP}$  determines the growth rates in equilibrium as an indicator to judge the quality of the endogenous-equilibrium.

Turning to the other connector, actual and endogenous data are directly related to the ratio of endogenous taxes to output in equilibrium;  $T_{AX}$  to  $Y$ , where  $Y_G = T_{AX}$ :

The size of government,  $Y_G / Y = T_{AX} / Y$ . (5)

The logic between endogenous taxes and government output is proved using the structure of the balance of payments and deficit in the real assets as follows:  $Y_G = C_G + S_G = W_G + \Pi_G$  holds in the endogenous model and system. As a result,  $S_G - I_G = T_{AX} - (C_G + I_G)$  is derived. Accordingly,  $T_{AX} = C_G + S_G$  holds so that  $T_{AX} = Y_G$  is proved as an accounting identity.

Eq. 5 determines the level of endogenous equilibrium and accordingly, the level of moderate balances between the government and private sector, each by country. This is because each ratio of net investment to output by sector,  $i = I / Y$ ,  $i_G = I_G / Y$ , and  $i_{PRI} = I_{PRI} / Y$ , are definitely determined in the endogenous-equilibrium, once the balance of payments and deficit are given. Each ratio of net investment is most tightly related to the rate of technological progress in equilibrium, where,

$$i = i_G + i_{PRI}. \quad (6)$$

The above base comes from,

$$(S - I) = (S_G - I_G) + (S_{PRI} - I_{PRI}), \quad (7)$$

where the balance of payments is  $S - I$  and  $S_G - I_G$  is deficit. And, the most important is the difference between private saving and net investment,  $S_{PRI} - I_{PRI}$ . Furthermore, the endogenous model and system, differently from the literature, typically proposes ‘net’ investment after capital consumption or economic depreciation.

$$K_{NET} = K_{GROSS} - D_{EPRE(stock)}, \quad (8)$$

$$\text{and } I_{NET} = I_{GROSS} - I_{DEPRE(flow)}, \quad (9)$$

where for national accounts, e.g., the Cabinet Office, Japan, and the Bureau of Economic Analysis, the US, each publish gross investment, depreciation, and net investment actual data. Then, it is possible to compare actual data with corresponding endogenous data. This work is important in that if actual net investment is higher than the endogenous net investment, the economy is balanced, and vice versa. *IFSY*, IMF, does not publish actual gross and net investment for some countries, particularly by sector. In these cases, the economic depreciation rate is set, e.g., 0.5 for developed countries and 0.6 or 0.7 for developing countries. The author needs a comment on Eq. 8, for actual capital stock. Actual capital stock is estimated differently from the measurement of endogenous capital stock, using the perpetual inventory method. Actual and endogenous capital differs significantly in some countries. Policy-makers must pay attention to the use of Eq. 8; actual versus endogenous.

Eq. 9 is definitely useful to policy-makers in that the following Eq. 10 influences the level of technological progress as a base. The endogenous rate of technological progress exists only when net investment is plus by sector.

$$i > 0, i_G > 0, \text{ and } i_{PRI} > 0. \quad (10)$$

Eq. 10 is interrelated to other equations but, expresses the primary tool for this paper’s intention. For example, extremely low-growth countries each have a



minus net investment. This fact is related to Eq. 5 and Eq. 7. The objects, targets, and policy proposals at section 3 are all related to this fact.

Let the author compare the above endogenous mechanics with that in the literature. Du Grauwe, Paul (6<sup>th</sup> Ed., 2005, 147) summarizes the relationship between government debt and budget deficit, referring to Buiter et al (1993) and Wickens (1993). Also, Du Grauwe (ibid., 225) explains the same relationship more plainly and wholly. Du Grauwe's equation is  $G - T + rB = dB/dt + dM/dt$  (Eq. B19.1), where government spending (including interest payments on the government debt),  $G = C_G + I_G$ , government debt,  $B$ , tax revenue,  $T = G + B$ , and the level of high-powered money or monetary base,  $M$ . Using the nominal interest rate,  $r$ , and,  $Y = GDP$ , the nominal growth rate of  $Y$ ,  $x$ , the ratio of  $G$  to  $Y$ ,  $G/Y$ , and  $b = B/Y$ ,  $\dot{b} = (g - t) + (r - x)$  (Eq. B19.5) and accordingly,  $(r - x)b = t - g$  at  $\dot{b} = 0$  are derived. Du Grauwe's equations connect  $M$  (as the financial assets) with the real assets, under plausible neutrality of the financial assets to the real assets. The author pays attention to his  $G/Y$  as an actual size of government, which corresponds with  $T_{AX}/Y$  at the above endogenous Eq. 5. It is natural but, the author indicates, Du Grauwe uses the interest rate. The difference between actual taxes and government spending is externally determined by 'the interest rate less the growth rate of output' multiplied by debt at  $\dot{b} = 0$ .

Contrarily, the endogenous model and system hold using endogenous data. It is more natural. It is preferable to compare endogenous data with related actual data such as actual taxes, net investment by sector and actual returns when actual wages are available by sector. A serious problem even today is that the actual rate of return at the government sector is not calculated, partly due to cash flow-in and flow-out treatment for budgetary deficit. If deficit is shown by  $S_G - I_G$ , connected with the balance of payments, then, actual data, even if these

data are limited as shown in *IFSY*, IMF, the whole picture will be revealed so that policy-makers could take a robust direction. Section 4 will present clear results to policy-makers.

Eq. 5 essentially clarifies empirical results by country (among 65 countries) and by area (Asia & Pacific, Euro, Non-Euro Europe, and Latin America, Near East, & Africa). In KEWT data-sets by country and sector,  $Y_G / Y$  is tentatively and repeatedly inserted at the first step. If  $Y_G / Y$  is a little bit higher than its moderate level,  $i_{PRI} = I_{PRI} / Y$  at the private (PRI) sector turns to an unbalanced level and, the equilibrium at the private sector does not holds. Since  $i = i_G + i_{PRI}$  holds, the balance between  $i_G$  and  $i_{PRI}$  is most important for attaining a moderate growth rate by sector in equilibrium. Equilibrium is preferred to a high growth of the government sector. Infrastructure and private net investment must be balanced as much as possible by year.

All the parameters and variables are interrelated with each other in the endogenous model and system, differently from the Du Gauwe's model. Therefore, this paper needs to clarify these interrelationships, focusing on endogenous taxes and growth in equilibrium. For example, from the viewpoint of real-assets policies, growth and tax-reduction never coexist, without increasing private net investment and decreasing government expenses other than net investment. Deflation is only a result of unbalanced Eq. 7,  $(S - I) = (S_G - I_G) + (S_{PRI} - I_{PRI})$ , at the sacrifice of net investment at the private sector.

Finally, for the relationship between the balance of payments and growth rate, the author compares Thirlwall and Hussain (1982) with the endogenous model and system. Thirlwall, A. P. and Hussain, N. M. (1982, 498–509), similarly to other Keynesian models, does not use the Cobb-Douglas production function. Instead, as a limited model, the article uses income elasticity of demand for imports and accordingly Harrod trade multiplier, with growth rates of income, exports, imports and relative price movements on income growth, where deficit

and investment by sector are not taken into account. Contrarily, the endogenous model and system never necessitate elasticity estimates since endogenous equations hold wholly. Note that the use of elasticity is not denied for the comparison analysis between actual and endogenous data.

### 3.2 *Two connectors between endogenous and actual data*

The differences between endogenous and actual data are shown by two connectors of (1) macro utility versus individual utility and (2) the speed years for convergence at the transitional path versus the *beta* symbolized in the literature although this *beta* is estimated only by using panel actual data.

#### 3.2.1 From individual utility to macro utility as preferences

The literature has used individual utility functions as a theoretical base. A problem lies in practice. It is difficult to directly estimate individual utility when it is applied to national accounts data. For this difficulty, the endogenous model and system set macro-based utility as the relative discounting rate of consumer goods to capital goods at the total economy,  $(\rho/r) = (\rho/r)_{TOTAL}$ ,  $(\rho/r)_G$  at the government sector, and  $(\rho/r)_{PRI}$  at the private sector. The author calls each of three ‘national taste’ and specifies preferences and culture by sector. National taste is estimated, empirically and universally, for international comparisons. National taste is deeply related to data settlement of KWET data-sets by country since the propensity to consume,  $c \equiv C/Y$ , is involved in it (for simplicity, hereunder, showing the case of the total economy). Set national taste is a function of national taste to the propensity to consume;  $(\rho/r)(c)$ . This function is shown by quadratic expression,  $(\rho/r) = D \cdot c^2 + E \cdot c + F$ , where each coefficient  $D$ ,  $E$ , and  $F$  of a country for many years must be adjusted thoroughly until all the plots of  $(\rho/r)$  by year could match a smooth curve of  $(\rho/r)(c)$ .

When the correlation coefficient  $R^2$  between national taste and the propensity to consume over years becomes closer to 0.8 to 0.85 at a country; take another country, apply the same quadratic equation, compare  $R^2$ , and adjust both coefficients until a common value is realized. Increase the number of countries and also years. This is because many countries step into advanced stages during many years and, the quadratic equation must absorb changes in economic stages. The author finally got  $(rho/r) = 13.301c^2 - 22.608c + 10.566$  for 55 countries and for the rest,  $(rho/r) = 1.8638c^2 - 2.4547c + 1.758$ . In many countries, each  $R^2$  shows 0.95 to 1.0. Exceptional are several countries in Asian saving-oriented and different national system-oriented countries like China.  $(rho/r)(c)$  is directly related to  $\alpha = 1 - (c/(rho/r))$ ;  $(r/w) = (\alpha/(1 - \alpha))/(K/L)$ ;  $r = \alpha / (K/Y)$  and participating in the determination of the qualitative level of the endogenous-equilibrium..

Finally, the author summarizes the stream of utility equations lying between literature's utility and macro-based utility as follows: Let the author introduce the concept of instantaneous utility by Cass David (1964, 4–5). Formulating each utility function of consumption and wages/compensation,

$$U(C) = C / rho = \sum_{t=0}^{\infty} \frac{C}{(1 + rho)} \text{ and } U(W) = W / r = \sum_{t=0}^{\infty} \frac{W}{(1 + r)} \text{ are derived, where}$$

$U(C) = U(W)$  holds. The author's  $1 - \alpha = c/(rho/r)$  was derived as shown above, where related definitions are  $(1 - \alpha) = W/Y$  and  $c = C/Y$ . The present value of  $U(C)$  or  $U(W)$  may be called social welfare as a stock. Cass David's use of  $U(C) = U(W)$  is a great gift to the endogenous model and system. As a result, the author's use of  $(rho/r)$  is justified.

### 3.2.2 From the *beta* in the literature to the speed years at the transitional path:

This section first roughly sketches the differences of the speed years for

convergence lying between the literature and the author's endogenous model and system and next, summarizes these differences using related equations and compares McQuinn and Whelan' (2007) method with the author's. The author stresses: The differences between the two sorts of the speeds for convergence well express the differences between the neoclassical growth model and the endogenous model and system.

For rough sketch; the literature has long pursued 'the speed for convergence' by using panel actual data. The endogenous model and system have pursued 'the speed years' for convergence as well, but using endogenous data by country. The equations used for the two different methods seem to be similar, but with essential difference of exogenous versus endogenous. The difference of the two equations must be clarified, dividing each equation into the numerator shown by number and the denominator shown by the convergence coefficient. First, exogenously, the numerator of the equation in the literature is shown, for example, as 0.69. This value is related to the denominator as the convergence coefficient,  $\beta$ , where  $0.69 = x$  in  $e^{-x} = e^{-\beta \cdot t}$ .<sup>8)</sup> Two problems are conceivable: 1) the speed for convergence is extremely slow and 2) the capital-labor ratio may reach its convergence at an infinite time, in the case of the Cobb-Douglas type. The above two problems differ from the textbook illustration that the capital-labor ratio reaches the steady state, where the capital-labor ratio remains unchanged together with related growth rates: i) the capital-labor ratio increases until the point of time when the capital-labor is lower and conversely, ii) the capital-labor ratio decreases until the point of time when the capital-labor is higher. Caused by 'under assumption,' the case of ii) may not exist; some contradictions may exist between assumptions and theory. The above 0.69 corresponds with the height of the capital-labor ratio when the current capital-

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8) Barro and Sala-i-Martin (2004, 2<sup>nd</sup> Ed., 56–59, 78–81, 111–119) basically followed 1<sup>st</sup> Ed., but with more supplemental equations.

labor ratio reduces by half (0.5).

On the other hand, the corresponding equation in the endogenous model and system is shown as 1.0 divided by the convergence coefficient,  $\lambda$ , where the speed years are  $1 / \lambda$ . The endogenous model and system use an accounting identity that  $1.0 = \text{the speed years} \times \text{an endogenous growth rate}$ . For example, if the growth rate of output is 0.03, the speed years are  $33.33 = 1.0 \div 0.03$ . The accounting identity, however, holds at single interest (not compound interest). The difference between single and compound interest rate is negligible when the growth rate is low and /or the speed years are fast (within ten years). This setting is acceptable just for avoiding tautology, complexity, and risk. If the growth rate is abnormally high as seen at young-developing stage countries, the difference is remarkable.

Nevertheless, the excuses to the above accounting identity are vanishing under the condition that the initial capital-output ratio is set equal to the capital-output ratio at convergence. The justifications are three: First, the capital-output ratio,  $\Omega$ , exposes a delicate relationship between the capital-labor ratio,  $k$ , and technology stock,  $A$ , using  $A = TFP = k^{1-\alpha} / \Omega$ . Second, the rate of return is connected with the market and proves the neutrality of ten year debt yield to the rate of return, under a derived condition that the initial rate of return equals the rate of return at convergence. Third, most importantly, the speed years at convergence,  $d\Omega(t) / dt = 0$ , are not infinite but within finite faster years, when one of endogenous data at KEWT is evenused. Samuelson, P. (1970, 1477–79) proved the constancy of the capital-output ratio mathematically. The author will precisely expose the above three points, by using related equations below.

Finally, the author compares the convergence coefficient stated by McQuinn and Whelan (2007) with the author's convergence coefficient. The neoclassical model essentially differs from the endogenous model and system and it is difficult to compare each other. A reason is that the rate of technological

progress is given externally at the neoclassical model while it is endogenously measured at the endogenous model and system. As a result, the capital-labor ratio and the capital-output ratio both converge at  $t^* = \infty$  when each is used as a leading parameter for the convergence coefficient. In the case of the endogenous model and system, contrarily, the above accounting identity,  $1.0 =$  the speed years  $\times$  an endogenous growth rate, is used for the speed years. Nevertheless, why does the author pay attention to McQuinn and Whelan (2007)? This is because the article exceptionally takes the capital-output ratio as a leading parameter, instead of the capital-labor ratio. Nevertheless, the article was unable to change the method and result of the convergence coefficient.

In detail, McQuinn, Kieran and Whelan, Karl (ibid., 159–184) follows Solow's (1956) external rate of technological progress. Their base equation, Eq. 4, is shown by  $y = \frac{Y}{L} = A \left( \frac{K}{Y} \right)^{\frac{\alpha}{1-\alpha}}$ , or differently  $A = \frac{k}{\Omega^{\frac{1}{1-\alpha}}}$ , where the capital-output ratio is  $K / Y$ . Then, their convergence coefficient,  $\lambda = (1 - \alpha) (g + n + \delta)$ , is directly connected with the capital-output ratio; such that  $g_{\Omega} = e^{-\lambda t} \Omega_0 + (1 - e^{-\lambda t}) \Omega^*$ . The last equation is expressed as  $g_{\Omega(t)} = \lambda (x^* - x(t))$ , where  $x(t) = \log x(t)$ . The convergence to the steady state will be much slower than that of the endogenous model and system. This shows a character of the neoclassical model in the case of Cobb-Douglas type (for definition, see the beginning at section 2).

### ***3.3 Comparison of the neoclassical model with the endogenous model and system***

#### **3.3.1 Brief version to exogenous versus endogenous**

The neoclassical model and endogenous model and system differ essentially in exogenous versus endogenous. This section shows its sketch by using the

speed of convergence and the relative share of capital briefly. First, for the speed of convergence or the speed years, reviewing how each be unsolved or solved. The literature used a concept of the speed of convergence, as shown in Sala-i-Martin, X. (1990a, b) and Barro, R. J., and Sala-i-Martin, X. (1995, Eq. 1.31, p. 36). This notion comes from the ‘continuous’ Cobb-Douglas production function, with a given growth rate of population, a given rate of technological progress, and a given rate of capital consumption or depreciation, and centers the growth rate of capital labor ratio,  $k = K / L$ , each in the neighborhood of the steady state. The convergence coefficient is designated by  $\beta$ . The  $\beta$  determines the speed years using the difference between the current  $\hat{k}$  and  $\hat{k}^*$  at convergence, each in the steady state. The convergence coefficient is shown by  $\beta = (1 - \alpha) \cdot (x + n + \delta)$ . The convergence coefficient  $\beta$  has three weak points, according to the author’s interpretation. First, Barro and Sala-i-Martin (ibid., p. 42) ‘assumes’ the steady state that the growth rate of per capita capital,  $g_k$ , equals the growth rate of per capita output,  $g_y$ , where  $y = Y / L$ . Second,  $k(t)$  may converge to  $k^*$  at an infinite time when the  $A(t)$  and  $K(t)$  in the Cobb-Douglas type are calculated using recursive programming, where a vague relative share of capital and a given rate of technological progress fall into a contradiction.<sup>9)</sup> Third, for empirical work to the Cobb-Douglas type, actual

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9) A power point function is measurable when the power is 0.2 or 0.3, where the relative share of capital must be constant. Similarly to Barro and Sala-i-Martin (1995; 2004), Jones, C. I. (1998, Eq. 2.13, p. 36; 2002, the same) uses a power function of  $y^*(t) = A(t) [s / (n + g + d)]^{\alpha/(1-\alpha)}$ . This equation includes the following implication.

The neutrality of technological progress was defined as the relationship between an external rate of technological progress and a constant relative share of capital. The author interprets this definition as it is peculiar and includes a keen problem. The neutrality, in the textbook, has been shown by Hicks’  $g_A$ , Solow’s  $g_A \cdot \alpha$ , and Harrod’s  $g_A (1-\alpha)$ , but both  $g_A$  and  $\alpha$  are unknown endogenously. If both are endogenously measured, the definition of the neutrality of technological progress ➤



panel data have to be used, instead of endogenous data by country, year, and sector.

The endogenous model and system, on the other hand, directly measure the endogenous-equilibrium by using the speed years, where all the assumptions disappear as shown in KEWT data-sets. Two typical examples under no assumption are: First, the marginal productivity of capital equals the rate of return and the marginal productivity of labor equals the wage rate in the endogenous-equilibrium;  $MPK = r$  and  $MPL = w$ . Second, the price level  $p$  is 1.0, by year and also in the transitional path by that year, where  $p \cdot Y = r \cdot K + w \cdot L$  holds. The endogenous model and system convert the above given rate of technological progress,  $x$ , to the endogenous rate technological progress in the endogenous-equilibrium,  $g_A^* = i(1 - \beta^*)$ , using the ‘discrete’ Cobb-Douglas production function (for the author’s  $\beta^*$ , see soon below). Therefore, the endogenous rate of technological progress constitutes one of two keys for solving problems in the endogenous model and system.

The other key for the endogenous model and system is the relative share of capital,  $\alpha = \Pi / Y$ , where  $\alpha$  is tightly related to the rate of technological progress. The ‘discrete’ Cobb-Douglas production function holds, helped by a constant  $\alpha = \Pi / Y$ , where  $\Pi$  is endogenous returns, in the transitional path by year. If the relative share of capital is not measured severely as a constant, the endogenous model and system shall require such assumption as  $MPK = r$  and  $MPL = w$  and cannot guarantee the perfect competition. The neoclassical model, contrarily, cannot accurately estimate the relative share of capital because the model holds under a given/external rate of technological progress. The endogenous model and system measure an endogenous rate of technological progress, which is guaranteed under a constant relative share of capital,  $\alpha$ .

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↘ disappears. Therefore, Solow had often tried to skeptically raise the problem related to the relative share of capital.

The discrete Cobb-Douglas production function, however, has to solve two difficulties caused by a constant  $\alpha$  at the transitional path by year: One difficulty; the production function has to hold at convergence. Then, the production function needs another parameter existing between the initial and at convergence, changing in parallel with a constant  $\alpha$ . The other difficulty; the production function must abandon the assumption of diminishing returns to capital (DRC). Then, the production function continues to mitigate DRC and realizes Constant returns to capital (CRC) at convergence in the transitional path. The two difficulties are related each other. The diminishing returns to capital (DRC) coefficient,  $\delta_0$ , solves the two difficulties and becomes essential to the endogenous-equilibrium.

Related to the above logic, the author refers to Solow (1961, 48–50). Solow (ibid., 7., 50) shows an impressive calculation using a simple example. The assumption and calculation are alive even today and the citation is the following:

It is obvious from this mode of argument that the model works the way it does only because wages are spent entirely on consumption and profits on machines. This is an extraordinarily powerful assumption, more powerful than many of its users realize. Here is a cute example of just how powerful it is: Constant returns to scale, competition, machines produced by labor alone, consumables produced by machines alone, wages consumed, profits saved. Exercise: prove that the relative share of wages in national income is exactly 1/2!

To the author's understanding, the relative share of capital,  $\alpha$ , is 0.5, by using Solow's equation of  $\frac{rK}{wL} = \frac{\alpha_{I\text{ sector}}}{1 - \alpha_{C\text{ sector}}}$ , since its right hand side is  $0.5 / (1 - 0.5)$ . Note that the endogenous model and system are an extension of Solow's model, converting external to endogenous technology. Using symbols and endogenous equations of the author's (though basically the same), Solow's above contents are shown as follows:  $W = C$  and  $S = \Pi$ ;  $c = 1 - \alpha$  and  $s = \alpha$ ; as a result,  $(rho/r) = 1.0$  holds, using national taste of the propensity to

consume,  $(\rho/r)(c)$ . Therefore, using  $(\rho/r) = 13.301c^2 - 22.608c + 10.566$ , if  $c = 0.5$ ,  $(\rho/r) = 2.58725$  holds and, if  $(\rho/r) = 1.0$ ,  $c = 0.7943$  holds, where there is no difference between one sector and two sector endogenous model and system and; under constant returns to scale,  $MPK = r$  and  $MPL = w$  in equilibrium, and thus, under perfect competition.

Solow (ibid., 8., 50) finally compares his exogenous model (1956) with a series of Uzawa's two sector model. Solow stresses that Solow's model takes saving as a fraction of aggregate income,  $s = S / Y$ , as an assumption (at one point Solow permitted the saving ratio to depend on the rate of return on capital). Solow indicates that if  $s = S / Y$  is applied to Uzawa's model, stability is no longer assured and the results become qualitatively like the one-sector model. The author indicates that the capital-output ratio of consumer goods should be lower than that of capital/investment goods, differently from Uzawa's (1964, 1965) two-sector model.<sup>10)</sup> The author intends to study how to measure the capital-output ratios at a two-sector model in the future. Tinbergen, Jan (1964) first bravely calculated the capital-output ratio but without actual capital-output ratio so that his capital-output ratio used for simulation was far higher than the capital-output ratio today.

The endogenous model and system love the use of the relative share of capital and the capital-output ratio rather than the capital-labor ratio, as shown below.

### 3.3.2 Whole exposures of mechanics by using equations

The mechanics of the endogenous model and system are explained using the

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10) It is impossible to prove Uzawa's (1964, 1–24; 1965, 18–31) two-sector model by using actual data, whose reason is that capital stock has not been consistently set up by country in international statistics. This is not the responsibility of statistics because the purpose of statistics is to leave records based on the SNA.

following equations:

$$\text{Starting with } \lambda^* = (1 - \alpha) + (1 - \delta^*) g_{A(ENDOG)}^*$$

$$\text{The convergence coefficient, } \lambda = (1 - \alpha) + (1 - \delta_0) g_A^* \quad (11)$$

$$\text{The capital-output ratio, } \Omega = K / Y: \Omega^* = \frac{\beta^* \cdot i(1 - \alpha)}{i(1 - \beta^*)(1 + n) + n(1 - \alpha)} \quad (12)$$

$$\text{The qualitative net investment coefficient, } \beta^* = \frac{\Omega^* (n(1 - \alpha) + i(1 + n))}{i(1 - \alpha) + \Omega^* \cdot i(1 + n)} \quad (13)$$

$$\text{The DRC coefficient, } \delta_0 = 1 + \frac{LN(\Omega^*)}{LN(B^*)} \text{ and } B^* = (1 - \beta^*) / \beta^* \quad (14)$$

$$\text{The elasticity of substitution } \sigma = 1.0 = \frac{\Delta k / k}{\left( \frac{\Delta r}{\Delta w} \right) / \frac{r}{w}} \text{ holds in the transitional path by year,} \quad (15)$$

simultaneously measuring the marginal productivities of capital and labor,  $MPK = r$  and  $MPL = w$  and deleting the assumption of perfect competition; under  $p \cdot Y = r \cdot K + w \cdot L$ , where the relative price level,  $p = 1.0$  (for sketch, see section 3.3.1).

$$\text{National taste and income share, } (1 - \alpha) = \frac{c}{(rho / r)} \text{ and, } \frac{K}{L} = \frac{(\alpha / (1 - \alpha))}{(r / w)}$$

$$\text{or } k = \frac{w \cdot \Omega}{1 - r \cdot \Omega} \quad (16)$$

Note that Eq. 16 is related to endogenous taxes,  $T_{AX} / Y$ , as discussed in section 3.1.

Between Eq. 11 to Eq. 16, ‘seven’ endogenous parameters cooperatively and wholly work: the relative share of capital  $\alpha$ ; the growth rate of population  $n$ ; the ratio of net investment to output  $i = I / Y$ ; the qualitative net investment coefficient  $\beta^*$ ; the diminishing returns to capital (DRC) coefficient  $\delta_0$ ; the capital-output ratio  $\Omega$ ; and, the ratio of government net investment to government output  $i_G = I_G / Y_G$ , or  $i_G = I_G / Y$  that corresponds with Eq. 5,  $Y_G / Y = T_{AX} / Y$ .

The author now explains endogenous mechanics using Eq. 12 to Eq. 14, and

focuses  $\delta_0$  as a core of endogenous mechanics. Eq. 11 represents a specified growth rate as a base for equilibrium. Eq. 11 spreads over other equations. Next, the author explains its background using Eq. 15 and Eq. 16. The background was exposed in section 3.2.1 but here, overwhelmingly. The author does not show the process to formulate each equation for the sake of simplicity. For the first appearance series for formulating each equation, after Feb, 2004, see References.

For measuring  $\delta_0$  at Eq. 14, Eq. 12 and Eq. 13 are prerequisite. Eq. 13,  $\beta^*$ , is measured only under the condition that the initial capital-output ratio equals the capital-output ratio at convergence;  $\Omega_0 = \Omega^*$ . The proof is: Eq. 12,  $\Omega^*$ , is not measured if  $\beta^*$  is known. Eq. 13,  $\beta^*$ , is not determined if  $\Omega^*$  is unknown. Thus, Eq. 13,  $\beta^*$ , is measured only when  $\Omega_0 = \Omega^*$  is presumed. The capital-output ratio so some extent changes along with time/years in the transitional path. The rate of change in capital-output ratio shall be zero at convergence, where the speed years spread between 5 to 200 years in the case of a moderate range of the endogenous-equilibrium. If the speed years is less than 5 years or more than 200 to 300 years, the situation turns to close to disequilibrium or falls into disequilibrium. These results are ascertained only when the relative share of capital is constant throughout the transitional path.

As a result, Eq. 14 is derived by using  $1 = \Omega^* \cdot B^{*1-\delta_0}$ , where  $1 = k^{\alpha-\alpha}$  holds at convergence and  $\beta^*$  is replaced by  $B^* \equiv (1-\beta^*)/\beta^*$ . The logic is that  $1 = \Omega^* \cdot B^{*1-\delta_0}$  is required for maintaining the Cobb-Douglas production function in the discrete case. Under the Cobb-Douglas production function,  $\Omega = \frac{k^{1-\alpha}}{A}$  generally holds (as proved by PhD thesis; Note 19, 38, 2003). Set a definition of  $B_{TFP}^* \equiv (B^*)^{1-\delta_0}$ .<sup>11)</sup> Then,  $\Omega = \frac{k^{1-\alpha}}{A}$  turns to  $\Omega = \frac{k^{1-\alpha}}{B_{TFP}^* \cdot k^{1-\delta_0}}$  or

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11) The form of  $y = B_{TFP} \cdot k$  is another expression of  $Y = AK$  model in Keynesian model (Thirlwall, A. P., 427–435, 2002), which does not use the C–D production function.

$\Omega = \frac{k^{\delta_0 - \alpha}}{B_{TFP}}$ . It is true that  $\alpha = \delta_0$  holds at convergence. Thus,  $\Omega^* = \frac{1}{B^{*1 - \delta_0}}$  and

accordingly, Eq. 14 is derived. It is necessary to obtain Eq. 14 for justifying a constant relative share of capital. When the initial value of  $\delta_0$  is higher than a constant  $\alpha$ , the initial diminishing returns to capital (DRC) is gradually adjusted by year towards  $\alpha$  at convergence. When the initial value of  $\delta_0$  is lower than a constant  $\alpha$ , the initial increasing returns to capital (IRC) is gradually adjusted by year towards  $\alpha$  at convergence. Further, after convergence, DRC turns to IRC and adversely, IRC turns to DRC, where the IRC after convergence implies that the capita-output reaches infinite and the DRC after convergence implies that the capita-output ratio reaches zero. A finite capital-output ratio is tested in the transitional path by year using KEWT data-sets by country and sector. In short, the endogenous model and system are based on the discrete Cobb-Douglas production function;  $\alpha$  changes by year and remains constant in recursive programming by year.

Turning to the background of the endogenous mechanics, the author wholly explains the background using Eq. 11, Eq. 15, and Eq. 16. The author compares the endogenous background with the neoclassical background by first using Meade James (1962) and then Resek, Robert, W. (1963). First, under the discrete Cobb-Douglas production function as an expression of the endogenous model and system, nine assumptions shown by Meade James (1962, 1–7; for nine assumptions), each by each disappear. And each assumption is replaced by endogenous equation or its extension, theoretically and empirically: Meade's assumptions are mostly alive even today. These assumptions are: 1) a closed economy; 2) perfect competition; 3) constant returns to scale (CRS); 4) the growing system in equilibrium; 5) no governmental budget (no taxation and no State expenditures but central monetary authority); 6) a constant cost-of-living

index (a constant money price of our single consumption); 7) full employment of labor and land; 8) all machines are alike or perfect malleability of machinery; 9) perfect substitution in production between capital goods and consumption goods).

The endogenous model and system measure the above assumptions and proves no assumption each by each: 1) + 5) the introduction of the balance of payments and deficit (Eq. 7); 2) + 6) (Eq.15 under  $p = 1.0$ ); 3) constant returns to scale (CRS) is a nature of the Cobb-Douglas production function; 4) the use of the endogenous-equilibrium, measurable (Eq. 11), as a supplemental use of the price-equilibrium (immeasurable, and causes are never shown from results); 7) full-employment with less inflation is guaranteed in a moderate equilibrium (unemployment is the last adjustment in the speed years, compared with the actual growth rate of population); 8) capital stock and its rate of return are measured simultaneously with other parameters and variables by country, year, and sector (Eq. 16); 9) the elasticity of substitution, *sigma*, is 1.0 in the transitional path (Eq. 15; but *sigma* changes by year at KEWT data-sets, which implies complete flexibility of capital and labor). The above facts are related each other overwhelmingly in the endogenous model and system.

Then, the author compares the endogenous model and system with Resek, R. W. (1963). Resek (ibid., 55–63) shows several illustrations, based on theory and its results, by case study and simulation. In particular, Resek (ibid., Charts 3 and 4; 59) compares the marginal rate of substitution (*MRS*, on the y axis) with the capital-labor ratio ( $k = K / L$ , on the x axis). If four cases each show a horizontal line, technical progress (by his terminology) is neutral to the capital-labor ratio. If different four cases each show a line rising to the right, technical progress is not neutral to the capital-labor ratio. The background theory is apparently supported by the neoclassical model in the price-equilibrium, where price of output, labor, factor material, and capital goods are used with each

corresponding quantity, calculating the risk rate of return required and the expected life of capital. But, behind the curtain, the relative share of capital is vague if the rate of technological progress is external; the author indicated a true implication for the neutrality of technical progress, at note 9 of this paper. One more; Resek (*ibid.*, Chart 6; 61) shows results simulated under different ranges of assumptions to the interest rate on the long-term, discount rate for future income, debt-equity ratio, and life of capital. The three (1919–29; 1930–39; 1959–59) lines each rise to the right and shift the capital-labor ratio to be higher from a low base. Resek (*ibid.*, 61) indicates ‘how growth in the output-labor ratio can be attributed to capital, technology, and an interaction.’ The background is represented by labor productivity,  $y = Y / L$ , and Its growth rate,  $g_y$ , under given external technology. Again, behind the curtain, the external technology exists.

What is an essential problem inherent in the neoclassical model from the viewpoint of the above respect? The author cites most pertinent words in Solow, R. M. (1958, 623) and this indication vividly exists even after fifty years later or more:

Government had to be dropped because our quaint accounting practices measure the value of its product by the compensation of its employees, so that by assumption no income is ever imputed to government-owned capital assets.

It implies that we cannot evaluate ‘deficit’ under the current accounting practices from the viewpoint of the rate of return at the government sector. It implies that deficit is independent of the growth rate of output at the private sector. In the case of the endogenous model and system, the rate of return and the growth rate of output are connected with the endogenous Phelps golden rule.

Wholly, the endogenous model and system, based on the discrete Cobb-Douglas production function, endogenously measure the rate of technological progress,  $g_A^* = i(1 - \beta^*)$ , the growth rate of per capita output,  $g_y^* = g_A^* / (1 - \alpha)$ ,



the growth rate of output,  $g_Y^* = n + g_Y^* (1 + n)$ , the rate of return,  $r_0 = r^* = \alpha / \Omega^*$ , and  $r^* = (\alpha / (i \cdot \beta^*)) g_Y^*$ , to confirm the endogenous Phelps golden rule coefficient,  $(\alpha / (i \cdot \beta^*))$ , based on Phelps, Edmund S. (1961, 1965).<sup>12)</sup> And, all the data are measured by sector, just using data before falling into the final stage at the SNA income. Typically, the relative discount rate of consumer goods to capital goods,  $(rho / r)$ , the relative share of labor,  $(1 - \alpha) = \frac{c}{(rho / r)}$ , the capital-labor ratio,  $\frac{K}{L} = \frac{(\alpha / (1 - \alpha))}{(r / w)}$ , and the elasticity of substitution,  $\sigma = 1.0 = \frac{\Delta k / k}{\left(\frac{\Delta r}{\Delta w}\right) / \frac{r}{w}}$ , are simultaneously measured (for each equation, see section 3.1).

In short, all the parameters and variables are involved in the endogenous model and its KEWT data-sets, supported by hyperbola equations, consistently by country, year, and sector, and over years, just like a phenomenon of the one dimensional reduction of space-time at two-dimensional in cosmos and quantum physics today. The author advocates that accounting information is typically homogenous information in this world just like rays and beams, diverging and converging. Hyperbolic equations hold each by reforming endogenous equations and spread over the endogenous model and system. This paper avoids the use of hyperbola equations for simplicity.

Finally, let the author compare the speed years at the endogenous model and system with the speed for convergence at the neoclassical model. The above endogenous convergence coefficient,  $\lambda = (1 - \alpha) + (1 - \delta_0) g_A^*$ , reduces to the exogenous one if  $\delta_0$  is replaced by  $\alpha$ : comparable to  $\beta = (1 - \alpha)(n + x)$  in Sala-

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12) Champernowne, D. G. (1962) raises a problem when saving equals profits/returns. Robinson, J. (1962) raises a problem when the rate of return is equal to the *MPK*. Solow, R. W. (1962, 255–257) comments these problems overwhelmingly in the neoclassical models. The endogenous model and system solves these problems starting with the relative discount rate of consumers goods to capital goods,  $(rho/r)(c)$ , and endogenously measuring  $MPK = r$  and  $MPL = w$  under perfect competition.

i-Martin (1990a, b). “Quantitative Aspects of Post-War European Economic Growth” edited by van Ark, Bart, and Nicholas Crafts (1996) shows several good researches for the speed of convergence at the EU countries based on Sala-i-Martin (ibid.), one of which is Javier Andres, Rafael Doménech and César Molinas (ibid., pp. 347–387). However, these empirical researches use econometrics based on actual panel data from statistics. The author advocates that actual and endogenous data comparisons are useful to sustainable policy-oriented analyses. If we obtain a few actual statistics data corresponding to endogenous data of KEWT data-sets, the neoclassical model and endogenous model and system will together open a new economics door. A few useful actual data are, for example, actual wages or compensation, actual taxes, and actual net investment by sector.

## **4. Country comparisons of results with policy implications**

### ***4.1 Brief explanations to empirical research***

Empirical results differ by country, based on the above mechanics and background. This section now needs to find facts and test empirical characteristics involved in Eq. 1 to Eq. 10, focusing on endogenous taxes as a size of government to influence net investment, technological progress, and the structure of the balance of payments,  $(S - I) = (S_G - I_G) + (S_{PRI} - I_{PRI})$ . The method for testing the results is divided into two; one is panel data analysis by area and the other is two-dimensional analysis between two items (i.e., cause and result) by country. Actual data have no endogenous equation to support by country so that econometrics needs actual panel data.

The endogenous data are KEWT version 5.11 for 65 countries, 1990–2009, by sector. These data cover 17 country Asia and Pacific area, 14 country Euro currency area, 15 country Non-Euro Europe area, and 19 country West Hemisphere/Latin America, Near East, and Africa area.

For both panel data analysis by area and two-dimensional analysis by country, the author selects the following seven data: 1) endogenous taxes,  $t_{AX(endo)} = T_{AX(endo)} / Y$ ; 2) endogenous net investment,  $i_{endo} = I_{endo} / Y$ ; 3) the actual growth rate of per capita output,  $g_{y(actual)}$ ; 4)  $lambda$  as an inverse number of the speed years,  $lambda = (1 - \alpha) + (1 - \delta_0) g_A^*$ ; 5) the balance of payments,  $bop = BOP / Y$ ; 6) the growth rate of per capita output in equilibrium,  $g_y^* = i(1 - \beta^*) / (1 - \alpha)$ ; 7) the actual growth rate of per capita  $GDP$ ,  $g_{GDP(actual)}$ . An absolute value is expressed by a ratio in a way that the absolute value is divided by endogenous output at the total economy,  $Y$ . A data-combination is two-dimensional and shown by  $a-b$  or  $a/b$  as an indicator, where  $a$  is on the y axis and  $b$  is on the x axis.

The author selects the following ten data-combinations, to test facts and characteristics useful to policy-makers, where each data-combination is ‘the y axis to the x axis’:

1.  $i_{endo}$  to  $t_{AX(endo)}$ , both data are endogenous.
2.  $g_{y(actual)}$  to  $t_{AX(endo)}$ , one actual and, the other endogenous.
3.  $i_{endo}$  to  $t_{AX(endo)} + \Delta d$ , one actual and, the other endogenous.
4.  $lambda$  to  $t_{AX(endo)}$ , both data are endogenous.
5.  $bop$  to  $t_{AX(endo)}$ , one actual and, the other endogenous.
6.  $i_{endo}$  to  $bop$ , one actual and, the other endogenous.
7.  $lambda$  to  $g_y^*$ , both data are endogenous.
8.  $bop$  to  $s_{PRI} - i_{PRI}$ , one actual and, the other endogenous.
9.  $g_y^*$  to  $g_{y(actual)}$ , one actual and, the other endogenous..
10.  $g_y^*$  to  $g_{GDP(actual)}$ , one actual and, the other endogenous.

The above data-combinations primarily pick up the relation between actual and endogenous data. It is most important for policy-makers to compare actual with endogenous data since actual data cannot be apart from endogenous data in the long run. Endogenous data reflect the result of leaders’ philosophy, theory,

and practice. And, endogenous data are controllable when the endogenous-equilibrium is within a moderate range by country. Remember that the balance of payments and deficit are actual data but, each component such as saving or net investment by sector becomes endogenous. If actual saving and actual net investment are available as in the examples of Japan and the US, 1960–2009, real-assets policies are evaluated much more clearly. The above data-combinations are useful to find facts that exist behind data and, to test characteristics and trends of facts.

The author used  $R^2$  for forming the national taste function of the propensity to consume, as clarified at its estimating method (see section 3.2.1). The author here adds some notes to the correlation coefficient,  $R^2$ . Figures used for this section do not each show the values of  $R^2$ . A reason here comes from the contents of research. First, let the author calculate  $R^2$  for panel data of  $bop$  to  $s_{PRI} - i_{PRI}$  by area;  $R^2 = 0.7232$  at Asia and Pacific area including 17 countries;  $R^2 = 0.6676$  at Euro area including 14 countries;  $R^2 = 0.8153$  at Non-Euro Europe area including 15 countries. Each result differs and includes some irregularities that occur before and after falling into close-to-disequilibrium. Assume that all countries by area each stay at a moderate range of endogenous equilibrium. Then, each value by area must be close to 1.0. Reality differs: because, close-to-disequilibrium results in a variety of divergence by area, which significantly lowers  $R^2$ . Second, let the author take  $\lambda$  to  $g_y^*$ ,  $g_y^*$  to  $g_{y(actual)}$  and  $g_y^*$  to  $g_{GDP(actual)}$  by country. Why not to stick to  $R^2$ ? Compared with the above area case, the number of plots is extremely small. As a result, a few time occurrences of close-to-disequilibrium for twenty years surprisingly damages  $R^2$ . The purpose of research is not the magnitude of  $R^2$  but the potential foundation, trends, and the whole shape/pattern/image to sustainability by country rather than the close-to-equilibrium itself. The market, of course, exaggeratedly reflects the circumstances.

## 4.2 Panel data analysis by area

Panel data analysis clarifies each character of eight data-combinations, 1990–2009, as shown by **Fig. 1., 2., 3., 4.** For information and comparison purposes, the author also presents six data-combinations at Japan and the US, 1960–2009, and tests the differences between the short and long run, as shown by **Fig. 5., 6.** Then, the author presents time-series wholly in detail at Japan and the US, 1960–2009, and steps into the wage rate and the rate of return, as shown by **Fig. 7., 8.**

The results of panel data analysis by area endogenously answer the current serious questions: 1) why a high level of plus balance payments reduces sustainable growth by country?; 2) why developed countries suffer from close-to-zero growth?; 3) why one side tax reduction decreases sustainable growth? The author plainly explains results of panel data analysis, with policy-implications.

1. Panel data shows the character of data-combinations differently by area. It implies that each area has its own characteristics supported by national taste and culture. Globalization cooperates with individuality by area. Diverse and freedom coexist, even though each level differs.
2. Two data-combinations,  $\lambda$  to  $g_y^*$  and  $bop$  to  $s_{PRI} - i_{PRI}$ , rigidly each show the same shape/pattern by area. It implies that the speed years are a surrogate for  $g_y^*$  and, the balance of payments is a surrogate for  $s_{PRI} - i_{PRI}$ . These two constitute a starting point for the cause-result analysis, where causes and results hold each other at the same time.
3. Two data-combinations,  $g_y^*$  to  $g_{y(actual)}$  and  $g_y^*$  to  $g_{GDP(actual)}$ , each show similar shape/pattern by area, although Euro currency area shows more convergent shape.
4. Data-combinations lead to finding invaluable facts. Typical data-combinations are  $i_{endo}$  to  $t_{AX(endo)}$  and  $i_{endo}$  to  $t_{AX(endo)} + \Delta d$ . Net investment and endogenous taxes are causes and results each other.

Countries by area each differently have a base-centre to  $t_{AX(endo)}$ . And the distribution of the range of  $t_{AX(endo)}$  differs by area. The same is true in the case of  $t_{AX(endo)} + \Delta d$ . The level of  $t_{AX(endo)} + \Delta d$  corresponds with the sum of macro demand, which equals macro supply.  $t_{AX(endo)} + \Delta d$  clarifies interrelated causes but macro demand=supply does not clarify the causes at the real assets. Endogenous net investment obtains the highest at the centre of  $t_{AX(endo)} + \Delta d$ .

5. Similarly,  $\lambda$  to  $t_{AX(endo)}$  or inversely, speed years to  $t_{AX(endo)}$  clarifies that the speed years need a pertinent level of endogenous taxes. It is not always necessary for policy-makers to raise endogenous taxes to obtain a moderate range of endogenous equilibrium. A fact or hypothesis is that there exists a pertinent point of endogenous taxes, between high and low levels, where endogenous net investment and the speed years are maintained effectively.
6. Further, endogenous net investment is maintained effectively when the balance of payments ranges between a few plus and minus range of  $bop = BOP / Y$ . The data-combination of  $i_{endo}$  to  $bop$  is most important for policy-makers to persuade people to understand true facts. Policy-makers and people believe that the higher the  $bop = BOP / Y$  the higher the growth rate is but, this is wrong. A true fact is that a high level of  $bop = BOP / Y$  reduces the engine to sustainable growth (see the right bottom by **Fig. 1.** to **4.**).

In short, common sense of policy-makers and people is wrong. Or, the converse is true. It is true that endogenous taxes and the balance of payments each have its moderate level to sustainable growth and moderate speed years. These are confirmed at two-dimensional analysis by country as below.

Next, for detailed information and comparison, the author presents six data-combinations at Japan and the US, 1960–2009, with time-series for the

relationship between the wage rate and the rate of return. This paper does not step into this relationship, yet the rate of return is another expression of the growth rate of output in equilibrium. Because, the author proved the endogenous Phelps golden rule coefficient,  $\alpha / (i \cdot \beta^*)$ , between the rate of return and the growth rate of output each in equilibrium. It is confirmed that Japan and the US, 1960–2009, each present the same fact to endogenous tax range. Japan presents the same fact as above for  $bop = BOP / Y$ . The US presents a little bit different fact for the balance of payments, since the US has sometimes overrun a moderate range of plus and minus  $bop = BOP / Y$ . Yet, it is true that when  $bop = BOP / Y$  decreases, three times in fifty years, the corresponding endogenous net investment steadily increases.

### 4.3 Two-dimensional analysis by country

Two-dimensional analysis by country focuses two data-combinations of (1)  $i_{endo}$  to  $bop$  and (2)  $g_{y(actual)}$  to  $t_{AX(endo)}$ , each using 47 figures for 44 countries and 3 area averages; 8 typical Asia & Pacific countries, 28 countries in Europe, and 8 typical other countries. The two data-combinations show ‘condensed’ causes and results from the viewpoint of policy-makers. The author has accumulated  $8 \times (65 + 3) = 548$  figures for rainy days. The author plainly explains results of two-dimensional analysis, with policy-implications as below.

(1)  $i_{endo}$  to  $bop$  (see **Fig. A1A. to A6A.** in Appendix):

1. A fact observed is that net investment increases steadily when the balance of payments,  $bop = BOP / Y$ , stays moderately between plus 3% and minus 3%, compared with unbalanced plus and minus, 5% to 10% or more. Net investment is one of two engines to determine the rate of technological progress and should always be plus. Many countries, for the last twenty years, have had serious experiences for net investment to fall into minus. It implies that the endogenous-equilibrium has not been maintained due to

some unbalanced policies.

2. A serious problem of the current policies is that policy-makers stick to the maintenance of the balance between macro demand and supply. A front solution is how to lead an economy, to accelerate the recovery of growth engine even under an unbalanced equilibrium. The author does not deny the market principle based on the financial/market assets but, the above problem often occurs in the price-equilibrium.
3. For robust net investment and growth, it is indispensable for policy-makers to direct the speed years towards a moderate range of equilibrium. The secret starts with the structure of the balance of payments, i.e.,  $(S-I) = (S_G - I_G) + (S_{PRI} - I_{PRI})$ . That is the policy to raise net investment, particularly at the private sector. In this respect, *bop to  $s_{PRI} - i_{PRI}$*  answers the question. This data-combination, as concluded at section 4.2, shows another fact that *bop* is a surrogate for the difference between saving and net investment at the private sector,  $s_{PRI} - i_{PRI}$ . The more moderate the balance of payments the more robust the balance of payments is. This is an everlasting fact.
4. When actual net investment is available in statistics, the level of robustness in net investment is strengthened by improving the difference between actual and endogenous net investment. Actual > endogenous shows 'be careful to bubble possibility' while actual < endogenous shows 'policies to economic sustainability are wrong.' The cause is an excessive deficit beyond a certain limit (see the endogenous taxes below). An actual ratio cannot last long, apart from the corresponding endogenous ratio. This is another fact observed among countries and over years. The market principle in the long-term is a reliable barometer. Yet, financial and market policies are supplemental and should not be used in the long run.
5. More important is the whole balance between the government and private sector under a certain level of endogenous-equilibrium. Particularly, from



the viewpoint of net investment, the balance between the two sectors moves violently by year compared with other causes or results. Some countries well manage in this respect while others are difficult to control. Seven endogenous parameters are interrelated behind, with national taste/preferences, history, and culture by country.

6. Conclusively, there exists much room for many countries to cooperate each other, rather in this global competition era today. The following fact or hypothesis works and so, participating countries increase net investment together.

A fact observed is that net investment increases steadily when the balance of payments,  $bop = BOP / Y$ , is moderate staying between plus 3% and minus 3%, compared with unbalanced plus and minus, e.g., plus and minus, 5% to 10% or more.

- (2)  $g_{y(actual)}$  to  $t_{AX(endo)}$  (**Fig. A1B. to A6B** in Appendix):

1. How can each economy control its equilibrium and net investment steadily and robustly? The answer is the size of government. This is measured by endogenous taxes,  $Y_G = T_{AX}$ , and  $Y_G / Y = T_{AX} / Y$ . Distribution of  $Y$  and disposition of  $Y$  are united, even if production of  $Y$  is differently shown after redistribution. Many countries, for the last twenty years, have experienced to have  $t_{AX} = T_{AX} / Y$  right and left or high and low geometrically, partly due to different government experiments and leaders' philosophies. Again here, there exist various basic policies among countries.
2. Similarly to the above case of  $i_{endo}$  to  $bop$ , the actual growth rate of per capita,  $g_{y(actual)}$ , does not always stay at a moderate range of the speed years by country. Different from the endogenous growth rate of per capita output,  $g_y^*$ , the actual growth rate of per capita output,  $g_{y(actual)}$ , sometimes suddenly and sharply falls less than zero. It implies that the economy lost a moderate equilibrium though finally recovering from close-to-disequilibrium

and/or disequilibrium. The author approves the work of the market and the market principle as a sign to disequilibrium since there is no other indicator reliable in the world. Nevertheless, the market cannot cure an economy, just temporarily shifting the economy to a safe place or a minimum range of equilibrium for a while.

3. Some countries maintain a high level of  $t_{AX} = T_{AX}/Y$  while others a low level of  $t_{AX} = T_{AX}/Y$ , to maintain a similar level of  $g_{y(actual)}$ . Assume that people are confident in give and take in their life-time under any social and pension system. Then, it seems to be acceptable. However, there is an absolute fact that if democracy or any other system by country does not think of the next generations, then facts shown by  $g_{y(actual)}$  to  $t_{AX(endo)}$  are never trustworthy of sustainability. And, this warning is often shown by a sudden extreme minus value of  $g_{y(actual)}$ .
4. When net investment at the private,  $i_{PRI} = I_{PRI}/Y$ , is minus even under a moderate level of the balance of payments, as in Japan, then, policy-makers and people must perceive that one-side tax reductions or subsidies as minus taxes never recover the growth rates, actual and endogenous, later. Some countries propose to stop deficit by writing in constitution by country. This may be a good compulsory means in West society yet, more important is people's eagerness to know everlasting facts and leaders' openness to facts, with think of the next generations and donation-minded to others.
5. Conclusively,  $i_{endo}$  to  $bop$  and  $g_{y(actual)}$  to  $t_{AX(endo)}$  are interrelated, through the endogenous-equilibrium or a moderate range of the speed years. Many countries have hitherto conquered the difficulties to bury an unbalance between macro demand and supply by country. This is one human fact but, resulting in unfair bubbles and sometimes with severe inflation. When actual data approaches endogenous data, policy-makers are able to avoid unfair bubbles that compel people to sacrifice their life, and world widely.

This should be another sustainable fact such that human is proud of.

## 5. Conclusions

The endogenous structure of the balance of payments at the real assets,  $(S - I) = (S_G - I_G) + (S_{PRI} - I_{PRI})$  is a starting base for the intention of this paper; i.e., whether or not the size of government as shown by endogenous taxes to output determines sustainable growth in a moderate equilibrium and guarantees a moderate speed years.

Endogenous taxes, the speed years, and the growth rates are tightly related each other in the endogenous-equilibrium. And, actual data stay within a certain range of endogenous data. These lead us to facts against common sense of policy-makers. This paper clarified facts against common sense, using 65 countries, 1990–2009, by sector, with the mechanics and background of the endogenous model and system, based on the discrete Cobb-Douglas production function. The differences between the discrete and continuous Cobb-Douglas production function appear typically when the rate of technological progress is precisely measured with seven endogenous parameters and when the relative share of capital is rigidly settled constant in the endogenous model and system. The differences between the endogenous model and system and the neoclassical growth and Keynesian models were compared, by referring to and citing representative articles in the literature. As a result, facts against common sense turned to real-assets facts, the author assures.

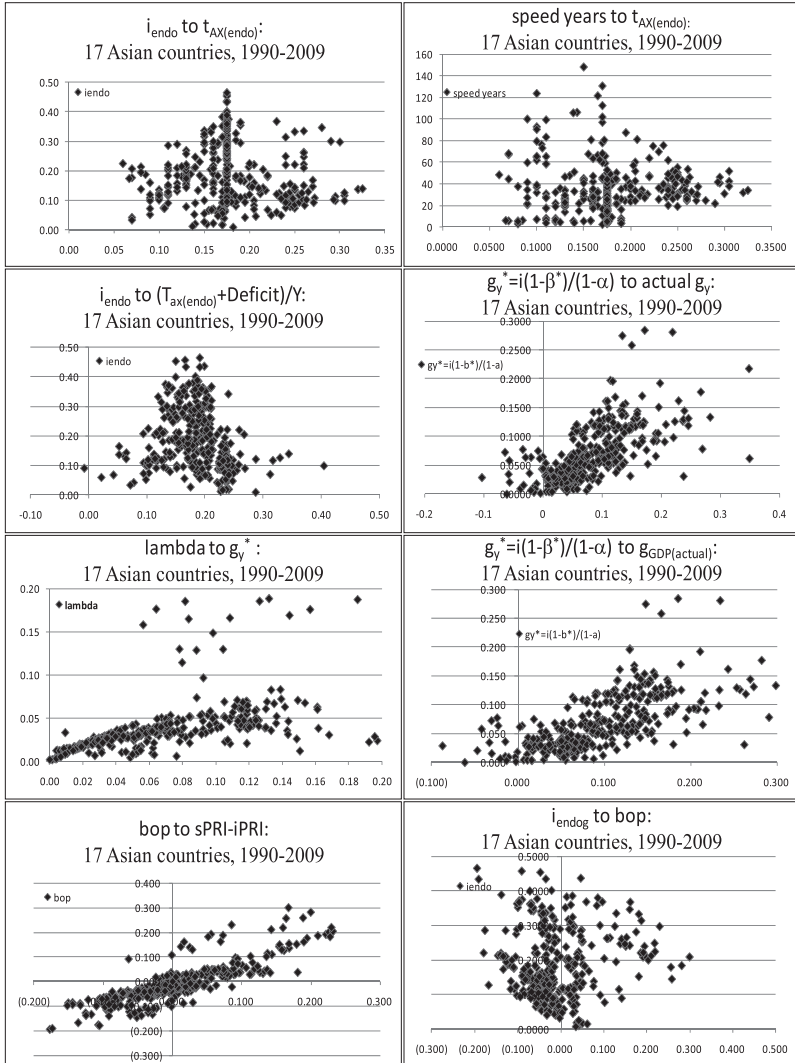
Typical real-assets facts are three:

First, Net investment increases steadily when the balance of payments,  $bop = BOP / Y$ , is moderate staying between plus 3% and minus 3%, compared with unbalanced plus and minus, e.g., plus and minus, 5% to 10% or more.

Second, if democracy or any other system by country does not think of the next generations, then the facts shown by  $g_y(actual)$  to  $t_{AX(endo)}$  are never

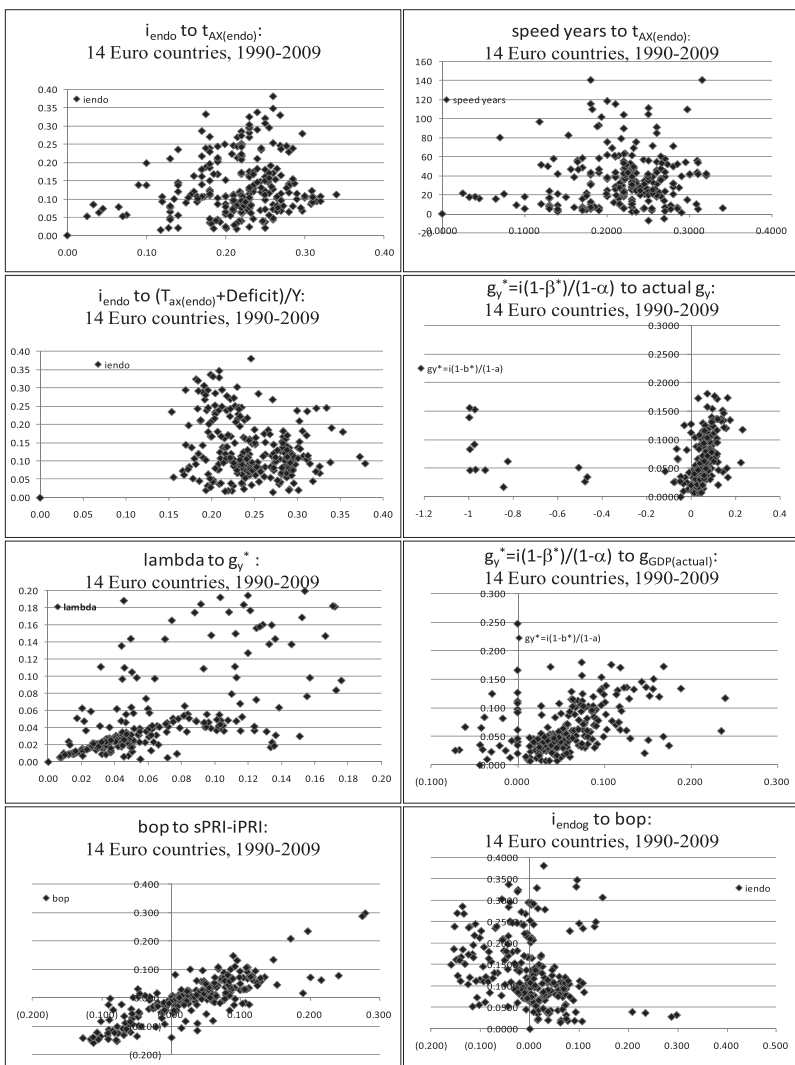
trustworthy of sustainability. And, this is often shown by a sudden extreme minus value of  $g_{y(actual)}$ . But, the sudden unbalance of macro demand and supply or unbalanced speed years by sector remain one half story.

Third, for the other half, if actual net investment is less than capital consumption at the private sector and, the growth rates of per capita output is less than the growth rate of population at the total economy, the situation is serious even the balance of payments is within a moderate range of plus and minus 3%. The true cause is huge deficit, which in turn brings about endless deflation until deficit level has a room for the next generations. One-side tax reductions and increases in subsidies as minus taxes decrease sustainable robustness of an economy. Against common sense is true at the real assets.



Data source: KEWT 5.11–1 for 17 Europe Area by sector, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. 1. Endogenous and actual data in terms of growth in equilibrium: 17 Asian and Pacific**



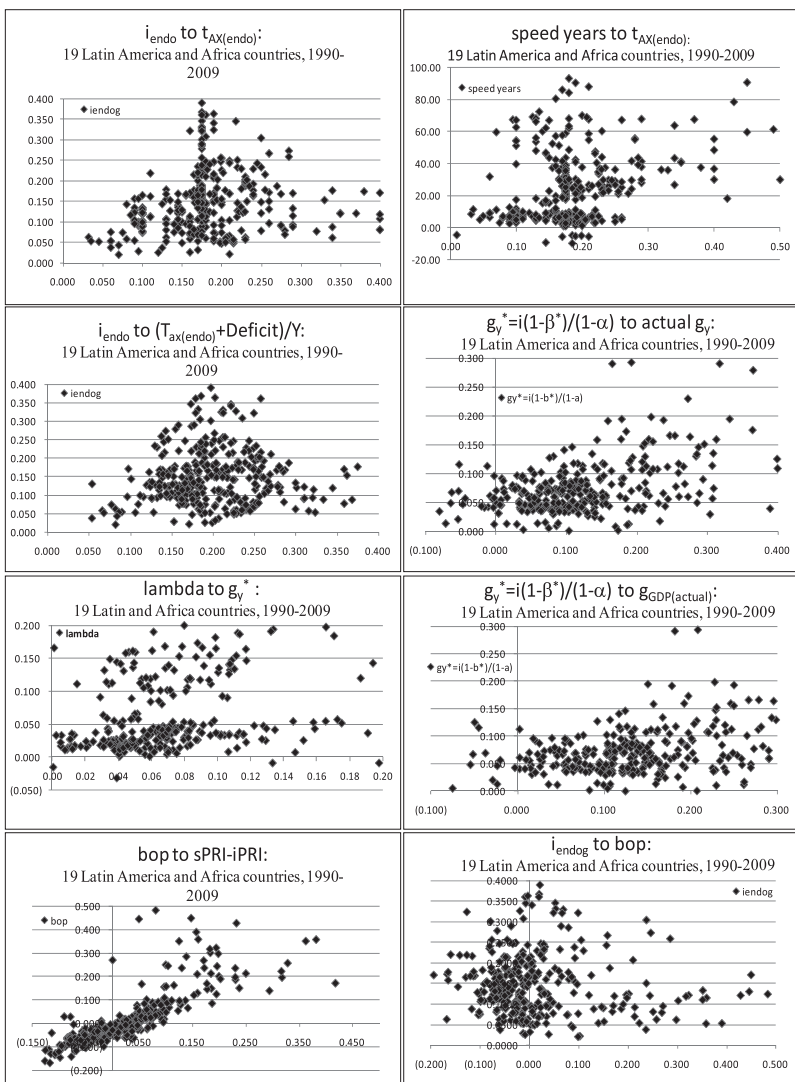
Data source: KEWT 5.11–2 for 14 Euro Area by sector, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. 2. Endogenous and actual data in terms of growth in equilibrium: 14 Euro**



Data source: KEWT 5.11–3 for 15 Euro Area by sector, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

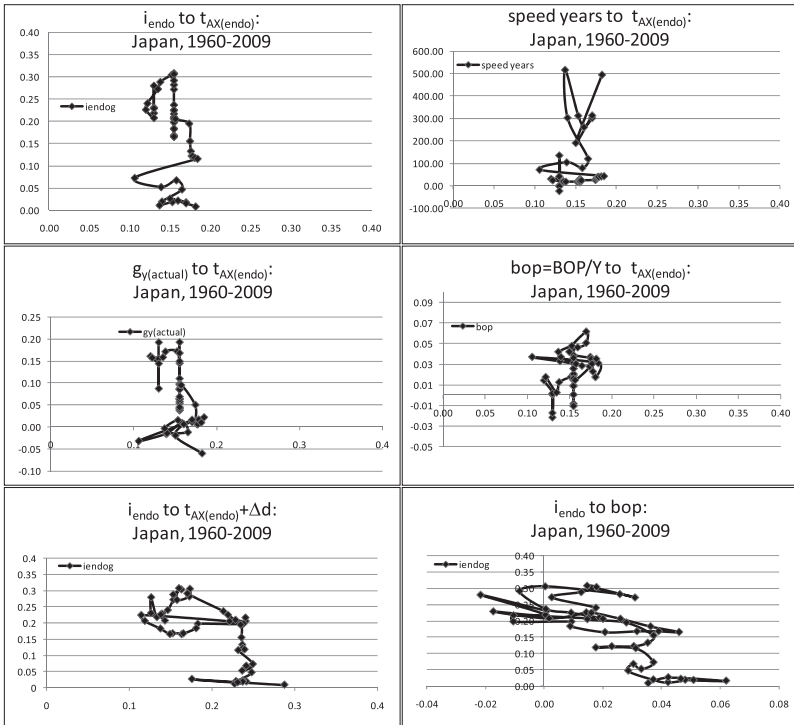
**Fig. 3. Endogenous and actual data in terms of growth in equilibrium: 15 Non-Euro Europe**



Data source: KEWT 5.11-4 for 19 Latin America and Africa Area by sector, 1990-2009, whose original data are *International Financial Statistics Yearbook*, IMF

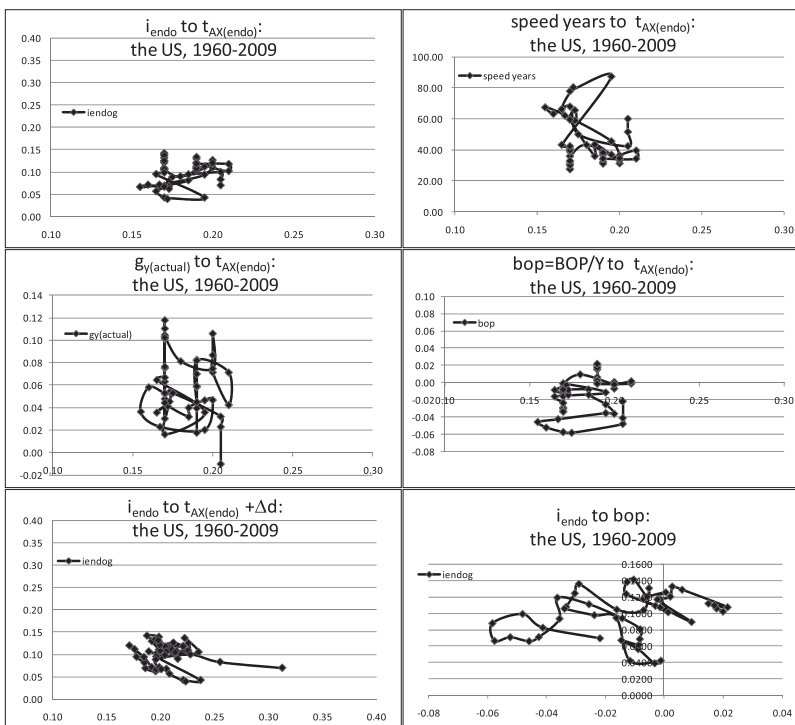
**Fig. 4. Endogenous and actual data in terms of growth in equilibrium: 19 Latin and Africa**





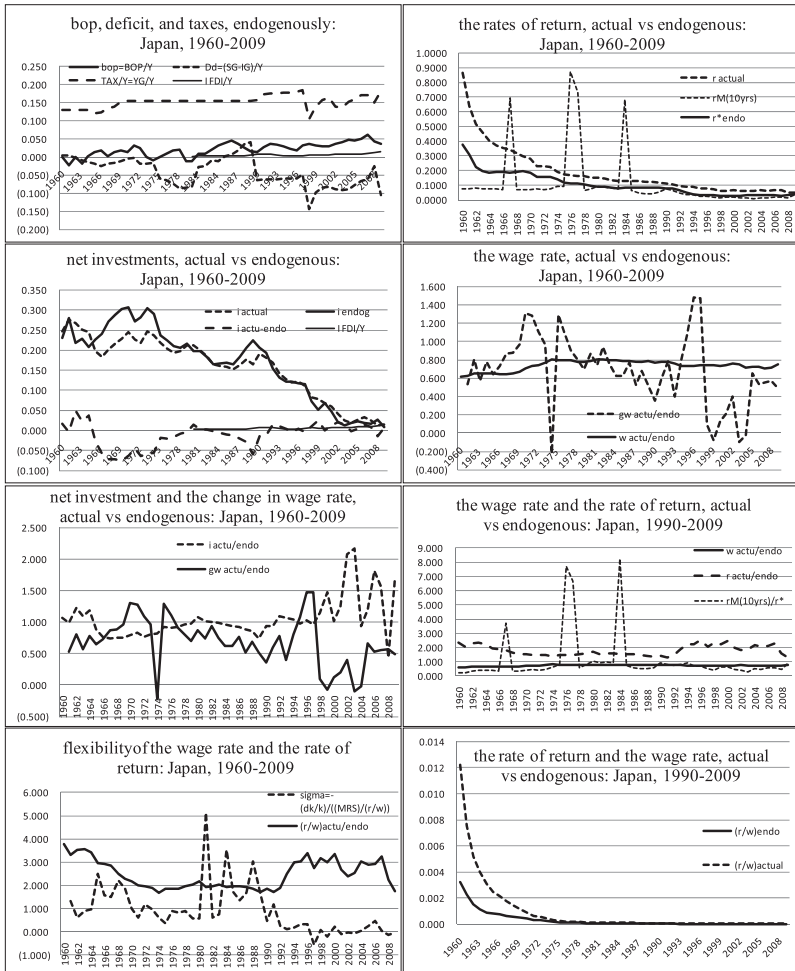
Data source: KEWT 5.11–6 for Japan and the US, 1960–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. 5.** Cause and result relationship between endogenous taxes, deficit, the balance of payments, the endogenous- equilibrium, the growth rate per capita, and net investment: Japan, 1960–2009



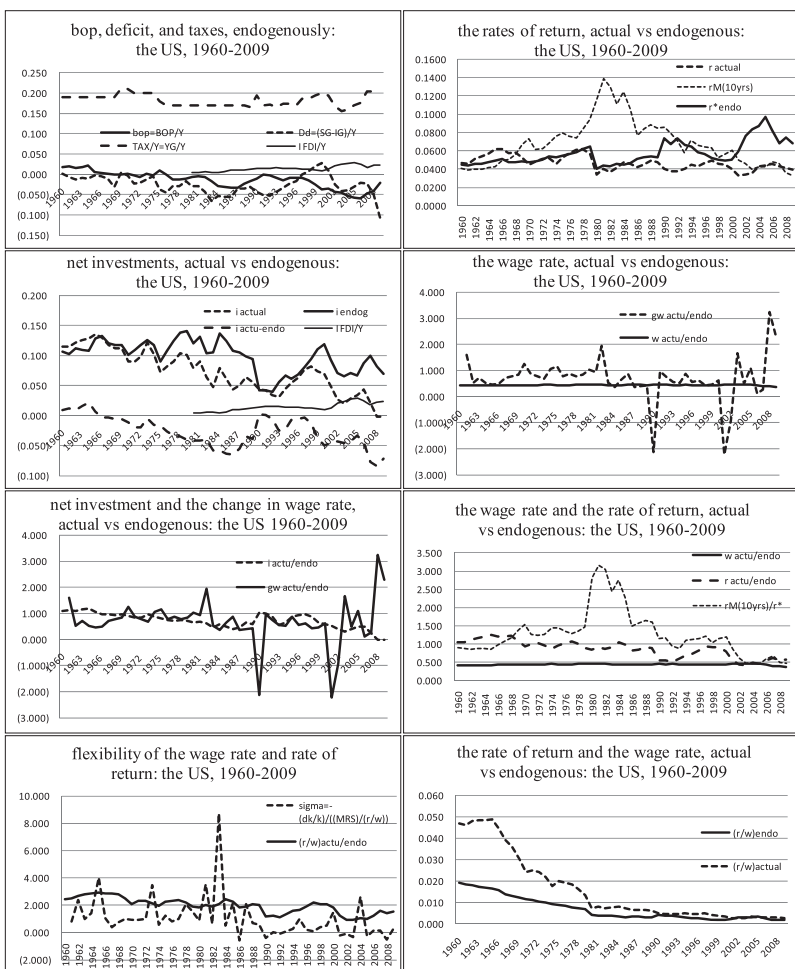
Data source: KEWT 5.11–6 for Japan and the US, 1960–2009, whose original data are  
*International Financial Statistics Yearbook*, IMF

**Fig. 6.** Cause and result relationship between endogenous taxes, deficit, the balance of payments, the endogenous- equilibrium, the growth rate per capita, and net investment: the US, 1960–2009



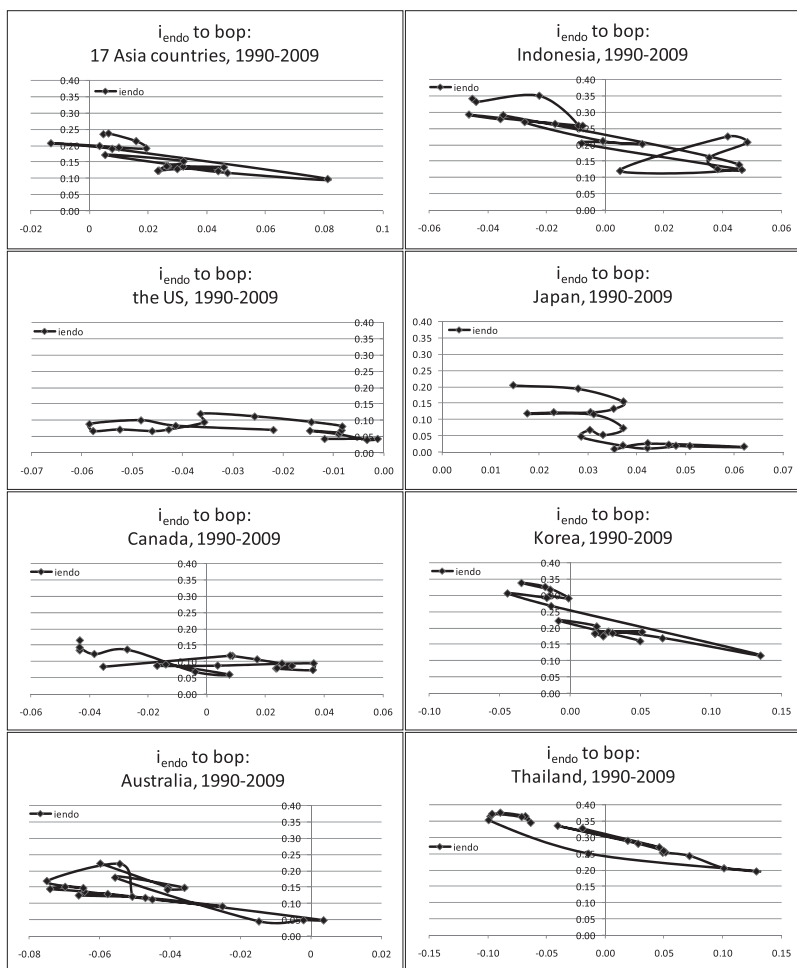
Data source: KEWT 5.11–6 for Japan and the US, 1960–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. 7.** Cause and result relationship between endogenous taxes, deficit, the balance of payments, the endogenous- equilibrium, the growth rate per capita, and net investment: Japan, 1960–2009



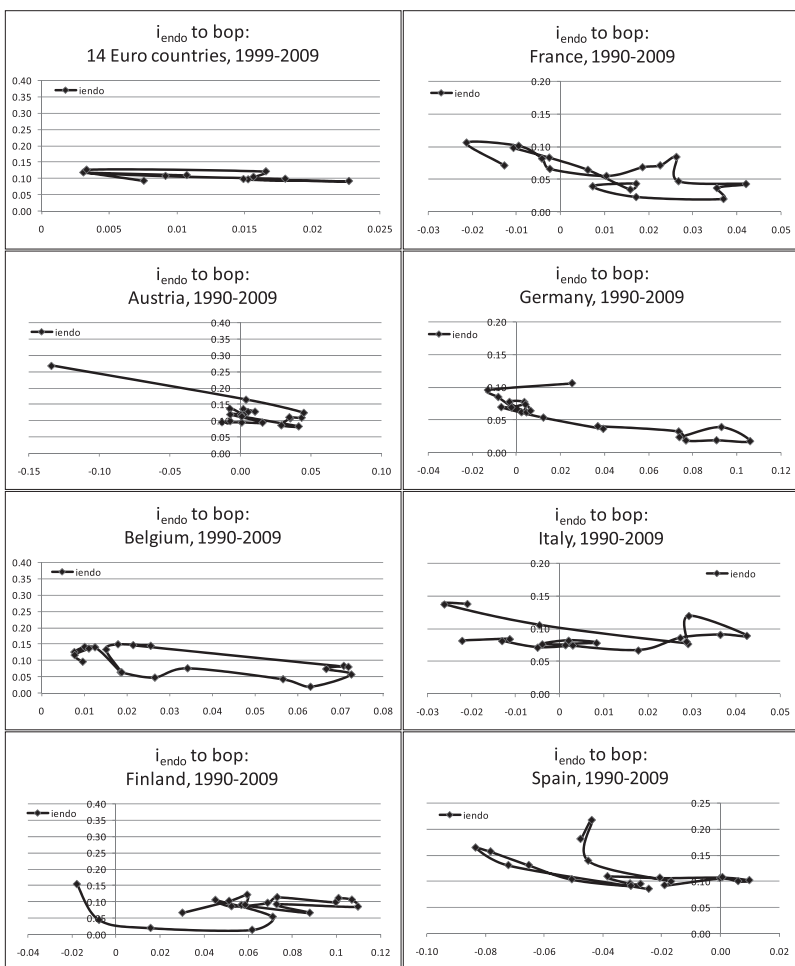
Data source: KEWT 5.11–6 for Japan and the US, 1960–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. 8. Cause and result relationship between endogenous taxes, deficit, the balance of payments, the endogenous- equilibrium, the growth rate per capita, and net investment: the US, 1960–2009**



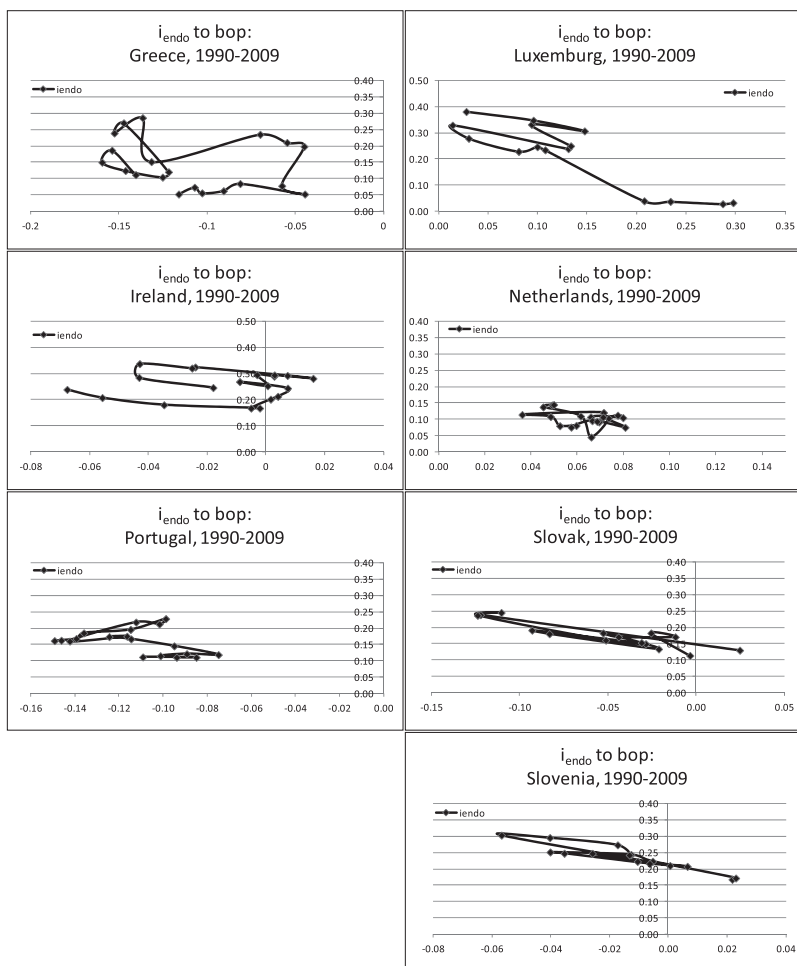
Data source: KEWT 5.11–1 for 17 Asia & Pacific, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A1A. Net investment ratio to the balance of payments in equilibrium, in 8 Asia countries: 1990–2009**



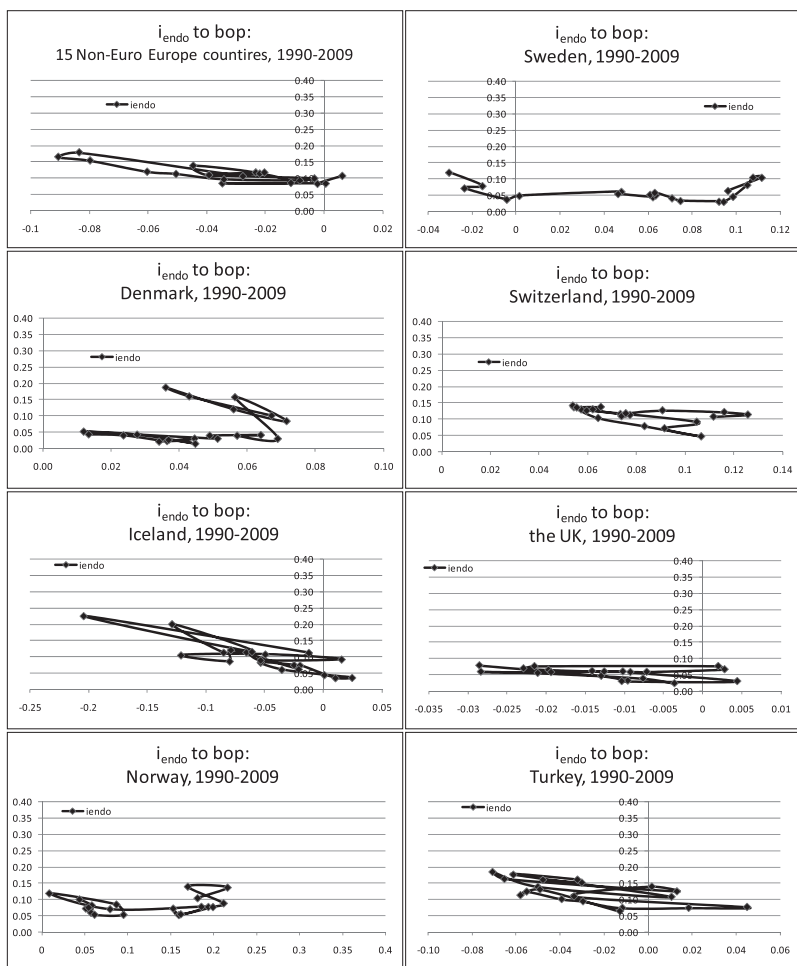
Data source: KEWT 5.11-2 for 14 Euro currency, 1990-2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A2A. Net investment ratio to the balance of payments in equilibrium: 8 Euro countries, 1990-2009**



Data source: KEWT 5.11–2 for 14 Euro currency, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

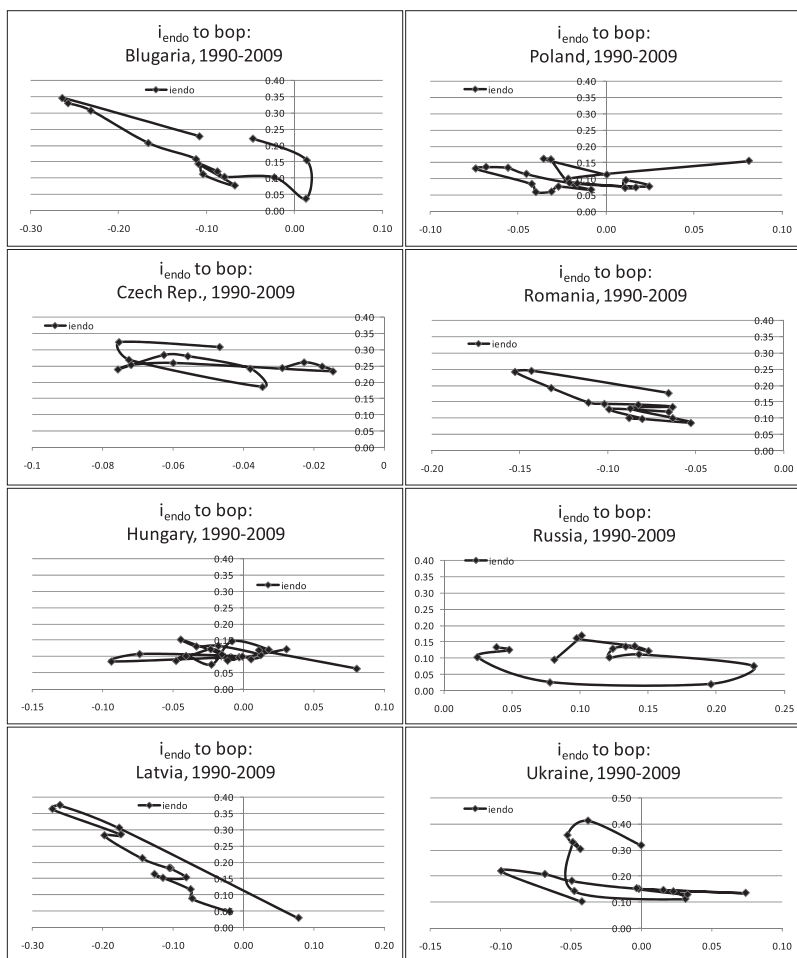
**Fig. A3A. Net investment ratio to the balance of payments in equilibrium: 7 Euro countries, 1990–2009**



Data source: KEWT 5.11-3 for Non-Euro Europe, 1990-2009, whose original data are *International Financial Statistics Yearbook*, IMF

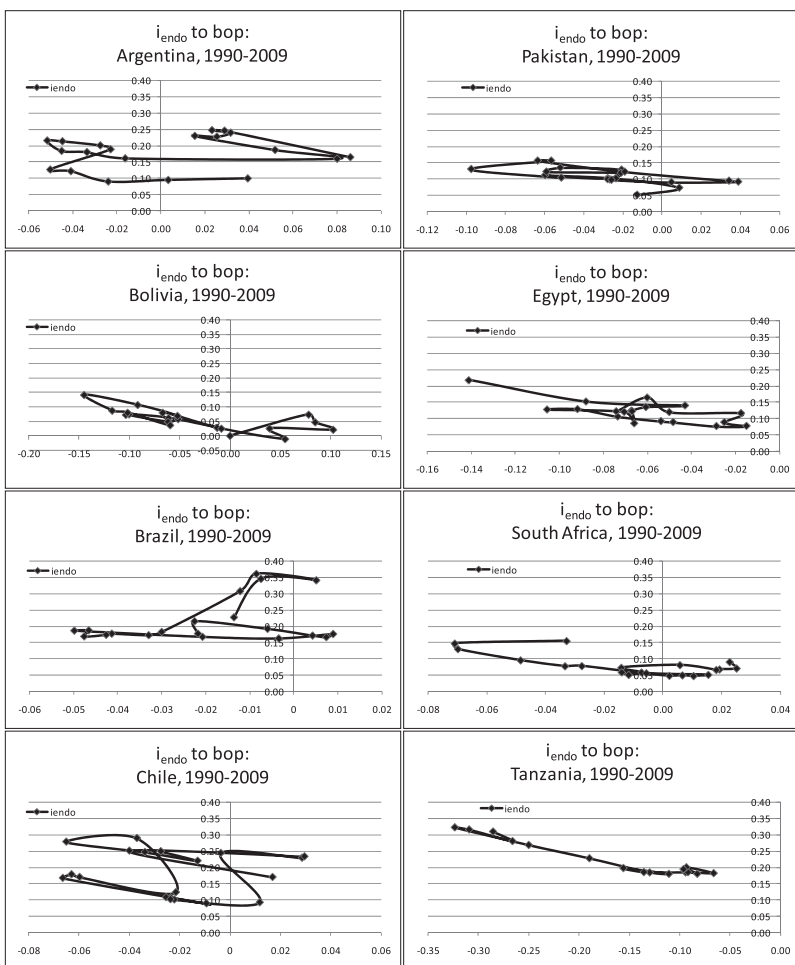
**Fig. A4A. Net investment ratio to the balance of payments in equilibrium: 8 Non-Euro Europe countries, 1990-2009**





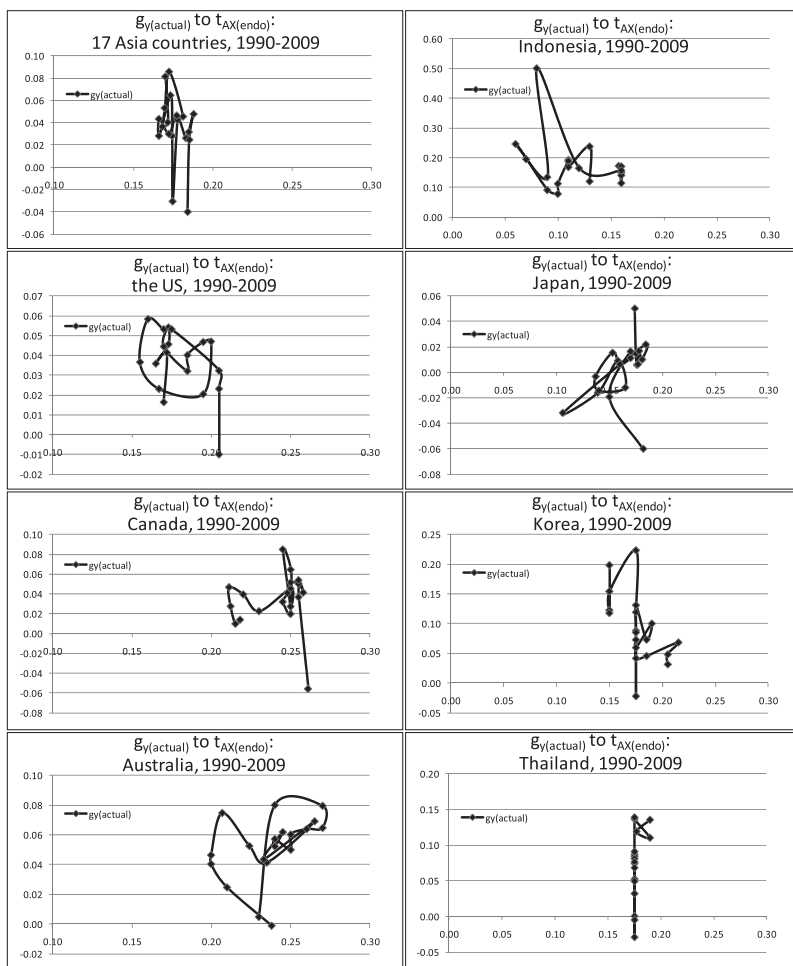
Data source: KEWT 5.11–3 for Non-Euro Europe, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A5A. Net investment ratio to the balance of payments in equilibrium: 8 Non-Euro Europe countries, 1990–2009**



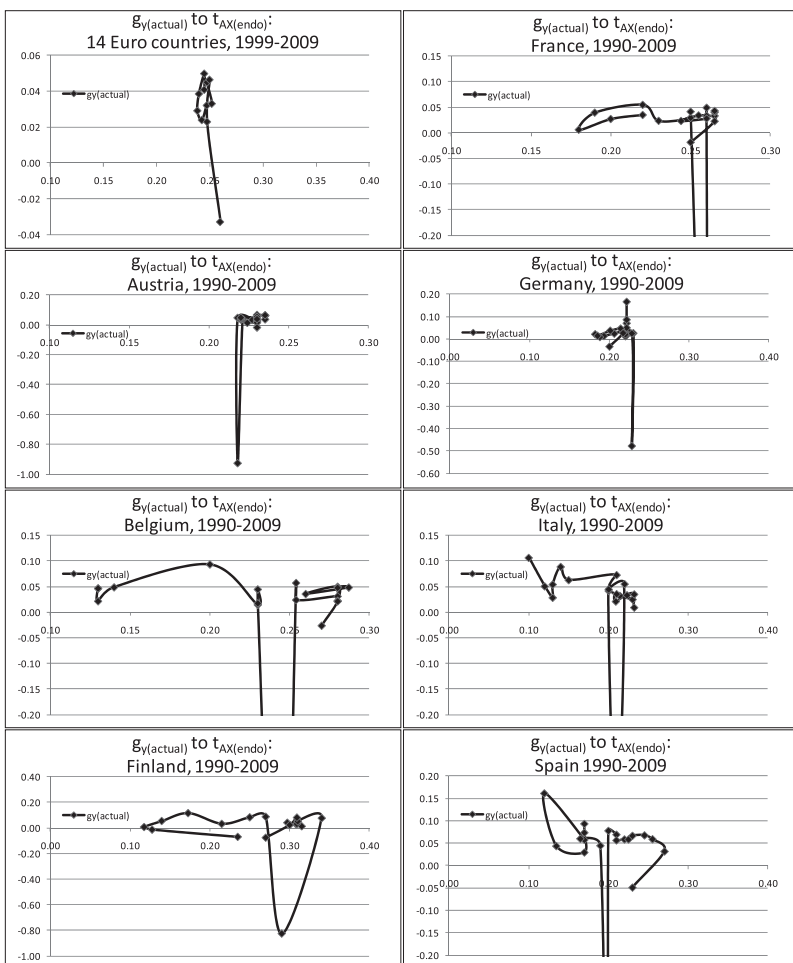
Data source: KEWT 5.11-4 for 19 Latin America, near East, and Africa, 1990-2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A6A. Net investment ratio to the balance of payments in equilibrium: 8 other countries, 1990-2009**



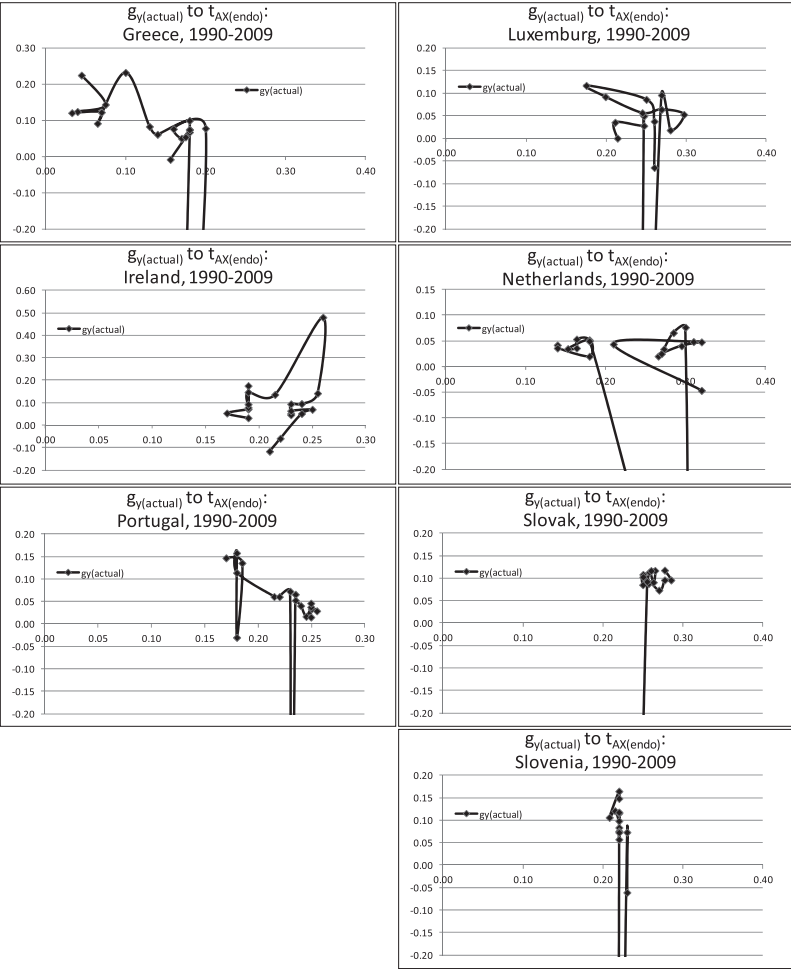
Data source: KEWT 5.11–1 for 17 Asia & Pacific, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A1B. Actual growth rate per capita to endogenous tax size in equilibrium: 8 Asia countries, 1990–2009**



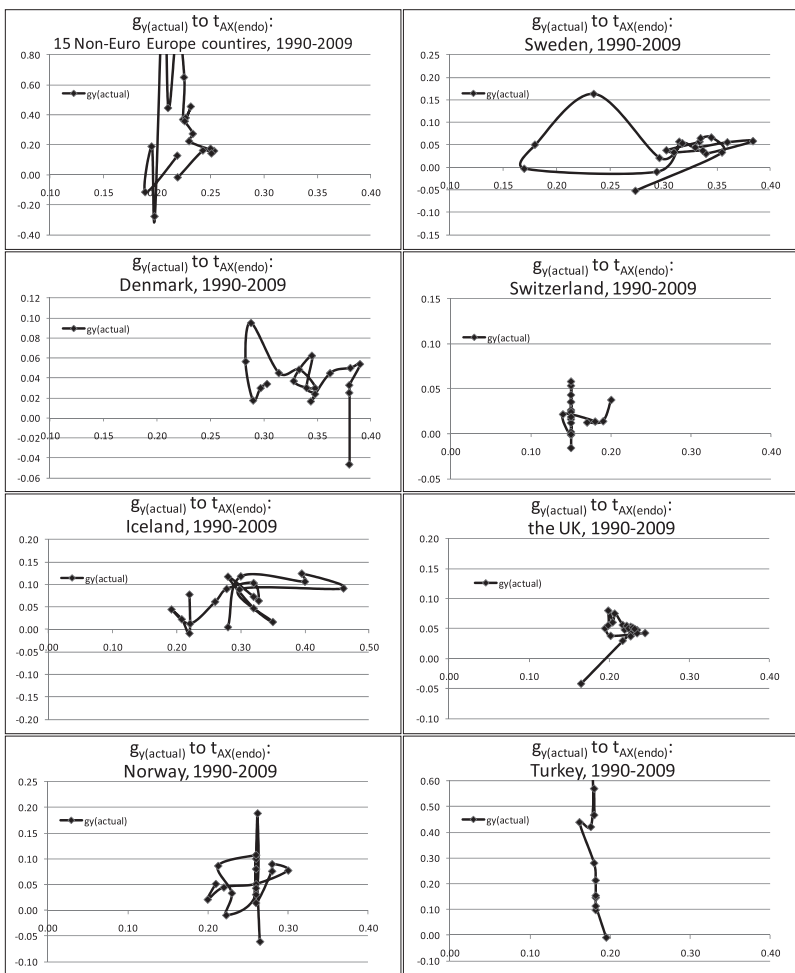
Data source: KEWT 5.11-1 for 14 Euro currency, 1990–2009, whose original data are  
*International Financial Statistics Yearbook*, IMF

**Fig. A2B. Actual growth rate per capita to endogenous tax size in equilibrium: 8  
Euro countries, 1990–2009**



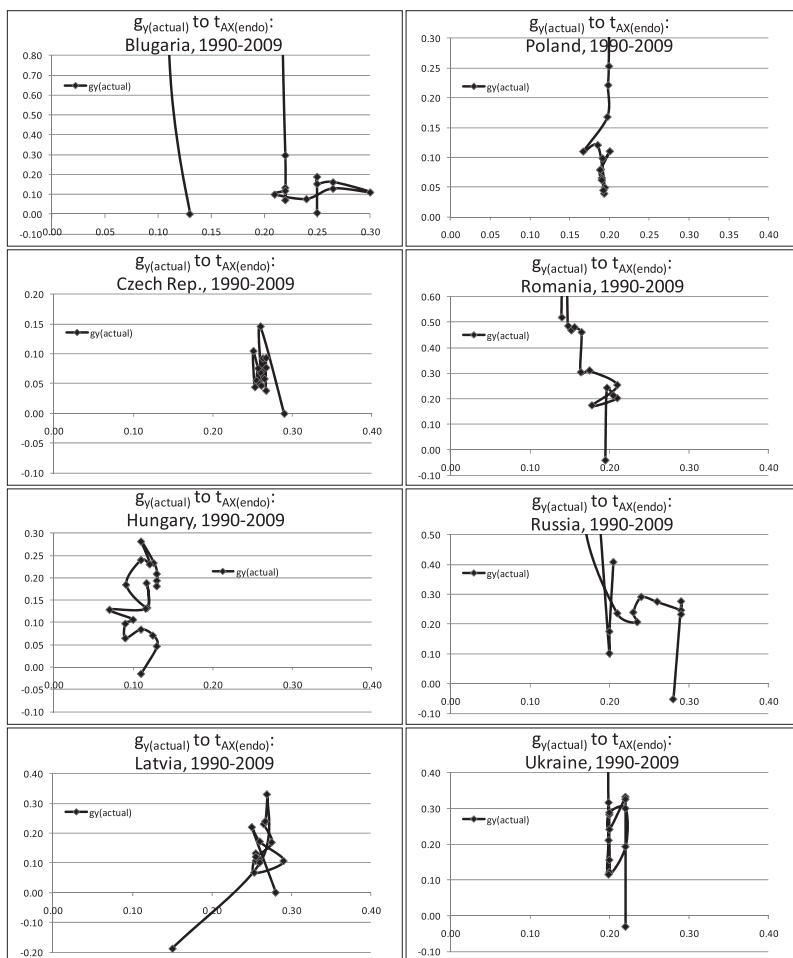
Data source: KEWT 5.11–1 for 14 Euro currency, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A3B.** Actual growth rate per capita to endogenous tax size in equilibrium: 7 Euro countries, 1990–2009



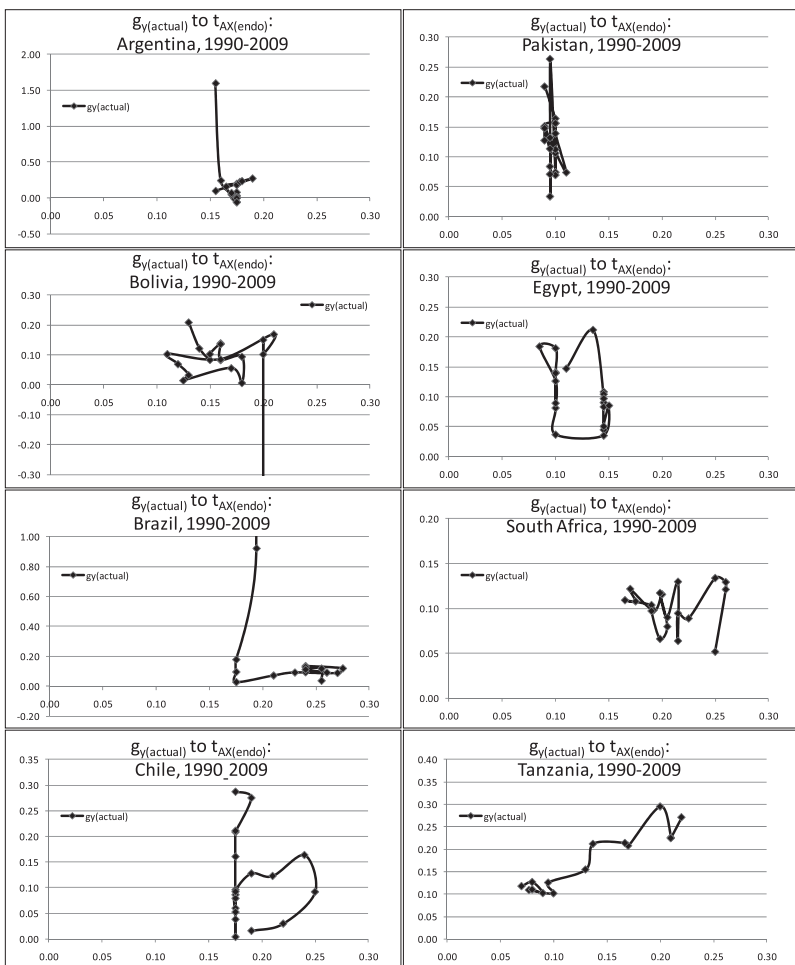
Data source: KEWT 5.11–3 for Non-Euro Europe, 1990–2009, whose original data are  
*International Financial Statistics Yearbook*, IMF

**Fig. A4B. Actual growth rate per capita to endogenous tax size in equilibrium: 8  
 Non-Euro Europe countries, 1990–2009**



Data source: KEWT 5.11–3 for Non-Euro Europe, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A5B. Actual growth rate per capita to endogenous tax size in equilibrium: 8 Non-Euro Europe countries, 1990–2009**



Data source: KEWT 5.11–3 for Latin America, near East and Africa, 1990–2009, whose original data are *International Financial Statistics Yearbook*, IMF

**Fig. A6B. Actual growth rate per capita to endogenous tax size in equilibrium: 8 other countries, 1990–2009**



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