A two-sector model of growth based on corporate finance (2): review of the investment ratio in the saving-side and the investment-side

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Abstract

In this paper, the author proposes first a two-sector model of growth that extends the two/one class growth model discussed in O'Connell [1995]. This model is a one-class (workers' own capital) and two-sector model under the balanced growth steady-state. It exhibits the same balanced condition as Harrod [1973] if capital stock is only owned by workers and the capital-output ratio is equal to one. General properties of this model include a result that the sensitivity of the changes in the growth rate in response to changes in the rate of profit depends crucially on the relative magnitude of the retention ratio (defined as undistributed profit/profit in the corporate sector) and the household propensity to save. However, this two-sector model is not supported enough by a panel data approach [Barro, 1991; De Long and Summers, 1991; Mankiw, Romer, D. and Weil, 1992] executed by the author who uses the national accounts of six countries, 1982-1994. The cause lies in how to introduce the wage and dividend propensity to save and the population growth rate into the growth rates of output and capital. The O'Connel's mode is based on constant labour productivity. The "improved" two-sector model is established in discrete time by using the financial structure of products and under necessary and sufficient conditions. This improved model is based on the initial financial structure of products and opens the door for the growth in labour productivity by introducing a given

1) I am thankful to Dr. Debasis Bandyopadhyay for his advice and for recommendation of articles suitable for my research. Also, I am oblidged to Dr. Tsutomu Tokimasa for his continuous review, Dr Masao Kawano for his interpretation of national accounts, and Dr. Yoshiomi Furuta for his mathematical advice and proofs. This paper differs from the previous one, the influence of corporate saving behavior on economic growth (1) [1997/March], which was based on Kaldor's framework, but did not fully integrate the saving of wages and dividends and the net investment which comes from these savings.

population growth rate and technological progress. The model starts with the review of the investment ratio, $\Delta K_P/Y_0$, which integrates the saving-side with the investment-side. As a result, using the initial predetermined variables of the relative share of profit and the capital-output ratio, important related variables in equilibrium are all endogenously mea-sured. The financial structure converges in equilibrium and diverges to some extent in disequilibrium, but disequilibrium quickly closes to equilibrium. This paper establishes conditions of equilibrium and compares them with those in equilibrium in literature.

1. Review of the two/one class growth model

1.1 Background and the relationship between two models: Two/one classes vs. Two-sectors

Prior to O'Connell, Baranzini [1975; correction 1991] justified both Pasinetti's model and Samuelson and Modigliani's model by using his own *quadratic* equation (5), but without using $s_p \equiv S_p/P$ (with some correction [1991]). The rate of profit r was solved as r_1 and r_2 from his equation. For this, O'Connell [1985, 1995] expressed the same quadratic equation (8) using s_p, s_c, and s_w (see the next section). She proves that the natural rate under full-employment steady-state growth is attained if $K_w/K=1$ (i.e., only if workers exist) and $r=r_2$. The two-sector model of the author's is justified by her proof and extends her model simply by eliminating dividend savings from that equation. The difference between the two/one class model and the two-sector model is whether dividend savings are added to worker savings (in the two/one class model) or the sum of consumption of dividends and capital gains is subtracted from worker savings (in the two-sector model). As a result, in the two-sector model, her quadratic equation reduces to a linear equation, and its solution is shown by $r=r_2$ as she proved. By this simplification, some new propositions are presented: e.g., changes to growth rate in response to changes in the rate of profit depend on the relative magnitude of the retention ratio and the household propensity to save.

However, the two-sector model holds under the balanced growth steady-state where the two-one class model holds. According to the author's panel data approach, it is not supported by empirical results. This is natural since the population growth rate is not equal to the growth rates of output and capital. Thus, this paper, furthermore, extends the two-sector model (under the balanced growth steady-state) to the improved two-sector model under the corporate-financed growth steady-state. The differences between underlying conditions are shown as follows:

1. The balanced growth steady-state is a condition such that the population growth rate, output, and capital are the same.
2. The corporate-financed growth steady-state is a condition such that the population growth rate is given as a parameter while the growth rate of output, g_Y, and the growth rate of capital, g_K, are endogenously measured and $g_Y=g_K$. A condition of $g_Y=g_K$ is guaranteed both under a necessary condition that the capital-output ratio and the relative share of profit are constant and under sufficient conditions that are described by assumptions.

This paper discusses only the above two states, and does not develop a condition that $g_Y \neq g_K$. Under a condition of $g_Y \neq g_K$, the capital-output ratio changes while the relative share of profit is given as a parameter (or vice versa; in a separate paper).[2)] The two-sector model does not include labour productivity as well as the two/one class model. However, the improved two-sector model can easily include labour productivity and its growth rate as an extension (see introduction of the coefficient of technological progress, below).

Conclusively speaking, the differences between the two/one class model and the two-sector model are briefly indicated as follows:

2) Kamiryo, [1997/Oct], Relationship between the growth rate of labour productivity and the rate of technological progress using discrete time in national accounts, at the 10th World Productivity Congress, Vina del Mar, Chile.

The two/one class model under the balanced growth steady-state:

1. Dividend savings are added to wage savings.
2. If $K_w/K=1$ ($K_c=0$), it reduces the Harrod-Domar condition where s_p is not directly expressed.

The two-sector model: under the balanced growth steady-state

1. The consumption of dividends and capital gains is subtracted from wage savings. It assumes that dividend savings are smaller than this consumption.
2. It reduces, if $K_w/K=1$, to the same Harrod-Domar condition where s_p is explicitly shown. There is no difference between the two models in the case of $K_w/K=1$ if the capital-output ratio, Ω, is shown as one.

What are the characteristics of the two-sector model under the balanced growth steady-state?

1. Use of a linear function instead of the quadratic function which produces two values r_1 and r_2.
2. Use of both dividends and capital gains (for simplicity and tentatively; the author, in the corporate-financed growth steady-state, abandons the use of capital gains needed for this simplicity and accepts the same assumption as the two/one class model).[3)]
3. Endogenous growth model which positively uses undistributed profit and its retention ratio s_p under a one class model ($K_w/K=1$) and Ω=a constant.
4. Pasinetti's statement of "$r_1=r$" is denied in the two-sector model. The "uniclass" balanced growth steady-state is expressed including s_p which was not used by the discussants of the two/one class model except Kaldor and O'Connell. It suggests that the use of undistributed profit is more important than the division of disposable income into two classes: workers' and capitalists' classes.

3) For the corporate sector, the use of capital gains is justified while SNA as a whole this use is not justified because of offset. I am thankful to Dr. M. Kawano [1997] for his illustration and confirmation.

5. The model, if $K_w/K=1$ and the capital-output ratio Ω=a constant, coincides with the Harrod-Domer model: $n=s_{S/Y}/\Omega$. However, even if it explicitly shows $\Omega\neq 1$ in the model (this is also an reduction of the O'Connell's model), this result, $n=s_{S/Y}/\Omega$, is not well accepted by the author's empirical study which uses a panel data approach.

The improved two-sector model: under the corporate-financed growth steady-state

1. The growth rate of output, g_Y, depends on the undistributed profit propensity to save, $s_{SP/Y}$. This is highly supported by the same above empirical study.
2. Review of the contents and of the two growth rates of output and capital: g_Y and g_K. Even if both growth rates are endogenously the same, the relationship between the wage and dividend propensity to save, $s_{SWD/Y}$, and the undistributed profit propensity to save, $s_{SP/Y}$, is clarified. The value of $s_{SWD/Y}$ is shown using $s_{SP/Y}$ and the capital-output ratio or the value of $s_{SWD/WD}$ is shown using $s_{SP/P}$ and the relative share of profit.
3. The investment ratio which is defined as net investment divided by output, I/Y^0, is expressed using the growth rate, $g_Y=g_K$, the capital-output ratio Ω, and the relative share of profit π (Equation 21). The inequality, $s_p>\frac{I}{Y^0}>s_w$, which had been discussed in 1960s, is clarified and proved (released from an assumption set by Pasinetti).
4. The structure of labour productivity is easily introduced into the model. The Harrod-Domer model under $n=g_Y=g_K$ introduces this structure by replacing n with the sum of the population growth rate and the growth rate of labour productivity. However, the above structure clarifies the relationship between the coefficient of technological progress, m* and the growth rate of labour productivity in terms of the above investment ratio I/Y^0.
5. The improved two-sector model does not depend on the marginal productivity of capital and the marginal utility of consumption. The financial structure of products assumes that the average productivity of capital equals the marginal

productivity of capital, but this is guaranteed under constant capital-output ratio. Also, the propensities to consume are expressed as dual to the propensities to save which are measured under the constant relative share of profit and the constant undistributed profit propensity to save.

1.2 Review of the two/one class growth model

Let the author start with **O'Connell [1995]**, where two classes (types) of people are treated. This model will be reviewed in the following sections.

1. Capitalists who do not supply any labour and owns most of capital stock
2. Workers who supply labour but also own part of shares

The typical household supplies his labour as worker but also owns the shares of corporate firms.

For comparison, in the two-sector model of growth, the author assumes that under current financial structure there are some incentives and constraints built within the system related to the fraction of dividends saved by the household. In particularly, that arrangement forces household to separate their decision on savings of dividends from their decision on savings of wages.

The framework is shown by definitions, assumptions, and propositions which leads to the solution step by step as follows:

Notations:

Y denotes net national income

W denotes disposable income of workers

W_w denotes wage income of workers

P denotes corporate profit

P_w denotes profit owned by workers

P_c denotes profit owned by capitalists

K denotes corporate capital stock

K_w denotes capital owned by workers

K_c denotes capital owned by capitalists

S_p denotes undistributed profit from profit

S_{ww} denotes wages saved by workers

S_{pw} denotes undistributed profit owned by workers

S_{pc} denotes undistributed profit owned by capitalists

S_{dw} denotes dividends saved by workers

S_{dc} denotes dividends saved by capitalists

D denotes total dividends from profit

D_w denotes dividends distributed to workers

D_c denotes dividends distributed to capitalists

C_{dw} denotes dividends consumed by workers (this is not used directly)

C_{dc} denotes dividends consumed by capitalists (this is not used directly)

n is denotes the population growth rate

The rate of profit (r) is defined as $r \equiv P/K$

The retention ratio (s_p) is defined as S_p/P

The fraction saved of capitalists' dividend income (s_c) is defined as $s_c \equiv S_{dc}/D_c$

The fraction saved of workers' wages (s_{ww}) is defined as $s_{ww} \equiv S_{ww}/W_w$

The fraction saved of workers' dividends (s_{dw}) is defined as $s_{dw} \equiv S_{dw}/D_w$

Note: when a one class model is introduced as capitalists' values are zero, the subscript "c" is deleted.

The balanced growth steady-state is defined as a condition such that the growth rates of population (n), capital stock K, and output Y are the same (as stated above). In other words, $n = \frac{\dot{K}}{K} = \frac{\dot{Y}}{Y}$.

Let the author also list the following identities (definitions).

1. $Y \equiv W + P$
2. $W \equiv W_w + P_w$
3. $K \equiv K_w + K_c$
4. $P \equiv P_w + P_c$

5. $P \equiv S_p + D$

6. $S \equiv S_w + S_c$

7. $P_w \equiv S_{pw} + D_w$

8. $P_c \equiv S_{pc} + D_c$

9. $S_p \equiv S_{pw} + S_{pc}$

10. $S_w \equiv S_{ww} + S_{pw} + S_{dw}$

11. $S_c \equiv S_{pc} + S_{dc}$

12. $D \equiv D_w + D_c$

13. $D_w \equiv S_{dw} + D_{cw}$

14. $D_c \equiv S_{dc} + D_{cc}$

Assumption 1 Profit is distributed according to a fixed proportion (r) of the ownership of capital stock, where the fixed proportion equals the rate of profit. In other words, $\frac{P_c}{K_c} = \frac{P_w}{K_w} = \frac{P}{K} = r.$

Assumption 2 Dividends are paid in proportion to the ownership of capital stock. In other words, $\frac{D_c}{K_c} = \frac{D_w}{K_w} = \frac{D}{K}.$

Assumption 3 Workers save a fixed proportion of wages and also save the same proportion of dividends. In other words, $s_w = \frac{S_{ww}}{W_w} = \frac{D_{sw}}{D_w}.$

Additional denotations are needed as follows:

Changes in the capitalists' capital stock by $\dot{K}_w$

Changes in the capitalists' capital stock by $\dot{K}_c$

Changes in the total capital stock by $\dot{K}$

Now, related variables, equations, and the solution are stated, following basically O'Connell's [1995] frameworks and steps and using my own above notations which are also used for the two-sector model of growth under one class. These are shown with corresponding equation numbers of her (with bold) as follows:

The model's endogenous variables are: S_c, S_w, $\dot{K}_w$, $\dot{K}_c$, $\dot{K}$, n, r

The following equations describe relationships among those variables.

1. $S_{dc} = f_1(r) = s_c(1 - s_p) r K_c$

2. $S_{ww}+S_{dw}=f_2(r)=s_w(Y-rK)+s_w(1-s_p)rK_w$

3. $\dot{K}=f_3(\dot{K}_c, \dot{K}_w)=\dot{K}_c+\dot{K}_w$

4. $\dot{K}_c=f_4(r)=s_c(1-s_p)rK_c+s_prK_c$

5. $\dot{K}_w=f_5(r)=s_w(Y-rK)+s_w(1-s_p)rK_w+s_prK_w$ Equation (**5**, p.364)

$\dot{K}=f_3(r)=s_w(Y-rK)+s_w(1-s_p)rK_w+s_c(1-s_p)rK_c+s_prK$

Equation (**6**, p.364, as a property of $\dot{K}=f_3(\dot{K}_c, \dot{K}_w)=\dot{K}_c+\dot{K}_w$)

6. $n=f_6(\dot{K})=\dot{K}/K$: this describes the balanced growth condition.

By solving the above equations 5 and 6, the following quadratic function is derived:

$a\{s_w+s_pZ(1-s_w)\}r^2-\{aZ+s_w+s_pZ(1-s_w)\}nr+n^2Z=0$ Equation (**8**, p.364)[4)]

where, $a\equiv s_c(1-s_p)+s_p$ and $Z\equiv rK/Y$

From this quadratic function, O'Connell finds two solutions of r: r_1 and r_2.

$r_1=f_7(n)=n/\{s_p(1-s_p)+s_p\}=n/a$, where $a\equiv s_c(1-s_p)+s_p$

$r_2=f_8(n)=nZ/\{s_w+s_pZ(1-s_w)\}$, where $Z\equiv rK/Y$

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4) The values of r_1 and r_2 are obtained using the following property of Equations (**5**) and (**6**). This is shown as a quadratic function .

$a\{s_w+s_pZ(1-s_w)\}r^2-\{aZ+s_w+s_pZ(1-s_w)\}nr+n^2Z=0$ Equation (**8**, p.364)

Now, if $K_c/K=0$ or $s_c=0$, then, the above Equation (**3**) reduces to $s=s_w+s_p(1-s_w)Z$. Since $Z=rs/n$, s is given as follows:

$$s=\frac{s_w}{1-s_p(1-s_w)r/n}=\frac{ns_w}{n-rs_p(1-s_w)}$$

If this s is introduced into the above Equation (**10'**), r_2 is proved to equal r as follows:

$$r_2=n(s-s_w)/ss_p(1-s_w)=\frac{rs_ps_w(1-s_w)}{s_ps_w(1-s_w)}=r$$

$r_2=nZ/\{s_w+s_pZ(1-s_w)\}$ Equation (**10**, p.364)

$r_2=n(s-s_w)/b$, where $b=ss_p(1-s_w)$ under $K_w/K=1$ and $K_c/K=0$ Equation (**10'**, p.364)

It implies that $r=r_2$ if $K_c=0$ or $s_c=0$ in the two/one-class model. This result is another expression of her own proof that $r=r_2$ if $K_c=0$ or $s_c=0$ using other equations (1, 7, and the above 10). The two sector model under $K_c=0$ follows her Proposition 2: $r=r_2$. The condition "anti-Pasinetti states prevail under $r=r_2$" was already proved in the previous section by obtaining the same full employment Harrod-Domar condition:

$r_2=r=\dfrac{n-s_w\frac{Y}{K}}{s_p-s_w}$. The only difference between two models was whether $\Omega=1$ or $\Omega\neq 1$.

The relationship between r, r_1, and r_2 was well analysed using Z (O'Connell, p.365) with the above equations.

1. $r=r_1<r_2$, $K_c>0$: Pasinetti equilibrium $s_w(1-s_pZ)<s_cZ(1-s_p)$
2. $r=r_1=r_2$, $K_c=0$ and $K_w=K$: Anti-Pasinetti states $s_w(1-s_pZ)=s_cZ(1-s_p)$
3. $r=r_2$, Anti-Pasinetti states $s_w(1-s_pZ)>s_cZ(1-s_p)$

It is stressed that if $K_w/K=1$ and $K_c/K=0$, then r equals r_2.

The results are, according to O'Connell [pp.364–365], shown as propositions 1 and 2 as follows:

Proposition 1 If $s_p=0$, then, $r_1=n/s_c$

The value of r_1 is the rate of profit derived by Pasinetti [1962] for the two-class economy, and $r=r_1$ implies that K_c/K is constant in the balanced-growth steady state.

Proposition 2 If $K_c=0$ ($K=K_w$), $Y/K=n/s$ and $r=r_2$

This is under $\dot{K}=s_wY+s_p(1-s_w)rK$; the full employment Harrod-Domar condition is obtained.

2. Contents of the two-sector model under the balanced growth steady-state

Next, let the author explain the two-sector model of growth using the same notations and assumptions as much as possible. However, the following three notations and definitions are tentatively added for simplicity even in the balanced growth steady-state:

C, defined as consumption of capital gains which is the amount derived from the difference between household savings S_w and "corporate investment $\dot{K}$ less undistributed profit S_p."

c, defined as the ratio of consumption of capital gains to capital gains (parameter)

G, defined as capital gains and is measured by C/c.

Now, assumptions of the author's are as follows:

Assumption 1 One class, namely worker class, is assumed: $P_c=K_c=0$. As a result, $S_{pc}=D_c=0$ and $s_c=0$. Also, capital gains are owned only by workers.

Assumption 2 Dividends saved by workers D_{sw} are zero.

This assumption is *tentative* for simplicity (needed for shifting a quadratic equation to a linear equation) and depends on "asset effect of capital gains." It is replaced by a statement that the consumption of capital gains and dividends is larger than savings of capital gains and dividends although asset effect does not increase output.[5] This is justified empirically, according to my empirical work [Kamiryo, 1996]. As a result, it is replaced, under no introduction of capital gains, by a statement that savings of dividends, D_S, are zero or dividends are all consumed; $D=D_{consum}$.

Therefore, dividend savings can be zero. This is an only difference between the O'Connell's model and the authors'. Assumption 2 is necessary to obtain a common solution for both models in the balanced growth steady-state. This assumption is withdrawn later in the corporate-financed growth steady-state and replaced by Assumption 3 (workers save a fixed proportion of wages and also save the same proportion of dividends) which was set by O'Connell. The framework is now shown much more simply.

1. $Y=W+P$
2. $W\equiv W_w+P_w$
3. $K\equiv K_w$
4. $P=P_w$
5. $P=S_p+D$
6. $S=S_w+S_p$

5) Masao Kawano [1997, April] proved the framework behind capital gains in terms of national accounts. Also, Kazuhiko Nishina showed the relationship between asset effect and capital gains in terms of consumption. The author is much obliged to the two professors.

7. $P_w=S_{pw}+D_w$

8. $S_p=S_{pw}$

9. $S_w=S_{ww}+S_{pw}$

10. $D=D_w=D_{consump}$

The framework is shown using the following parameters, variables, and equations, and the solution which corresponds with $r=r_2$ proved by O'Connell.

Parameters:[6)]

Y, defined as national income$\equiv$W+P

W, defined as household income

P, defined as corporate profit

s_p, defined as the retention ratio of corporate profit

s_w, defined as the fraction saved of household income

r, defined as the rate of profit in the balanced growth steady-state

Ω, defined as the capital-output ratio and measured as by K/Y (Y/K is defined as the average productivity of capital)

Variables:

S_p, defined as corporate undistributed profit

S_w, defined as household savings

$\dot{K}_p$, defined as changes in corporate capital stock by undistributed profit$\equiv S_p$

$\dot{K}_w$, defined as changes in corporate capital stock by household savings$\equiv S_w$

$\dot{K}$, defined as changes in the aggregate stock of corporate capital

n, defined as the natural rate in *the balanced growth steady-state* and measured by $\dot{K}/K$

Equations:

1. $S_p=f_1(r)=s_p rK$

2. $\dot{K}_p=S_p$

6) It is allowed not to set Y, W, and P as parameters if s_p, s_w, r, and Ω are already parameters.

3. $S_w = f_3(r) = s_w(Y - rK)$

4. $\dot{K}_w = S_w$

5. $\dot{K} = f_5(\dot{K}_p, \dot{K}_w); f_5(\dot{K}_p, \dot{K}_w) = \dot{K}_p + \dot{K}_w$ (or $\dot{K} = \dot{K}_p + \dot{K}_w$)

6. $f_6(\dot{K}) = \dot{K}/K$; $n = f_6(\dot{K})$

As implicit functions: (S_p, S_w, $\dot{K}_p$, $\dot{K}_w$, $\dot{K}$, n)

1.	$F_1(\dot{K}, S_p)=0$	$F_1(x_1, x_3)=0$
2.	$F_2(r, \dot{K}_p)=0$	$F_2(x_3, x_7)=0$
3.	$F_3(\dot{K}_w, S_w)=0$	$F_3(x_2, x_4)=0$
4.	$F_4(r, \dot{K}_w)=0$	$F_4(x_4, x_7)=0$
5.	$F_5(\dot{K}_p, \dot{K}_w, \dot{K})=0$	$F_5(x_3, x_4, x_5)=0$
6.	$F_6(\dot{K}, n)=0$	$F_6(x_5, x_6)=0$

How to solve:

7. By 2, solve for $\dot{K}_p$: $\dot{K}_p = s_p K r$

8. By 4, $\dot{K}_w = s_w Y - s_w r K$

9. By 5, $\dot{K} = s_p K r + s_w Y - s_w r K$ $\quad \frac{\dot{K}}{K} = s_p r + s_w \frac{Y}{K} - s_w r$

10. By 6 and 9, the solution is given as $n = s_p r + s_w(\frac{Y}{K}) - s_w r$

(1) or, $n = \frac{\dot{K}}{K} = s_p \cdot r + s_w(\Omega - r)$

The value of n is shown only using parameters, r, s_p, s_w, Ω, where $\Omega \equiv K/Y$:

$n = \varphi(r, s_p, s_w, \Omega)$

This solution coincides with the result of O'Connell which reduces to the Harrod-Domar condition if $K_c=0$ and if $\Omega=1$ as proved below:

1. The two/one class model

$Y/K = n/s$, as earlier proved by Harrod [1973].

The value of s was proved by O'Connell [1985, p.116] as,

$$s = s_w + s_p(1-s_w)Z + (s_c - s_w)(1-s_p)ZK_c/K \text{ and } Z = rK/Y$$

Harrod's value was proved by O'Connell [1995, p.366] as,

$$s = s_w + s_p(1-s_w)r(K/Y) + (s_c - s_w)(1-s_p)r(K_c/Y)$$

If $=s_c=0$, then, $s_w+s_p(1-s_w)r(K/Y)$

Therefore, $Y/K=n/\{s_w+s_p(1-s_w)r(K/Y)\}$

or $\mathbf{n=s_w+s_p(1-s_w)r}$

2. The two-sector model

$\mathbf{n=r(s_p-s_w)+s_w(Y/K)}$ as shown above solution (10)

If $Y=K$, then, $n=s_w+s_p(1-s_w)r$ is obtained which is exactly the result of her model.

This shows that my model holds if $Y/K=K/Y=1$ under Harrod-Domar model.

Propositions using the solution

The above solution presents some propositions at once using partial derivatives. The propositions 1-4 below are a little "general" since they express K/Y under $K_c=0$. These propositions are specified below by using an assumption $\Omega=1$ which characterises the two-sector model under Harrod-Domar condition.

Starting with the solution $n=s_pr+s_wY/K-s_wr=r(s_p-s_w)+s_w/\Omega$ or $n=\varphi(r, s_p, s_w, \Omega)$,

Partial derivatives of n with respect to each parameter are as follows:

1. $\frac{\partial n}{\partial r}\geq 0$ iff $s_p\geq s_w$
2. $\frac{\partial n}{\partial s_p}=r\geq 0$
3. $\frac{\partial n}{\partial s_w}=-r+\frac{Y}{K}\geq 0$ iff $Y/K\geq r$
4. $\frac{\partial n}{\partial \Omega}=\frac{-s_w}{\Omega^2}<0$

Propositions are stated as follows:

Proposition 1 Changes in the growth rate n in response to changes in the rate of profit r depends on the relative magnitude of s_p and s_w.

Proposition 2 Changes in the growth rate n in response to changes in the retention ratio s_p is larger only if $s_p>s_w$. (if $s_p<s_w$, then $\partial n/\partial r<0$)

Proposition 3 Changes in the growth rate n in response to changes in household propensity to save s_w depends on the relative size of r and $1/\Omega$.

Proposition 4 Changes in the growth rate n in response to changes in Ω is negatively related to s_w and Ω^2.

Proposition 1 implies that the larger the corporate retention ratio the larger the growth rate. This is justified in the real world since the household propensity to save is low and stable compared with the corporate retention ratio.

Proposition 2 shows the relationship between s_p and s_w in terms of n. This relationship was discussed using an inequality, $s_w<I/K<s_p$, by Kaldor [1955-56, 1962] and Pasinetti [1966].

Proposition 3 is new since the two-sector model shows Y/K (the average productivity of capital) or K/Y (the capital-output ratio) as a parameter. Since Y/K is considerably larger than the rate of profit in the real world, this proposition implies that the growth rate depends on the productivity of capital.

Proposition 4 implies that the larger the growth rate the smaller the household propensity to save s_w. It also implies that the larger the growth rate the smaller the capital-output ratio Ω. This may come from the accumulation of capital. Capital stock needs a suitable level for the growth rate: if capital stock is too small ($\Omega<1$), then the growth rate is large, but at the sacrifice of full employment, and vice versa. It is noted the higher the population growth rate, the lower the improvement of human capital which discourage investment based on human capital and technological progress.[7)]

These results are consistent with some well-known propositions. However, an assumption is needed if the two-sector model completely corresponds with one case of the two/one class model. This assumption is "if $\Omega=1$." It is clear that if $\Omega=1$

7) Of course, this is also related to the contents of population and the population growth rate. As proposed by Lucas [1988], technological progress is accelerated when a higher proportion of workers' time is spent in education.

$n=g_Y=g_K$ and thus Harrod-Domar's model is right. Furthermore, this will be extended in a separate paper which discusses the relationship among economic depreciation/capital consumption, the growth rate, the costs of capital, and the rate of profit. Let the author now discuss the solution of $n=r(s_p-s_w)+s_w$ whose $\Omega=1$. Partial derivatives of n with respect to each parameter are as follows:

1. $\frac{\partial n}{\partial r}\geq 0$ iff $s_p\geq s_w$

2. $\frac{\partial n}{\partial s_p}=r\geq 0$

3.* $\frac{\partial n}{\partial s_w}=-r+1\geq 0$ (3*≠above 3)

4.* $\frac{\partial n}{\partial \Omega}=-s_w<0$ (4*≠above 4)

Also, $\mathbf{n=r(s_p-s_w)+s_w(Y/K)}$ is rearranged under $\Omega\neq 1$ and $\Omega=1$ as follows:

Under $\Omega\neq 1$

$$\frac{Y}{K}=\frac{n-r(s_p-s_w)}{s_w}$$

$$s_p=\frac{n-(\frac{Y}{K}-r)s_w}{r}$$

$$s_w=\frac{n-rs_p}{\frac{Y}{K}-r}$$

Under $\Omega=1$

$$1=\frac{n-r(s_p-s_w)}{s_w}$$

$$s_p=\frac{n-(1-r)s_w}{r}$$

$$s_w=\frac{n-rs_p}{1-r}$$

Propositions 3 and 4 are simplified under the full-employment Harrod-Domar condition as follows:

Proposition 3* Changes in the growth rate n in response to changes in household propensity to save s_w depends on the magnitude of $1-r$.

Proposition 4* Changes in the growth rate n in response to changes in Ω is negatively related to s_w.

Also, the following propositions are shown **if Ω is one.**

Proposition 5* If Ω is equal to one, then $n=g_Y=g_W=g_P=g_K$ in the balanced growth steady-state. The value Ω generally defined *as constant* in the balanced growth steady-state is now specified *as one* by this proposition.

Proposition 6* If Ω is equal to one, then the average productivity of capital APK equals the marginal productivity of capital MPK: **APK=MPK=1.**

Proposition 7* If Ω is equal to one, then the relative share of corporate profit to net national income π equals the rate of profit r:

$$\pi=\frac{P}{Y}=r=\frac{P}{K}\text{, since } Y=K$$

Propositions 1, 2, 3*, and 4* are now expressed using π as a parameter instead of r. Proposition 7* simplifies a crucial problem in economic debates between π and r.

3. The two-sector model under the corporate-financed growth steady-state

3.1 Direction of takeoff

The two-sector model of growth started with the two/one growth model which O'Connell developed based on the Harrod-Domer model. The results of this two-sector model do not contradict with the two/one growth model. A contribution may lie in a separation of $\Omega \neq 1$ from $\Omega=1$. However, both models are based on the balanced growth steady-state, whose condition is such that the growth rate of output, the growth rate of capital, and the population growth rate are the same. The solution was:

(1) $n=s_p r+s_w Y/K-s_w r=r(s_p-s_w)+s_w/\Omega$

Equation 1 reduces to $n=s_{SP/Y}/\Omega$, where $s_{SP/Y}\equiv S_P/Y$. It implies that $n=s_{SP/Y}/\Omega$ holds if $n=g_Y=g_K$. However, it leads to a simple equation that $g_Y=g_K=s_{SP/Y}$, and it is difficult to support both models by empirical works (see a panel data approach below). The two-sector model can easily release its condition by separating the population growth rate from the other two. This is an intention of this section.

If the growth rate of output g_Y equals the growth rate of capital g_K, and also they equal the population growth rate n (under the balanced growth steady-state), then, propositions stated in this paper hold. However, under $g_Y \neq g_K \neq n$, these propositions do not hold. How is this condition changed to approach the real world? There are three steps to improve the model by setting;

1. the population growth rate g^{e}_{NE} as a constant/parameter ($n \neq g^{e}_{NE}$)
2. the capital-output ratio Ω as a constant/parameter whose value differs from one ($\Omega \neq 1$)
3. or the relative share of profit π as a constant/parameter.

As a result, two conditions are clarified as follows:

1. $g_Y = g_K$ under constant Ω and π
2. $g_Y \neq g_K$ under constant π (Ω varies)[8)]

$g_Y = g_K \neq g^{e}_{NE}$ is called a condition "under the corporate-financed growth steady-state," in equilibrium. What conditions (assumptions) are needed for $g_Y = g_K$? This is discussed in this paper using the theoretical real financial structure of products as a clue of national accounts in discrete time. The expected real financial structure of products in disequilibrium, where $g_Y \neq g_K$, holds if the capital-output ratio varies under less sufficient assumptions (one of three sufficient conditions, $g_Y = c_{DI}$, is excluded). The expected real financial structure of products converges to the theoretical real financial structure of products as discussed in another paper.

3.2 Additional notations

The financial structure of products is **first** illustrated as a version for the improvement of the two-sector model which belongs to a one class model. Second, additional notations, and third, necessary and sufficient conditions, are intentionally specified as eight assumptions. The financial structure of products is distinguished from that in the two/one class model of O'Connell's. However, this structure was de-

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8) This condition needs the introduction of the coefficient of technological progress between the investment ratio and the growth rate of labour productivity (see below; also Kamiryo [1997/Oct]). It is noted that this coefficient is measured assuming that Ω and π are constant even after they have changed by net investment and accordingly the investment ratio.

veloped after reviewing the two/one class model and by thoroughly clarifying relationships between parameters and variables in a one class model. The structure is composed of three parts; the initial given (predetermined) values, unknown real values, and the difference. Real values imply those under fixed price level. The initial values at the end of a period are originally nominal, but as a base for unknown real values which are obtained under eight assumptions below and after one period. Thus, for two periods, values are compared as real.

Initial given (predetermined) values:

W^0	$W^0=S^0_W+C^0_W$	$C^0=C^0_W+C^0_{DI}$
$P^0=D^0_I+S^0_P$	$D^0_I=S^0_{DI}+C^0_{DI}$	$S^0_{WD}=S^0_W+S^0_{DI}$
	$S_P=\Delta K_{SP}$	$S^0=S^0_P+(S^0_W+S^0_{DI})$
$Y^0=W^0+P^0$		$Y^0=C^0+S^0$
	$K^0_P=K^0_{SP}+K^0_{WD}$	
	$K^0_P=K^0_N-K^0_{W+G}$	

Unknown real values

W	$W=S_W+C_W$	$C=C_W+C_{DI}$
$P=D_I+S_P$	D_I $\quad D_I=S_{DI}+C_{DI}$	$S_{WD}=S_W+S_{DI}=\Delta K_{WD}$
	$S_P=\Delta K_{SP}$	$S=S_P+(S_W+S_{DI})=\Delta K_P$
$Y=W+P$		$Y=C+S$
	$K_P=K_{SP}+K_{WD}$	$\Delta K_P=\Delta K_{SP}+\Delta K_{WD}$
	$K_P=K_N-K_{W+G}$	$\Delta K_P=\Delta K_N-\Delta K_{W+G}$

Increase of each value and the same endogenous growth rate:

e.g., for Y; $\Delta Y\equiv Y-Y^0$ $\quad g_Y=\dfrac{Y-Y^0}{Y^0}$

$g_Y=g_{KP}$ $\quad g_Y=g_W=g_P=g_{SP}=g_{DI}$ $\quad$ under constant $\pi=\Omega_P\cdot\rho$

$g_{KP}=g_{KSP}=g_{KWD}$

Let the author **second** denote and define the propensities to save and other variables to improve the two-sector model of the author's. Released from the two-one model, some notations are simplified and others takes into consideration additional concepts:

1. Propensities to save for Y:

$s_{SP/P}\equiv S_P/P$ $s_{SP/Y}\equiv S_P/Y$ $s_{SWD/WD}\equiv(S_{SW}+S_{DI})/(W+D_I)$ $s_{SWD/Y}\equiv(S_{SW}+S_{DI})/Y$

$s_{S/Y}\equiv(S_P+S_{SW}+S_{DI})/Y=S/Y$ $s_{S/Y}=s_{SP/Y}+s_{SWD/Y}$

These propensities to save in the two-sector model should be connected with $s_p\equiv S_p/P$ and $s_w\equiv S_w/W$ in the two/one model: s_p corresponds with $s_{SP/P}$ and s_w corresponds with $s_{SWD/WD}$. The improved two-sector model takes advantage of the relationship between propensities to save by using the capital-output ratio and the relative share. Propensities to save whose denominator is Y are additive and convenient.

2. D_I is defined as dividends paid by the corporate sector. Dividends paid and dividends received are offset as a nation, but D_I plays an important role in a national economy in terms of the retention ratio $s_{SP/P}$ and the investment ratio $I/Y^0\equiv\Delta K_P/Y^0$. The model intends to reveal this role which D_I plays.[9]
3. K_P is defined as corporate capital, K_{W+G} is defined as household and governmental capital, and K_N id defined as capital as a nation: $K_N=K_P+K_{W+G}$. This paper treats K_P (which is unique in this paper), but the ratio of K_P/K_N influences the growth rate of output. This is shown empirically. K_P is also defined as the sum of K_{SP} (which comes from internal savings; undistributed profit) and K_{SWD} (which comes from external savings).
4. The corporate capital-output ratio Ω_P is used instead of Ω in the two/one class model. Ω_P is defined as $\Omega_P\equiv\Omega_N\cdot\frac{K_P}{K_N}$ and the national capital-output ratio Ω_N corresponds with Ω in the two/one class model. For a sustainable growth of output, the larger the K_P/K_N and the smaller the Ω_P, the better. For social capital and welfare, a sustainable growth is sacrificed to some extent.

9) Economic depreciation is defined as capital consumption and denoted as $D_{EP:E}$. Accounting depreciation is denoted as $D_{EP:A}$. These are discussed together with the cost of profit, the rate of profit, and the discount rate below. Thus, notation, D, is not used for dividend.

5. The relative share of profit is expressed as ρ instead of r (which was used in the two/one class model and the old two-sector model). This is because the improved two-sector model takes into consideration the discount rate, $r_{SP}=r_{DI}=r_P=r-g_Y$, the cost of capital, r, and the dividend valuation value and its valuation ratio (see below), where $\rho\equiv\frac{P}{K_P}=\frac{P^0}{K_P^0}$, $r\equiv\frac{P}{K_P^0}$, and $r=\rho(1+g_{KP})$ by time difference since $K_P=K_P^0(1+g_{KP})$.
6. Surplus of the nation buries the difference between savings and net investment. The model is designed as a closed system, but surplus of the nation is at once reviewed in terms of the investment ratio and the coefficient of technological progress.
7. The population growth rate or the growth rate of workers is denoted as g_{NE}^e instead of n in the two/one class model. g_{NE}^e is a parameter while n is a variable which equals the growth rates of output and capital; $n=g_Y=g_K$.
8. The coefficient of technological progress, m*, and its growth rate, g_m^*, are introduced between the investment ratio, I/Y^0, and the growth rate of labour productivity, $g_{Y/NE}$:

$$m^*\equiv\frac{g_{Y/NE}}{\Delta K_P/Y^0} \quad \text{and} \quad g_m^*\equiv\frac{m^{*1}-m^{*0}}{m^{*0}}$$

Without taking into consideration m* and g_m^*, savings calmly equal net investment and there is no room for disturbing this equality. Instead of output maximization, a sustainable growth of labour productivity is aimed as a target and supported by capital-saving (augmenting) technological progress.

The two-sector model of growth adds the above notations and definitions for its improvement.

3.3 Most fundamental idea and assumptions

The most fundamental idea behind assumptions is **first** stated as follows: This idea is how to establish the relationship between the capital-output ratio Ω_P, the retention ratio $s_{SP/P}$ (or the undistributed profit propensity to save $s_{SP/Y}$), and the growth rate of output g_Y which equals the growth rate of corporate capital g_{KP}.

1. The ratio of dividends to undistributed profit is shown as a ratio of $\Phi \equiv (1-s_{SP/P})/s_{SP/P}$, where $s_{SP/P}$ is the retention ratio.
2. The net present value of future undistributed profit in an infinite periods of time equals the initial corporate capital K^0_P, which is required for maintaining capital. Dividend payment assumes to be allowed after capital maintenance.
3. The growth rate of output g_Y is defined as a minimum rate needed for capital maintenance. Then, the numerator of the growth rate of output is required to be undistributed profit.
4. The ratio of dividends to the initial capital is defined as one after capital maintenance in an infinite periods of time: $c_{DI} \equiv D_I/K^0_P$.
5. What is the relationship between Ω_P and Φ? Or, what is the relationship between g_Y and c_{DI}? Let the author show an equality, $\Omega_P \equiv \frac{K^0_P}{Y^0} = \frac{D_I}{S_P} \equiv \Phi$

If $\Omega_P > \Phi$, then, $g_Y > c_{DI}$.

If $\Omega_P = \Phi$, then, $g_Y = c_{DI}$.

If $\Omega_P < \Phi$, then, $g_Y < c_{DI}$.

Conclusively speaking, the relationship between Ω_P and Φ converges $\Omega_P = \Phi$, and, $g_Y = c_{DI}$ by assuming a sustainable balance between capital maintenance and dividend payment. What does $g_Y = c_{DI}$ imply? This shows that the growth rate of output for capital maintenance, g_Y, equals the ratio of dividends to the initial capital which corresponds with g_Y [10) and implies a condition at a point of balance between capital maintenance and dividend payment.

6. Thus, an underlying equality is shown as $\Omega_P = \Phi$. This leads to

$$\Omega_P = \frac{1-s_{SP/P}}{s_{SP/P}} \quad \text{or} \quad s_{SP/P} = \frac{1}{\Omega_P + 1}. \tag{1}$$

This relationship constitutes a most fundamental base for the financial structure

10) The concept of c_{DI} is directly connected with the initial capital. The concept of the dividend cost of capital, r_{DI}, is directly connected with the valuation of dividend $V^0_{DI} \equiv D_I/(r-g_Y)$, where $c_{DI} = r - r_{DI}$, and $r_P \equiv P/K^0_P$ and $r_{DI} = r - g_Y$ (see valuation below).

of products and enables the model to measure variables endogenously in discrete time. As a result, the model does not need any help from such as the marginal productivity of capital and the marginal utility theory[11)] whose measurement is not easy.

The same growth rate in the financial structure of products does not require the use of the above theories which are based on continuous time. Instead, the same growth rate, $g_Y = g_K$, specifies the relationship between propensities to save as will be shown below. In discrete time, the structure of average and marginal [Kamiryo, 1989] is used. Also, it is possible differently to introduce the elasticity of substitution using the relationship [Allen, 1975] between the Laspeyres quantity index and the Paasche price index.

Finally, what is the relationship between the population growth rate (the growth rate of workers/employees), g^e_{NE}, and the financial structure of products? The financial structure does not necessarily need g^e_{NE} in a narrow sense, but it needs g^e_{NE} in a broad sense to complete the financial structure.

In detail, this g^e_{NE} is basically given as a parameter, and also can be expressed as a function of the growth rate of output. This is needed for expressing more freely the balanced growth steady-state where $n = g_Y = g_K$ in literature. The number of workers N_E and its growth rate g^e_{NE} are not directly related to the financial structure of products. However, these are needed when labour productivity, the growth rate

11) e.g., Bertola [1993, p.1196, p.1197], after discussing marginal utility of consumption, points out as follows:

1. A relaxation of the representative-individual assumption to allow for heterogeneous income sources reveals a striking similarity between these models and post-Keynesian models of income distribution and growth, where different saving propensities for different classes of income earners were assumed rather than derived from utility-maximizing behavior.

2. Growth will tend to be fast, in the absence of (lumpsum) redistribution, when political attention is focused on the investment-enhancing policies suggested by the recent literature on endogenous growth at suboptimal rates.

of labour productivity, and the coefficient of technological progress are introduced to complete the model under the balanced growth steady-state.

Second, necessary and sufficient conditions are shown as assumptions and as follows:

Assumption 1 Capital is owned by workers (the same as Assumption 1 in the balanced growth steady-state).

It is justified in order to show, more simply and explicitly, the undistributed profit propensity to save $s_{SP/Y}$ in the model. It implies that "two-sector" is more important than "two/one class."

Assumption 2 The initial basic values equal those at the end of the previous period and are all known. The changes of these values in the following one period are unknown, but determined endogenously by the initial values and under some assumptions.

Assumption 3 Savings equal net investment **by** sector. This paper treats the relationship between savings (used for corporate net investment) and corporate net investment. The savings (used for household and government net investment) and the household and government net investment are equal. In an open economy, the surplus of the nation in national accounts corresponds with the balance between savings and net investment as well as national budget deficit.

Assumption 4 In any two periods under discrete time, the capital-output ratio Ω_P is a constant,[12] and the relative share of profit π is a constant. These constitute necessary conditions for the model. As a result, the rate of profit ρ is also a constant, where $\pi=\Omega_P \cdot \rho$.

Assumption 5 In any two periods under discrete time, all kinds of propensities to save (or consume) are constant. This condition is more strict than a condition that

12) It generally implies that the marginal productivity of capital, MPK, equals the average productivity of capital, APK (see Proposition 6*). The author states the structure of average and marginal as Equation 5.

the relative share of profit π is constant.

By Assumptions **4** and **5**, all kinds of growth rates are the same, and the componendo theorem (see below) holds. Also, the financial structure for two periods is shown as real.

Assumption 6 The increase of output (real net national income) ΔY equals undistributed profit S_P.

It implies that this increase is minimum for capital maintenance. Why is this justified? This is because the growth rate of output g_Y equals the growth rate of corporate capital g_{KP} only under this condition. If ΔY equals the sum of undistributed profit and the savings which come from the household sector, then, $g_Y \neq g_{KP}$.

Assumption 7 The growth rate of output $g_Y \equiv \frac{\Delta Y}{Y^0}$ equals the ratio of dividends to the initial capital $c_{DI} \equiv \frac{D_I}{K_P^0}$.

It implies a unique balance between undistributed profit and dividend payment under capital maintenance. If $g_Y > c_{DI}$, then, capital maintenance is more important than dividend payment, and if $g_Y < c_{DI}$, then, dividend payment is more important than capital maintenance. As a result, finally, $g_Y = c_{DI}$ is derived and justified.

Under Assumptions 6 and 7, dividends D_I equal corporate net investment ΔK_P, and also corporate capital which comes from as an accumulation of undistributed profit, K_{SP}, equals output Y: If $\Delta Y = S_P$ and $g_Y = c_{DI}$, then, $D_I = \Delta K_P$ and $K_{SP} = Y$ (see below). These three conditions constitute **sufficient** conditions of the model. Assumptions 4 and 5, where $\pi = \Omega_P \cdot \rho$ are constant in any two periods, constituted **necessary** conditions. The second and third sufficient conditions, $D_I = \Delta K_P$ and $K_{SP} = Y$, are proved below.

Assumption 8 The wages propensity to save $s_{SW/W}$ equals the dividend propensity to save $s_{SDI/DI}$: $s_{SW/W} = s_{SDI/DI} = s_{SWD/WD}$.

From savings: $s_{SWD/WD} = s_{SWD/Y} / (1 - s_{SP/Y})$

or, from consumption: $c_{SWD/WD} = (1 - s_{SWD/Y} - s_{SP/Y}) / (1 - s_{SP/Y})$

The denominators are not Y, but each W and D_I: $s_{SW/Y} \neq s_{SDI/Y}$. This is a supplemental condition needed for maintaining the same growth rate for each item. This is the same as Assumption 2 in the two/one class model. However, the improved two-sector model specifies the relationship between propensities to save by this assumption as will be discussed below.

3.4 Proofs of two sufficient conditions using $\Delta Y = S_P$

Let the author prove two sufficient conditions, $K_{SP}=Y$ and $D_I=\Delta K_P^0=S_P+S_{WD}$, using Assumption 6 which states the first sufficient condition, $\Delta Y=S_P$, and Assumptions 7 and 8 which leads to the above two conditions. Assumptions are justified by clarifying the contents of g_Y and g_{KP}, and the equality, $g_Y=g_{KP}$.

First, $K_{SP}=Y$ as a sufficient condition, is proved without using Assumption 7, $g_Y=c_{DI}$ which connect π with Ω_P, together with other important relationships and equalities.

As stated already, corporate capital stock K_P^0 or K_P and net investment ΔK_P are each divided into two components: undistributed profit S_P^0 or S_P and savings from wages and dividends S_{WD}^0 or S_{WD}.

1. $K_P^0=K_{SP}^0+K_{SWD}^0$ or $K_P=K_{SP}+K_{SWD}$ (2)
2. $\Delta K_P=\Delta K_{SP}+\Delta K_{SWD}=S_P+S_{WD}$, where $\Delta K_{SP}=S_P$ (3)
3. From a constant Ω_P (by assumption),

$$\Omega_P^\Delta=\frac{\Delta K_P}{\Delta Y}=\Omega_P=\frac{K_P^0}{Y^0}=\frac{K_P}{Y} \text{ and as a result, } \frac{\Delta Y}{Y^0}=\frac{\Delta K_P}{K_P^0} \tag{4}$$

The marginal capital-output ratio is defined as Ω_P^Δ. Define the growth rates of output and capital, g_Y and g_{KP}:

1. $g_Y \equiv \Delta Y / Y^0$
2. $g_{KP} \equiv \Delta K_P / K_P^0$

Then, it implies that $g_Y=g_{KP}$ under a constant Ω_P.

What is each numerator of g_Y and g_{KP}? The equality, $\Delta K_P \equiv \Delta K_{SP}+\Delta K_{SWD}=S_P+S_{WD}$, is unquestionable under Assumption 6. If so, what is a sufficient condition for ΔY and K_{SP} both under $g_Y=g_K$ and constant Ω_P and π? Let the author first raise the structure of weighted average between two periods.

Structure of the weighted average

$$\underset{\Omega_P^1}{\frac{K_P^1}{Y^1}} = \underset{\Omega_P^0}{\frac{K_P^0}{Y^0}} \times \underbrace{\boxed{\frac{Y^0}{Y^1}}}_{\text{weight for average}} + \underset{\Omega_P^\Delta}{\frac{\Delta K_P}{\Delta Y}} \times \underbrace{\boxed{\frac{Y^1-Y^0}{Y^1}}}_{\text{weight for marginal}} \tag{5}$$

This structure holds even under constant Ω_P: $\Omega_P^0=\Omega_P^\Delta=\Omega_P^1$, since $\frac{Y^0}{Y^1}+\frac{Y^1-Y^0}{Y^1}=1$ [Kamiryo, 1989, p.159].

It is the denominator's item that determines the weights for average and marginal.

Now, the denominators of the growth rates of output and capital are discussed as follows:

$g_Y=g_{KP}$ is derived under constant Ω_P and π

Recall $\quad g_Y\equiv\frac{\Delta Y}{Y^0}\quad$ and $\quad g_{KP}\equiv\frac{\Delta K_P}{K_P^0}$

where, $\Delta K_P=\Delta K_{SP}+\Delta K_{SWD}$ and $K_P^0=K_{SP}^0+K_{SWD}^0$

By Assumption 4, constant Ω_P and π, any growth rate in the financial structure of products is the same:

$$\frac{\Delta Y}{Y^0}=\frac{\Delta K_P}{K_P^0}=\frac{\Delta K_{SP}}{K_{SP}^0}=\frac{\Delta K_{SWD}}{K_{SWD}^0}=\frac{\Delta K_{SP}+\Delta K_{SWD}}{K_{SP}^0+K_{SWD}^0} \text{ by the componendo rule}$$

Thus, $g_Y=g_{KP}$ holds under constant Ω_P and π. However, it implies that the ratio of $g_Y=g_{KP}$ is still unknown. A sufficient condition, $K_{SP}=Y$, is required in order to determine the ratio of $g_Y=g_{KP}$. Using Assumption 6, $\Delta Y=S_P$, the equation is expressed as,

$$g_Y=\frac{S_P}{Y^0}=\frac{\Delta K_P}{K_P^0}=\frac{\Delta K_{SP}}{Y^0}=\frac{S_{WD}}{K_{SWD}^0} \text{ and } g_{KP}\equiv\frac{S_P+S_{WD}}{Y^0+K_{SWD}^0} \text{ by the componendo rule.}$$

As a result, the capital-output ratio Ω is expressed, proving $K_{SP}=Y$,

$$\Omega_P=\frac{K_{SP}^0+K_{SWD}^0}{Y^0}=\frac{Y^0+K_{SWD}^0}{Y^0}=\frac{S_P+S_{WSD}}{\Delta Y}=\frac{Y+K_{SWD}}{Y}.$$

Repeating, the following equations are derived:

1. $g_Y\equiv\frac{\Delta Y}{Y^0}=\frac{S_P}{Y^0}$ (6)

2. $g_{KP} \equiv \frac{\Delta K}{K_P^0} = \frac{S_P+S_{WD}}{Y^0+K_{SWD}^0}$ (7)

3. or, $\frac{S_P}{Y^0} = \frac{S_P+S_{WD}}{Y^0+K_{SWD}^0} = \frac{S_{WD}}{K_{SWD}^0}$ (8)

Using these equations as a base, the equations under the corporate-financed growth steady-state will be solved endogenously step by step (if necessary, with a given population growth rate, g_{NE}^e) as follows:

1. $g_Y \equiv \frac{S_P}{Y^0} = \frac{Y^0(1+g_Y)\cdot s_{SP/Y}}{Y^0} = s_{SP/Y}(1+g_Y)$

   As a result, $g_Y = \frac{s_{SP/Y}}{1-s_{SP/Y}}$ (9)

2. The relationship between g_Y and g_{KP}:

$$g_{KP} = \frac{S_P+S_{WD}}{K_P^0} = \frac{Y^0}{K_P^0}\left(\frac{S_P\cdot K_P^0}{K_P^0\cdot Y^0} + \frac{S_{WD}\cdot K_P^0}{K_P^0\cdot Y^0}\right) = \frac{1}{\Omega_P}(g_Y + s_{SWD/Y}(1+g_Y)), \quad (10)$$

   where, $g_Y = g_{SWD}$ and also $g_{KP} = g_Y$ under constant π and Ω_P

3. Then, $g_Y = \frac{1}{\Omega_P}(g_Y + s_{SWD/Y}(1+g_Y))$

   As a result, $\Omega_P \cdot g_Y = g_Y + s_{SWD/Y}(1+g_Y)$, where $g_Y = s_{SP/Y}/(1-s_{SP/Y})$

   By dividing both sides by g_Y,

$$\Omega_P = 1 + s_{SWD/Y} + \frac{s_{SWD/Y}}{g_Y} = 1 + s_{SWD/Y} + \frac{(1-s_{SP/Y})s_{SWD/Y}}{s_{SP/Y}} = \frac{s_{SWD/Y}+s_{SP/Y}}{s_{SP/Y}} = \frac{S_{WD}+S_P}{S_P} \quad (11)$$

or $\Omega_P = \frac{s_{S/Y}}{s_{SP/Y}} = 1 + \frac{s_{SWD/Y}}{s_{SP/Y}}$

or, $s_{SED/Y} = s_{SP/Y}(\Omega_P - 1)$ $\quad S_{WD} = S_{SP}(\Omega_P - 1)$ (12)

These results show that $s_{SWD/Y} = 0$ if $\Omega_P = 1$. Equation 12 implies that the unbalanced growth model approaches the balanced growth steady-state regardless of the value of Ω_P. The author distinguishes "unbalanced" or "balanced" from "disequilibrium" or "equilibrium." The former does not take into consideration technological progress while the latter takes into consideration technological progress.

4. Relationships among the propensities to save and the capital-output ratio

   Using $s_{SWD/Y} = s_{SP/Y}(\Omega_P - 1)$,

$$s_{(SWD+SP)/Y}=s_{S/Y}\equiv s_{SWDY}+s_{SP/Y}=s_{SP/Y}(\Omega_P-1)+s_{SP/Y}=s_{SP/Y}\cdot\Omega_P \tag{13}$$

$c_{(W+DI)/Y}=1-s_{SP/Y}\cdot\Omega_P$, which is dual to $s_{SP/Y}\cdot\Omega_P$ and not used in this paper.

Second, $D_I=\Delta K_P=S_P+S_{WD}$ as a sufficient condition, is proved with using Assumption 7, $g_Y=c_{DI}$, which connect π with Ω_P, together with other important relationships and equalities.

The ratio of dividends to profit, D_I/Y

$$\frac{D_I}{Y}=\frac{Y}{Y}-\frac{S_P}{Y}-\frac{W}{Y}=1-s_{SP/Y}-(1-\pi)=\pi-s_{SP/Y} \tag{14}$$

Equation 13 does not directly derive $\mathbf{D_I=\Delta K_P}$. The following table shows the final relationship between π and Ω_P after deriving $D_I=\Delta K_P$ or $\pi-s_{SP/Y}=s_{SP/Y}\cdot\Omega_P$. It seems that π and Ω_P are separately treated: π is related to distribution while Ω_P is related to savings and investment. How can π be related to Ω_P in the financial structure of products? The author stresses this point before proving $D_I=\Delta K_P$ as a sufficient condition.

The relationship profit and savings

$Y(1-\pi)=W$	$1-s_{SP/Y}(\Omega_P+1)$	$1-s_{SP/Y}(\Omega_P+1)+s_{SP/Y}\cdot\Omega_P=1-s_{SP/Y}$
D_I	$s_{SP/Y}\cdot\Omega_P$	$s_{SWD/Y}+s_{SP/Y}=s_{SP/Y}\cdot\Omega_P$
S_P	$s_{SP/Y}$	$\pi=s_{SP/Y}\cdot\Omega_P+s_{SP/Y}=s_{SP/Y}(\Omega_P+1)$

where, $W+D_I=C_{W+DI}+S_{WD}+S_P$, and $S_{WD}=S_P(\Omega_P-1)$ and $S_{WD}+S_P=S_P\cdot\Omega_P$

A question is why D_I equals "$S_P\cdot\Omega_P$." Or, why is "$\pi=s_{SP/Y}(\Omega_P+1)$" derived?

Let raise the author the same question in more detail. $S_{WD}=S_P(\Omega_P-1)$ was already derived using $\Delta Y=S_P$, and $K^0_{SP}=Y^0$ or $K_{SP}=Y$ as sufficient conditions as above, but $Y=W+P$ in the LHS does not directly related to $Y=C+S$ in the RHS in this table. Let the author first review **the same propensity to save both for wages and dividends** as Assumption 8, $\mathbf{s_{SWD/WD}}=s_{SW/W}=s_{SDI/DI}$.

$$W=C_W+S_W=(1-s_{SWD/Y})Y(1-\pi)+s_{SWD/Y}\cdot Y(1-\pi)=(1-\pi)Y$$

$$D_I=C_{DI}+S_{DI}=(1-s_{SWD/Y})\cdot Y(\pi-s_{SP/Y})+s_{SWD/Y}\cdot Y(\pi-s_{SP/Y})=(\pi-s_{SP/Y})Y,$$

since $\pi(1-s_{SP/P})=\pi\left(1-\frac{s_{SP/Y}}{\pi}\right)=\pi-s_{SP/Y}$, where $s_{SP/P}=s_{SP/Y}/\pi$

However, as seen above, both W and D_I cannot be expressed using the value of Ω_P. For dividends, $D_I/Y=\pi-s_{SP/Y}$, and for savings, $s_{SWD/Y}+s_{SP/Y}=s_{SWD/Y}(1-\pi)+s_{SWD/Y}(\pi-s_{SP/Y})=s_{SWD/Y}+s_{SWD/Y}\cdot s_{SP/Y}=s_{SWD/Y}(1+s_{SP/Y})=s_{SP/Y}(\Omega_P-1)+s_{SP/Y}=s_{SP/Y}\cdot\Omega_P$. Readers may think that corporate profit P should be equal to corporate net investment ΔK_P. Then, dividends paid D_I must be equal to savings of wages and dividends S_{WD}: $P=\Delta K_P$ and $D_I=S_{WD}$. However, this is a specified case that wages are consumed and dividends are saved, and the same propensity to save both for wages and dividends does not hold. If so, Assumption 8 must be abandoned. There seems to be no way to connect $\pi-s_{SP/Y}$ with $s_{SP/Y}\cdot\Omega_P$ (or π with $s_{SP/Y}(\Omega_P+1)$).

Now, the sufficient condition, $\mathbf{D_I=\Delta K_P=S_{WD}+S_P}$ (or $\pi-s_{SP/Y}=s_{SP/Y}\cdot\Omega_P$ or $P=\Delta K_P+S_P=S_P(\Omega_P+1)$), is derived and proved using Assumption 7 as follows:

According to Assumption 7, the growth rate of output $g_Y\equiv\frac{\Delta Y}{Y^0}$ equals the dividend cost of corporate capital $c_{DI}\equiv\frac{D_I}{K_P^0}$ and $g_Y=c_{DI}$. It is necessary for readers not to mix C_{DI} up with $g_Y=\frac{s_{SP/Y}}{1-s_{SP/Y}}$. The proof only comes from $\Omega_P\equiv\frac{K_P^0}{Y^0}=\frac{D_I}{S_P}\equiv\Phi$.[13] It derives $\Omega_P=\frac{1-s_{SP/P}}{s_{SP/P}}$ or $s_{SP/P}=\frac{1}{\Omega_P+1}$. (15)

Since $\frac{1-s_{SP/P}}{s_{SP/P}}=\frac{\pi-s_{SP/Y}}{s_{SP/Y}}$,

$\pi-s_{SP/Y}=s_{SP/Y}\cdot\Omega_P$ or

$\pi=s_{SP/Y}(\Omega_P+1)$ (16)

13) Prof. Furuta, Y. [1997, May] proves this condition supporting the author's intention as follows:

$s_{SP/P}=Y/(K_P+Y)$, and $s_{SP/P}\cdot\pi(\Omega_P-1)=P(K_P-Y)/Y(K_P+Y)$.

On the other hand, $S_P=P\cdot Y/(K_P+Y)$, and $S_{WD}=P(K_P-Y)/Y(K_P+Y)$.

Therefore, $D_I=S_P+S_{WD}$.

As a result, $D_I = S_{WD} + S_P$ since $S_{WD} + S_P = S_P \cdot \Omega_P$

From this sufficient condition, the value of wage and dividend propensity to save, $\mathbf{s_{SWD/WD}}$, is derived (see Assumption 8) as follows:

Under a condition of $D_I = \Delta K_P = S_{WD} + S_P$,

For D_I/Y, $\pi - s_{SP/Y}$

For savings, $S = S_P + S_W + S_{DI}$, $s_{S/Y} = s_{SP/Y} + \mathbf{s_{SWD/WD}}(1-\pi) + \mathbf{s_{SWD/WD}}(\pi - s_{SP/Y})$

Then, $\pi - s_{SP/Y} = s_{SP/Y} + \mathbf{s_{SWD/WD}}(1-\pi) + \mathbf{s_{SWD/WD}}(\pi - s_{SP/Y})$

As a result, $\pi = 2s_{SP/Y} + \mathbf{s_{SWD/WD}}(1 - s_{SP/Y})$

Therefore, $s_{SP/Y} = \dfrac{\pi - s_{SWD/WD}}{2 - s_{SWD/WD}}$ (17)

or, $s_{SWD/WD} = \dfrac{\pi - 2s_{SP/Y}}{1 - s_{SP/Y}}$ (18)

Furthermore, by using Assumptions 7 and 8, the following equations are derived using π, Ω_P, ρ, and $s_{SP/Y}$:

$$\rho \equiv \frac{P}{K_P} = s_{SP/Y}\left(1 + \frac{Y}{K_P}\right) = \frac{\pi}{\Omega_P + 1}\left(\frac{\Omega_P + 1}{\Omega_P}\right) = \frac{\pi}{\Omega_P} \tag{19}$$

$$g_Y = \frac{s_{SP/Y}}{1 - s_{SP/Y}} = \frac{\dfrac{\pi}{\Omega_P + 1}}{1 - \dfrac{\pi}{\Omega_P + 1}} = \frac{\pi}{\Omega_P + 1 - \pi} \tag{20}$$

3.5 Related some propositions

Finally, propositions will be stated in comparison with literature as follows:

Proposition 1 The growth rate of output equals the growth rate of capital if the increase in output equals undistributed profit ($\Delta Y = S_P$), corporate accumulated undistributed profit equals output ($K_{SP} = Y$), and dividends equal corporate net investment ($D_I = \Delta K_P$), under constant Ω_P and π (under the corporate-financed growth steady-state where the population growth rate is finally given as a parameter).

Proposition 2 The savings from wages and dividends are zero if the capital-output ratio is one and fixed, if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ under the corporate-financed growth steady-state.

Proposition 3 The growth rate of output, g_Y, is not related to the wage and dividend propensity to save, $s_{SWD/Y}$, but only related to the undistributed profit propensity to save, if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ under the corporate-financed growth steady-state.

According to the structure of average and marginal (see Equation **5**), the values of the denominator of Ω_P, i.e., Y^0 and Y^1, are used for the weights when Ω_P changes. It also supports that the growth rate of output is determined by S_P and Y (not by S_{WD} and K_{SWD}). If the growth rate of capital differs from the growth rate of output, then, there is a room for depending on the wage and dividend propensity to save, $s_{SWD/Y}$. However, even so, this extent will depend on the coefficient of technological progress and its growth rate and the investment ratio (see below). It implies that the use of savings from wages and dividends is risky compared with undistributed profit whose base is real.

Proposition 4 The higher the relative share of profit and the lower the capital-output ratio, the higher the growth rate of output if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ under the corporate-financed growth steady-state: $g_Y = \dfrac{\pi}{\Omega_P + 1 - p}$.

Proposition 5 The growth rate of corporate capital, g_{KP}, is, likewise, not related to the wage and dividend propensity to save, $s_{SWD/Y}$, but only related to the undistributed profit propensity to save, if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ under the corporate-financed growth steady-state.

It is because $g_Y = g_{KP} = \dfrac{S_P}{Y^0} = \dfrac{S_P + \Delta K_{SWD}}{Y^0 + K^0_{SWD}}$ (see Equations 6, 7, and 8). It seems that the higher the wage and dividend propensity to save, $s_{SWD/Y}$, the higher the growth rate of corporate externally-financed capital, g_{KSWD}, but this is exactly equal to the growth rate of output, if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ under the corporate-financed growth steady-state.

Proposition 6 The higher the use of external savings the higher Ω_P, but this helps the (real) growth rate $g_Y = g_{KP}$ to decrease, if $\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$ in the dynamic equilibrium. Furthermore, the higher the use of external savings the higher the expected growth rate of corporate capital g^e_{KP} in the disequilibrium, where $g^e_Y \neq g^e_{KP}$.

The leverage is defined as external savings/undistributed profit S_{WD}/S_P and shown as $\frac{s_{SWD/Y}}{s_{SP/Y}}=(\Omega_P-1)$ (see Equation 12).

In short, behind the above propositions, three sufficient conditions exist: $\Delta Y=S_P$, $K_{SP}=Y$, and $D_I=S_P+S_{WD}$. And, $K_{SP}=Y$ and $D_I=S_P+S_{WD}$ were proved and justified using $I=\Delta K_P=\Delta K_{SP}+\Delta K_{SWD}$ and $\Delta K_{SP}=\Delta Y=S_P$ in Assumption 6, and $g_Y=c_{DI}$ in Assumption 7, together with Assumption 8, $s_{SWD/WD}=s_{SW/W}=s_{SDI/DI}$.

4. Review of the two/one class models

4.1 Overall review of the balanced steady-state in literature

The following Equation 1, which was discussed at the beginning, under the balanced steady-state is reviewed briefly using the relationships between propensities to save. This suggests a way for the two-sector model to improve. Notations in literature are compared with those in the author's model each by each.

(1) $$n=s_p r+s_w\left(\frac{Y}{K}\right)-s_w r \qquad \text{or,} \quad n=\frac{\dot{K}}{K}=s_p\cdot r+s_w(\Omega-r)$$

Set $s_p\equiv S_p/P=s_{SP/P}$ and $s_w\equiv(S_w+S_{DI})/(W+D_I)=s_{SWD/WD}$ in Equation 1 (by assuming that only workers exist). Then, Equation 1 is rewritten using $s_{S/Y}=s_{SP/Y}+s_{SWD/Y}$ and π or $r=\rho=\pi/\Omega$ as follows:

$$s_{SP/P}=s_{SP/Y}/\pi \qquad s_{SWD/WD}=s_{SWD/Y}/(1-\pi)$$

(2) $$s_p-s_w=\frac{s_{SP/Y}}{\pi}-\frac{s_{SWD/Y}}{1-\pi}=\frac{s_{SP/Y}(1-\pi)-s_{SWD/Y}}{\pi(1-\pi)}$$

Therefore, Equation 1 is shown as,

(3) $$n=r(s_p-s_w)+s_w/\Omega=\rho\left(\frac{s_{SP/Y}(1-\pi)-s_{SWD/Y}}{\pi(1-\pi)}\right)+\frac{\pi}{\Omega}\left(\frac{s_{SWD/Y}}{\pi(1-\pi)}\right)=\rho\left(\frac{s_{SP/Y}}{\pi}\right)=\frac{s_{SP/Y}}{\Omega}.$$

The value of n in Equation 3 shows the growth rate of output in the case that does not introduce population and its growth rate. It implies that the growth rate of labour productivity is zero. Equation 3 suggests the following points:

1. If $\Omega=1$ under the balanced growth steady-state, then, $n=s_{SP/Y}$.
2. If $\Omega\neq1$ under the balanced growth steady-state, then, $n=s_{SP/Y}/\Omega$.
3. Pasinetti [1962] concludes that $\frac{P}{K}=\frac{n}{s_c}$ in his Equation 3'. This is shown as $n=r\cdot s_c$ or $n=\rho\cdot s_{SP/P}$ and corresponds with the above Equation 3. This solution holds under $n=g_Y=g_K$, but it cannot clarify the relationship between g_Y and g_K.
4. Samuelson and Modigliani [1966] shows an equation $\frac{K}{Y}=\frac{n}{s_w}$ as a theory dual to the above Pasinetti theorem $\frac{P}{K}=\frac{n}{s_c}$. Their equation, if expressed by the author's notations, corresponds with

$$n=\frac{\pi}{\Omega}\left(\frac{s_{SP/Y}(1-\pi)-s_{SWD/Y}+s_{SWD/Y}}{\pi(1-\pi)}\right)=\frac{1}{\Omega}\left(s_{SP/Y}-\frac{s_{SWD/Y}}{1-\pi}+\frac{s_{SWD/Y}}{1-\pi}\right)$$

Thus,

$$n=\frac{1}{\Omega}(s_{SP/Y}-s_{SWD/WD}+s_{SWD/WD})=\frac{s_{SP/Y}}{\Omega}$$

Then, does $\rho\cdot s_{SP/P}$ equal $\frac{s_{SP/Y}}{\Omega}$?

This equality holds since $\frac{\pi}{\Omega}\cdot\frac{s_{SP/Y}}{\pi}=\frac{s_{SP/Y}}{\Omega}$. As a result, $\frac{K}{Y}=\frac{n}{s_w}$ raised by Samuelson and Modigliani [1966] is unsolved in the two/one class model.

5. As a conclusion, the above literature differs from the author's. Their discussions do not clarify the relationships between a variety of propensities to save, including the common propensity to save, $s_{SWD/WD}=s_{SW/W}=s_{SDI/DI}$, owing to their limited framework. The improved two-sector model of the author's approaches the same balanced steady-state (under the corporate-financed growth model) when the population growth rate is not introduced. Yet, it makes it possible to measure basic all variables endogenously, particularly by using Assumptions 6, 7, and 8 in an balanced growth steady-state.

6. Furthermore, the improved two-sector model additionally introduces the population growth rate together with the growth rate of labour productivity and the coefficient of technological progress (see below). The intention raised by Harrod will be clarified with technological progress.

4.2 Review of Kaldor, Pasinetti, and Samuelson and Modigliani

The author started with the two-one class model and simplified it as a one class model. Then, the two-sector model of the authors extended to the improved two-sector model under the corporate-financed growth steady-state where $g_Y = g_{KP}$ and g^e_{NE} is a parameter. One of remarkable results is the reveal of the wage and dividend propensity to save, $s_{SWD/WD}$, which equals the wage propensity to save, $s_{SW/W}$, and the dividend propensity to save, $s_{SDI/DI}$. This ties the two/one class model with the improved two-sector. The existence of $s_{SWD/WD}$ needs a condition that $D_I = \Delta K_P$. By using this sufficient condition with other sufficient conditions ($\Delta Y = S_P$, and $K_{SP} = Y$), the improved two-sector model completes and makes it possible endogenously to measure the propensities to save and the growth rate of output or capital which are not related to the population growth rate.

Kaldor [1955-6] presents that $\frac{P}{K} = \frac{n}{s_p}$ if $s_w = 0$. Pasinetti [1962, 1974] presents that this Kalddor' equation holds if $1 \geq s_p \geq s_w \geq 0$. Kaldor and Pasinetti advocate, without using the marginal productivity theory, that it is corporate undistributed profit and its propensity to save to determine the growth rate n. The improved two-sector model of the authorís shows that it is corporate undistributed profit and its propensity to save to determine the growth rate $g_Y = g_{KP}$ under necessary conditions (Ω_P and π are constant) and suficient conditions ($\Delta Y = S_P$, $K_{SP} = Y$, and $D_I = \Delta K_P$). This section intends to clarify the above Pasinetti's theorem (which is more general than Kaldor's) with the authors. A key for this comparison lies in the relationship between $s_p \equiv S_P/P = s_{SP/P}$, the investment ratio, I/Y^0, and $s_w \equiv S_{WD}/(W + D_I) = s_{SWD/WD}$.

Pasinetti [1974, p.106] shows the following equation:

(4) $s_p > \frac{I}{Y} > s_w$

Let the author discuss this inequality thoroughly in this section. Is this an assumption or a result? Pasinetti sets it as an assumption and asserts that $\frac{P}{K} = \frac{n}{s_p}$ holds under this assumption. The author thinks that Equation (4) is derived under the above necessary and sufficient conditions. The process for it is explained step by step. Equation (4) is shown using the author's notations as

$$\frac{I}{Y^0} = \frac{\Delta K}{K^0} \cdot \frac{K^0}{Y^0} = g_K \cdot \Omega_P = g_Y \cdot \Omega_P = \frac{\Omega_P \cdot \pi}{\Omega_P + 1 - \pi} \tag{21}$$

Also, using the results in Equations 15 and 16,

$$\mathrm{s_p} = \mathrm{s_{SP/P}} = \mathrm{s_{SP/Y}}/\pi = \frac{1}{\Omega_P + 1} \quad \text{or} \quad s_{SP/Y} = \frac{\pi}{\Omega_P + 1}$$

$$\mathrm{s_w} = \mathrm{s_{SWD/Y}}/(1-\pi) = \frac{s_{SWD/Y}}{1-\pi} = \frac{s_{SP/Y}(\Omega_P - 1)}{(1-\pi)} = \frac{\frac{\pi(\Omega_P - 1)}{\Omega_P + 1}}{(1-\pi)} = \frac{\pi(\Omega_P - 1)}{(1-\pi)(\Omega_P + 1)} \tag{22}$$

Then, by replacing the original inequality (Equation 22) using the above three equations, the following inequality is derived:

$$\frac{1}{\Omega_P + 1} > \frac{\Omega_P \cdot \pi}{\Omega_P + 1 - \pi} > \frac{\pi(\Omega_P - 1)}{(1-\pi)(\Omega_P + 1)} \tag{23}$$

This inequality is divided into two as follows:

1. $\frac{1}{\Omega_P + 1} > \frac{\Omega_P \cdot \pi}{\Omega_P + 1 - \pi}$:
2. $\frac{\Omega_P \cdot \pi}{\Omega_P + 1 - \pi} > \frac{\pi(\Omega_P - 1)}{(1-\pi)(\Omega_P + 1)} = \frac{\Omega_P \cdot \pi - \pi}{\Omega_P + 1 - \pi - \pi \cdot \Omega_P}$:

Do the two inequalities hold. It depends on the values of Ω_P and π.[14]

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14) Let the author simplify the inequality by replacing $\Omega_P + 1 - \pi$ with $\Omega_P + 1$.

$$1 > \Omega_P \cdot \pi > \frac{\pi(\Omega_P - 1)}{(1-\pi)} \quad \text{or} \quad 1 - \pi > \Omega_P \cdot \pi(1-\pi) > \pi(\Omega_P - 1)$$

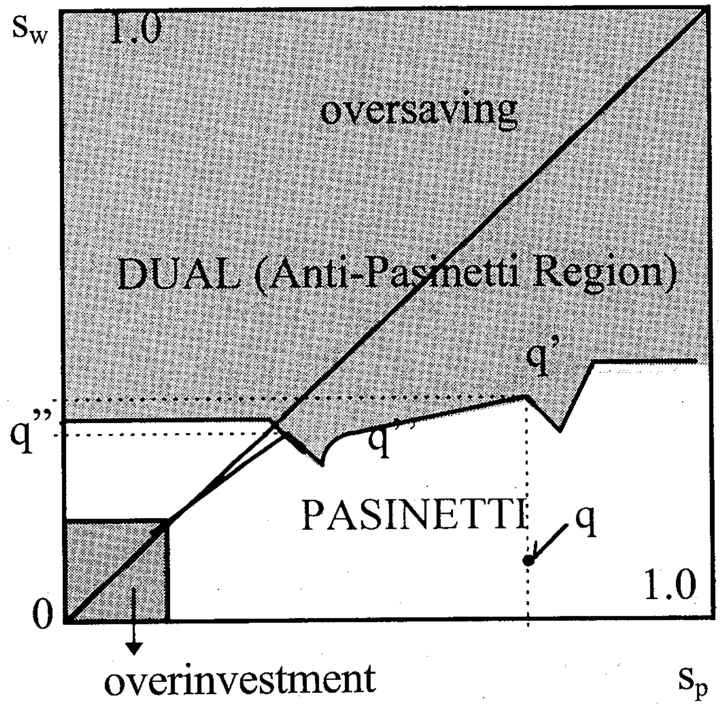
Assume that the value of π is small and replace $1-\pi$ with 1. Then, ↗

Then, once more, how is the above inequality, $s_p > \frac{I}{Y} > s_w$ (or using the author's notation, $s_p > \frac{I}{Y^0} > s_w$) derived? Let the author, alternatively and directly, use the propensities to save, $s_p = s_{SP/P}$ and $s_w = s_{SWD/WD}$, in Equations 17 and 18:

$$s_p = s_{SP/P} = \frac{s_{SP/Y}}{\pi} = \frac{\pi - s_{SWD/WD}}{\pi(2 - s_{SWD/WD})} \quad \text{and} \quad s_w = s_{SWD/WD} = \frac{\pi - 2s_{SP/Y}}{1 - s_{SP/Y}} = \frac{\pi - 2s_{SP/P} \cdot \pi}{1 - s_{SP/P} \cdot \pi}$$

First, let the author introduce a figure drawn by Samuelson and Modigliani [1966, Figure 2, p.325] (see **Figure 1** in this paper). This version suggests a lighthouse

Figure 1 Pasinetti Region versus Anti-Pasinetti Region: r=n/s$_c$ and A=n/s$_w$



Source: Samuelson and Modigliani [1966, p.325]; q'−q satisfies dual and Pasinetti

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↘ $1 > \pi \cdot \Omega_P > \pi(\Omega_P - 1)$

Does this inequality hold? What are conditions necessary for this inequality?

1. $1 > \pi \cdot \Omega_P$ between $s_p = s_{SP/P}$ and I/Y^0
2. $\pi \cdot \Omega_P > \pi(\Omega_P - 1)$ between I/Y^0 and $s_w = s_{SWD/WD}$

The above first, $1 > \pi \cdot \Omega_P$, is a condition, but the above second does not need a condition. For example, if $\pi = 0.06$ and $\Omega_P = 2.0$, then, this condition is satisfied. The relationship between Ω_P and π is shown as $s_{SP/Y} = \frac{\pi}{\Omega_P + 1}$ (see Equation 24): for this case, $s_{SP/Y} = 0.06/3 = 0.02$. This is a rough tentative result. The author needs a more definite result. If the value of $\pi \cdot \Omega_P$ is derived, it is meaningful.

for studying the relationship between propensities to save under $n \neq g_Y = g_{KP}$. Samuelson and Modigliani show a square whose x axis is s_p (0 to 1.0) and y axis is s_w (0 to 1.0). The diagonal line shows $s_p = s_w$. Pasinetti's Region is shown as a bottom part of this square whose boundary is a line almost parallel to the x axis. The region above this boundary line is set as Anti-Pasinetti's Region which is "dual to Pasinetti" and indicates a region of oversaving. A small square in Pasinetti's Region whose origin is the same as the origin of the above square indicates a region of undersaving. In Figure 1, q (=Q) or q−q' is the only point or line which satisfies Pasinetti and Anti-Pasinetti as dual, but in the author's case, this is shown by a region (see Figure 3). The author endogenously measures, using Ω_P and π, the theoretical relationship between s_p ($=s_{SP/P}$) and s_w ($=s_{SWD/WD}$) in a dynamic equilibrium[15] where $g_Y = g_{KP}$ and shows a specified case of $s_p = s_w$. The approach by Samuelson and Modigliani differs from the author's and connects the ratio of profit to capital $r = n/s_c$ with the output-capital ratio $A = n/s_w$, where the relative share of profit $\alpha = r/A$ (which corresponds with $\pi = \rho \cdot \Omega_P$ in the author's). However, they show the diagonal line of $s_p = s_w$ (or $n/s = n/s_p = n/s_w$), whose changes are clarified by using the author's proof that $\pi \cdot \Omega_P = 1$ if $s = s_p = s_w$[16] as will be discussed below.

Second, $s_{SWD/WD}$ (Equation 18) is shown as a hyperbolic form and its character is summarized as follows (see **Figures 2 and 3**):

$$s_{SWD/WD} = \frac{\pi - 2s_{SP/P} \cdot \pi}{1 - s_{SP/P} \cdot \pi} = 2 + \frac{1 - \dfrac{2}{\pi}}{-s_{SP/P} + \dfrac{1}{\pi}} \tag{24}$$

---

15) This paper concentrates on the dynamic equilibrium which is defined as a condition that $g_Y = g_{KP}$. The dynamic disequilibrium is defined as a condition that $g_Y \neq g_{KP}$. The comparison reveals business cycle. Both dynamic equilibrium and disequilibrium introduce technological progress under a given g^e_{NE} as a parameter.

16) If $\pi \cdot \Omega_P = 1$ under $s = s_p = s_w$ is right, then, the diagonal line of $s_p = s_w$ shown by Samuelson and Modigliani [1966, p.325] must indicate that the output-capital ratio is one since $\alpha = r/A = s_w/s_p = 1$ and A must be one.

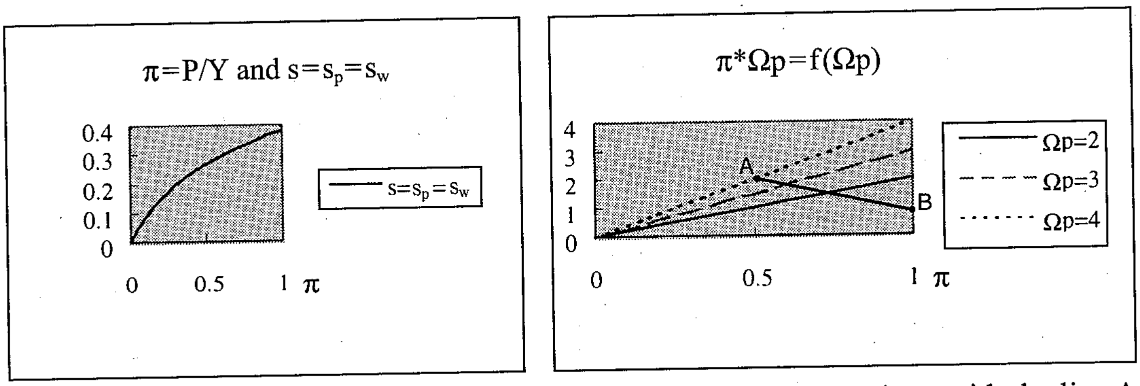
1. The hyperbolic curve of $s_{SWD/WD}$ slightly decreases to the right (see, 4 and 5; it approaches from π to zero between $s_{SP/P}=0$ and $s_{SP/P}=1$).
2. The horizontal asymptote is equal to 2.
3. The vertical asymptote is equal to $1/\pi$.
4. If $s_{SP/P}=0$, then, $s_{SWD/WD}=\pi$.
5. If $s_{SP/P}=1$, then, $s_{SWD/WD}=0$.
6. If $s_{SP/P}=s_{SWD/WD}$, then, $s_{SP/P}=\dfrac{\pi-2s_{SP/P}\cdot\pi}{1-s_{SP/P}\cdot\pi}$.

$$s_{SP/P}(1-s_{SP/P}\cdot\pi)=\pi-2s_{SP/P}\cdot\pi$$

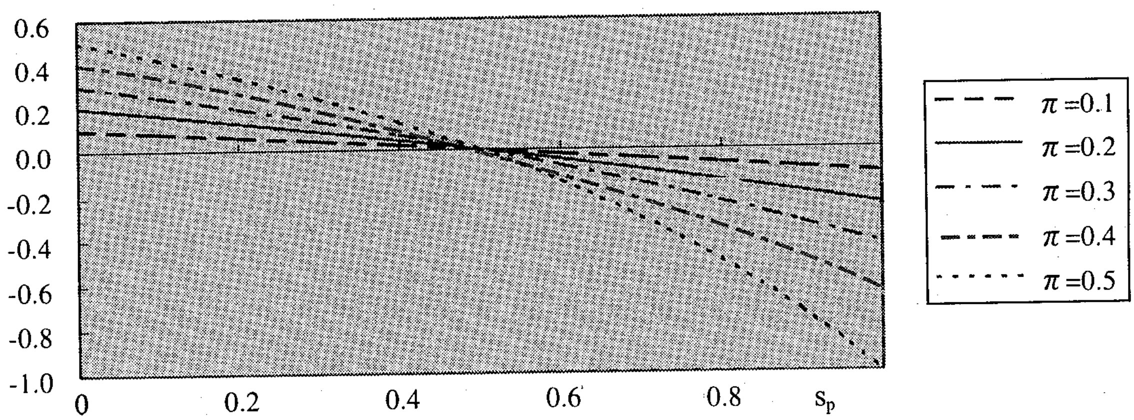
$$\text{or, } -s_{SP/P}{}^2\cdot\pi+s_{SP/P}(1+2\pi)-\pi=0 \quad (25)$$

[17]

Figure 2 Basic relationship between propensities to save, π, and Ω_P: Under $n\neq g_Y=g_{KP}$

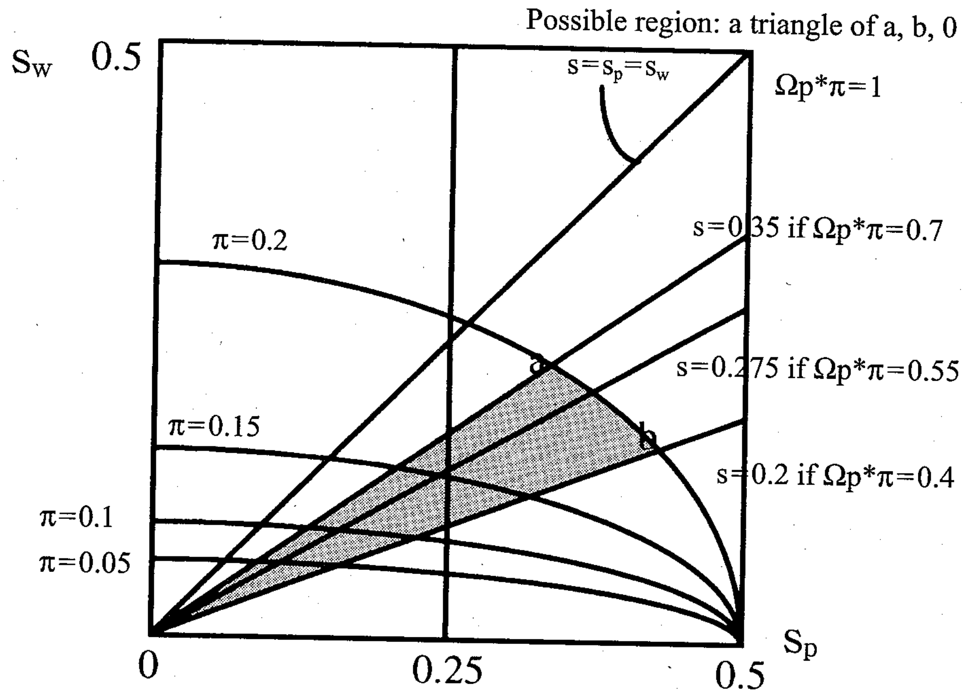


$s=s_p=s_w$ under $\Omega_p\cdot\pi=1$ along with the line A to B



17) For the quadratic equation, $A\cdot x^2+B\cdot x+C$, there are two solutions that $x=\dfrac{-B\pm\sqrt{B^2-4A\cdot C}}{2A}$.

Figure 3　Relationship between s_p and s_w by π and Ω_P



s_w by π	$=s_{SWD/WD}$ by $s_{SP/P}$					
s_p	0.0010	0.1000	0.2000	0.3000	0.4000	0.5000
$\pi=0.1$	0.0998	0.0808	0.0612	0.0412	0.0208	0.0000
$\pi=0.2$	0.1996	0.1633	0.1250	0.0851	0.0435	0.0000
$\pi=0.3$	0.2995	0.2474	0.1915	0.1319	0.0682	0.0000
$\pi=0.4$	0.3994	0.3333	0.2609	0.1818	0.0952	0.0000
$\pi=0.5$	0.4992	0.4211	0.3333	0.2353	0.1250	0.0000

NOTE: Under a condition of $D_I=\Delta K_P=S_{WD}+S_P$,

For D_I/Y, $\pi-s_{SP/Y}$

For savings, $S=S_P+S_W+S_{DI}$, $s_{S/Y}=s_{SP/Y}+s_{SWD/WD}(1-\pi)+s_{SWD/WD}(\pi-s_{SP/Y})$

Then, $\pi-s_{SP/Y}=s_{SP/Y}+s_{SWD/WD}(1-\pi)+s_{SWD/WD}(\pi-s_{SP/Y})$

As a result, $\pi=2s_{SP/Y}+s_{SWD/WD}(1-s_{SP/Y})$

Therefore, $s_{SP/Y}=\dfrac{\pi-s_{SWD/WD}}{2-s_{SWD/WD}}$ (17) or, $s_{SWD/WD}=\dfrac{\pi-2s_{SP/Y}}{1-s_{SP/Y}}$ (18)

A solution is $s_{SP/P}=s_{SWD/WD}=\dfrac{-(1+2\pi)+\sqrt{1+4\pi}}{-2\pi}$ (26)

under $(1+2\pi)^2-4\pi=1+4\pi>0$ as a discrimination.

Therefore, a solution is $s_{SP/P}=s_{SWD/WD}=\dfrac{-(1+2\pi)+\sqrt{1+4\pi}}{-2\pi}$ (26)

under $(1+2\pi)^2-4\pi=1+4\pi>0$ as a discrimination.

The above results are shown as **Figures 2 and 3** as follows:

1. The diagonal line shows that $s_{SP/P}$ equals $s_{SWD/WD}$.
2. This diagonal line crosses the hyperbolic curve, whose intersection point, s_{INTC}, is important in that
   - 2.1 Region of $s_{SP/P}<s_{INTC}$ shows that $s_{SP/P}<s_{SWD/WD}$
   - 2.2 Region of $s_{SP/P}>s_{INTC}$ shows that $s_{SP/P}>s_{SWD/WD}$.
   - 2.3 Region of $s_{SWD/WD}>s_{INTC}$ shows that $s_{SWD/WD}>s_{SP/P}$
   - 2.4 Region of $s_{SWD/WD}>s_{INTC}$ shows that $s_{SWD/WD}<s_{SP/P}$
3. Region of 2.3 ($s_{SWD/WD}>s_{INTC}$) is much smaller than Region 2.4 ($s_{SWD/WD}<s_{INTC}$) although the intersection point is determined by the quadratic equation whose parameter is the relative share of profit π. It implies that $s_{SWD/WD}<s_{SP/P}$ can be accepted.
4. At the intersection point, the value also equals the investment ratio, I/Y^0, which is expressed as an average propensity to save, $s_{(SWD+SP)/Y}=s_{S/Y}$.
5. Except this intersection point, Region of $s_{SP/P}<s_{SWD/WD}$ or $s_{SP/P}>s_{SWD/WD}$ shows that the the average propensity to save lies between $s_{SP/P}$ and $s_{SWD/WD}$.
6. As a result, following inequality holds except Region of 2.3 ($s_{SWD/WD}>s_{INTC}$).

$$s_{SP/P}>\frac{I}{Y^0}>s_{SWD/WD} \tag{27}$$

The above conclusion was derived under the corporate-financed growth steady-state where $g_Y=g_{KP}$ and without using the population growth rate. Note that this inequality holds using π (if I/Y^0 equals $s_{S/Y}$, then, without using Ω_P; see below).

In the above conclusion, I/Y^0 was set as $s_{(SWD+SP)/Y}=s_{S/Y}$. However, this I/Y^0 is exactly shown as $\dfrac{I}{Y^0}=g_{KP}\cdot\Omega_P=\dfrac{s_{SP/P}\cdot\pi}{1-s_{SP/P}\cdot\pi}\cdot\Omega_P=\dfrac{s_{SP/Y}}{1-s_{SP/Y}}\cdot\Omega_P.$ (28)

This is because $I^0/Y^0=I/Y=s_{S/Y}$, but $I/Y^0\neq s_{S/Y}$. Then, $I/Y^0=s_{S/Y}(1+g_Y)$. If so, what is the relationship between $s_{S/Y}$ and $g_{KP}\cdot\Omega_P=g_Y\cdot\Omega_P$? In another word, does $I/Y^0=s_{S/Y}(1+g_Y)=g_Y\cdot\Omega_P$ hold? This is proved as follows:

$$\frac{g_Y}{1+g_Y}=\frac{\dfrac{s_{SP/Y}}{1-s_{SP/Y}}}{\dfrac{1-s_{SP/Y}+s_{SP/Y}}{1-s_{SP/Y}}}=s_{SP/Y}$$

$$s_{S/Y}=s_{SWD/Y}+s_{SP/Y}=s_{SP/Y}\cdot\Omega_P \tag{29}$$

Therefore, the above equation $I/Y^0=s_{S/Y}(1+g_Y)=g_Y\cdot\Omega_P$ was proved.

As a result, $s_{SP/P}>\frac{I}{Y^0}>s_{SWD/WD}$ holds by the relationship of

$$s_{S/Y}=s_{SP/P}\cdot\pi\cdot\Omega_P,\ \text{since}\ s_{SP/Y}=s_{SP/P}\cdot\pi. \tag{30}$$

This is derived under a condition of $\pi\cdot\Omega_P$**<1** in the real world which is the following second case (see **Figure 3**):

1. If $\pi\cdot\Omega_P>1$, then, $s_{S/Y}>s_{SP/P}$ A condition of $\pi\cdot\Omega_P>1$ is almost impossible.
2. If $\pi\cdot\Omega_P<1$, then, $s_{S/Y}<s_{SP/P}$ A condition of $\pi\cdot\Omega_P<1$ holds.

Third, let the author review the relationship between Pasinetti's $\frac{P}{K}=\frac{n}{s_p}$ and Samuelson and Modigliani's $\frac{K}{Y}=\frac{n}{s_w}$, where $s_p=s_{SP/P}$ and $s_w=s_{SWD/WD}$. For this discussion, the author is rather in favour of Pasinetti' theorem, and not in favour of Samuelson and Modigliani since the author makes much of undistributed profit. However, if the above solution of the quadratic equation is introduced (see Equation 25), $s_{SP/P}=s_{SWD/WD}=\frac{-(1+2\pi)+\sqrt{1+4\pi}}{-2\pi}$,[18] then, $\frac{n}{s_p}=\frac{n}{s_w}$ is derived and shown as a diagonal line. However, in this case, the rate of profit is to be equal to the capital-output ratio. This is not plausible and contradictory. The cause is traced back to the difference between $n=g_Y=g_{KP}$ and $n\neq g_Y=g_{KP}$.

If so, **fourth**, what is the difference between Pasinetti's $\frac{P}{K}=\frac{n}{s_p}$ and the author's rate of profit? Conclusively speaking (see below, the dividend valuation value and the dividend valuation ratio),

18) However, this case has a solution, $x=\frac{-B+\sqrt{B^2-4A\cdot C}}{2A}$ (not as $x=\frac{-B-\sqrt{B^2-4A\cdot C}}{2A}$).

$$g_Y = \frac{D_I}{K_P^0} = \frac{(1-s_{SP/P})\,P^0\,(1+g_Y)}{K_P^0} = \rho(1-s_{SP/P})(1+g_Y) = r(1-s_{SP/P}) \tag{31}$$

Compare this result with the above Pasinetti's:[19)]

1. n can be replaced by g_Y.
2. P/K can be replaced by ρ.
3. Then, Pasinetti's equation is shown as $\rho = g_Y/s_{SP/P}$ if $n = g_Y = g_{KP}$, where $s_w < I/Y$ and $s_c > I/Y$ are set as two restrictions [p.106] in order to limit the validity of the mathematical formulations to the range in which they have an economic meaning.
4. The author's equation, on the other hand, is shown as $\rho = \dfrac{g_Y}{(1-s_{SP/P})(1+g_Y)}$ if $n \neq g_Y = g_{KP}$ under necessary and sufficient conditions.
5. Nevertheless, it is remarkable that Pasinetti intends to use the retention ratio for the growth rate.

Pasinetti, however, finally presents another equation (3.3), which is, likewise, expressed using the author's notations as $\Omega_P = \frac{s_{S/Y}}{g_Y}$ or $g_Y = s_{S/Y} \cdot \Omega_P$. The difference between Pasinetti's and the author's is $g_Y = s_{S/Y} \cdot \Omega_P$ versus $g_Y = r_{SP} \cdot \Omega_P$ (see Equation 44, below). In the financial structure of products, the capital-output ratio Ω_P is directly related to $s_{S/Y}$ since $\Omega_P = (s_{SP/Y} + s_{SWD/Y})/s_{SP/Y}$, but g_{KP} is related to $s_{S/Y}$ while g_Y is only related to $s_{SP/P}$ or $s_{SP/Y}$ even under $g_Y = g_{KP}$. Pasinetti's case, thus, have to reduce to Harrod-Domer's under $n = g_Y = g_{KP}$ (see, the two/one class model of O'Connell's). In short, Pasinetti's theorem is not an endogenous growth model

19) Pasinetti's [1974, p.107] final equation is shown as (note that s_c is shown as s_p in this paper)

$$\frac{P}{Y} = \frac{1}{s_p - s_w} \cdot \frac{I}{Y} - \frac{s_w}{s_p - s_w} \quad \text{or} \quad \frac{P}{K} = \frac{1}{s_p - s_w} \cdot \frac{I}{K} - \frac{s_w}{s_p - s_w} \cdot \frac{Y}{K}.$$

However, these two comes from a basic equation that

$$I = s_w \cdot W + s_p \cdot P = s_w \cdot Y + (s_p - s_w)P.$$

This base is the same as the author's. The author sets necessary and sufficient conditions as assumptions, which lead to a endogenous growth model. $\Omega_P \neq 1$ does not mean $g_Y \neq g_{KP}$.

while the author's opens a door for an endogenous growth model under $g_Y = g_{KP}$. The use of n might prevent him to extend.

5. Further developments: valuation ratios and labour productivity

5.1 Capital stock and its valuation ratios

The relationship between corporate capital, its valuation values, and the valuation ratios are derived endogenously using the results of the previous section. Let the author briefly discuss this issue.

First, the valuation value is obtained usually as shown by [Khoury and Parsons, 1981]:

$$V_0 = \sum_{t=1}^{\infty} \frac{D^e}{(1+r^M)^t} = \frac{D^e}{r^M} \quad \text{or} \quad V_0 = \sum_{t=1}^{\infty} \frac{D^e(1+g)^t}{(1+r^M)^t} = \frac{D^e}{r^M - g} \tag{32}$$

where, D^e is expected dividends and r^M is the market cost of capital which is given from the stock market.

Also, the cost of capital "r" is distinguished from the rate of profit "ρ" as follows:

$$r = \frac{P^0(1+g_P)}{K_P^0} \quad \text{versus} \quad \rho = \frac{P^0(1+g_P)}{K_P} = \frac{P^0(1+g_P)}{K_P^0(1+g_K)}, \tag{33}$$

where $g_P = g_Y = g_{KP}$ and $r = \rho(1+g_Y)$ or $r > \rho$[20] (34)

Second, for the three valuation values (see below), a common cost of capital and a common discount rate are used. The cost of capital, r, is defined as the ratio of profit to the initial corporate capital. The discount rate, $r - g_Y$, is defined as the rate which is used for measuring each valuation value and equals the cost of capital less the growth rate of output. The three discount rates are: the discount rate of undistributed profit (for capital maintenance), r_{SP}, the discount rate of paying dividends, r_{DI}, and the discount rate of the sum of undistributed profit and dividends, r_P. The

20) However, it does not mean that $r \neq \rho$ since the difference comes from the growth rate of output lying between two periods.

relationships between the cost of capital and the discount rate are shown as follows:

1. The cost of capital $r \equiv P/K_P^0 = (S_P + D_I)/K_P^0$
2. The discount rate $r_{SP} = r - g_Y$ $r_{DI} = r - g_Y$ $r_P = r - g_Y$
3. As a result, $r = r_{SP} + g_Y$ $r = r_{DI} + g_Y$ $r = r_P + g_Y$

The relationship between r and $r_{SP} = r_{DI} = r_P$ is expressed simply as

$$\frac{r_{SP}}{r} = \frac{r_{DI}}{r} = \frac{r_P}{r} = \frac{1}{\Omega+1} = s_{SP/P}. \tag{35}$$ [21)]

Third, three corresponding valuation values are the valuation value of undistributed profit, V_{SP}^0, the valuation value of dividends, V_{DI}^0, and the valuation value of profit, V_P^0, where $V_P^0 = V_{SP}^0 + V_{DI}^0$. Using the common discount rate and following the form of the NPV, $V^0 = \sum_{t=1}^{\infty} \frac{X^e(1+g)^t}{(1+r)^t} = \frac{X^e}{r-g}$, where X^e is S_P, D_I, or P, three valuation values are derived as follows:

1. $$V_{SP}^0 = \frac{S_P}{r-g_Y} = \frac{s_{SP/P} \cdot P^0(1+g_Y)}{r-g_Y} = \frac{s_{SP/P} \cdot P}{r-g_Y} = \frac{s_{SP/Y} \cdot Y}{r_{SP}} \tag{36}$$
2. $$V_{DI}^0 = \frac{D_I}{r-g_Y} = \frac{(1-s_{SP})\, P^0\,(1+g_Y)}{r-g_Y} = \frac{(1-s_{SP/P})P}{r-g_Y} = \frac{(\pi - s_{SP/Y})Y}{r_{SP}} \tag{37}$$
3. $$V_P^0 = V_{SP}^0 + V_{DI}^0 = V_{SP+DI}^0 = \frac{P}{r-g_Y} = \frac{P^0(1+g_Y)}{r-g_Y} = \frac{\pi \cdot Y}{r_{SP}} \tag{38}$$

Note that V_{SP}^0 and V_P^0 are used here only for showing the relationship between π, $s_{SP/Y}$, and three valuation values in terms of V_{DI}^0. The dividend valuation value, V_{DI}^0, is finally chosen as a unique valuation value (for discussion, see below).

Fourth, let the author once more discuss here a sufficient condition that $D_I = \Delta K_P$

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21) $g_Y = \frac{\pi}{\Omega_P + 1 - \pi}$ and $r_{SP} = \frac{\pi}{\Omega_P(\Omega_P + 1 - \pi)}$, and thus, $r = r_{SP} + g_Y = \frac{\pi(\Omega_P + 1)}{\Omega_P(\Omega_P + 1 - \pi)}$.

As a result, $\frac{r_{SP}}{r} = \frac{1}{\Omega_P + 1} = s_{SP/P} = \frac{s_{SP/Y}}{\pi}$.

which was already proved using Assumption 7, $g_Y = c_{DI}$, where the growth rate of output $g_Y \equiv S_P/Y^0$ equals the ratio of dividends to the initial capital $c_{DI} \equiv D_I/K_P^0$ under Assumption 6, $\Delta Y = S_P$: $g_Y = \frac{S_P}{Y^0} = \frac{D_I}{K_P^0}$. This is proved as follows:

1. From this $\frac{S_P}{Y^0} = \frac{D_I}{K_P^0}$, $\Omega_P = \frac{K_P^0}{Y^0} = \frac{D_I}{S_P} = \frac{D_I^0(1+g_Y)}{S_P^0(1+g_Y)} = \frac{1-s_{SP/P}}{s_{SP/P}}$ (39)

2. Equation 15 was already shown as $s_{SP/P} = s_{SP/Y}/\pi = \frac{1}{\Omega_P+1}$ or $\Omega_P = \frac{1-s_{SP/P}}{s_{SP/P}}$.

3. Above 1. and 2. show exactly the same result. Therefore, the condition of $D_I = \Delta K_P$ is proved to be equal to the condition of $g_Y = \frac{S_P}{Y^0} = \frac{D_I}{K_P^0} = c_{DI}$.

   As a result, furthermore, the following relationships are derived.

1. $g_Y = \frac{D_I}{K_P^0} = \frac{(1-s_{SP/P})\,P^0\,(1+g_Y)}{K_P^0} = \rho(1-s_{SP/P})(1+g_Y) = r(1-s_{SP/P})$ (40)

since $r = \rho(1+g_Y)$ by Equation 34.

2. $r_{SP} = r_{DI} = r_P = r - g_Y = r - r(1-s_{SP/P}) = r \cdot s_{SP/P}$ (the same as Equation 35), thus,

$$r = \frac{g_Y}{1-s_{SP/P}} = \frac{\pi(\Omega_P+1)}{\Omega_P(\Omega_P+1-\pi)} \tag{41}$$

$$g_r = \frac{s_{SP/Y}}{1-s_{SP/Y}} = \frac{\pi}{\Omega_P+1-\pi} \tag{42}$$

$$s_{SP/Y} = \frac{g_Y}{1+g_Y} = \frac{\pi}{\Omega_P+1} \quad \text{(the same as Equation 24)}$$

$$\rho = \frac{s_{SP/Y}}{1-s_{SP/P}} = \frac{\pi}{\Omega_P+1} \Big/ \frac{\Omega_P+1-1}{\Omega_P+1} = \frac{\pi}{\Omega_P} \tag{43}$$

The differences between each variable come from the differences of combinations of π and Ω_P. There have been discussions for π, Ω_P, ρ, $s_{SP/Y}$, and g_Y for many years since Kaldor [1955-56]. However, these differences were now clarified under the corporate-financed growth steady-state.

Thus, the implication of $D_I = \Delta K_P$ is finally arranged in the following statements, and thus, justified more definitely.

1. A sufficient condition, $D_I = \Delta K_P$, is important in that $D_I = \Delta K_P$ connects the growth rate of output, g_Y, with the ratio of dividends to the initial corporate capital, $D_I/$

K_P^0, as well as $D_I = \Delta K_P$ connects π with Ω_P.

2. If $g_Y > D_I/K_P^0$, the discount rate becomes smaller, but at the sacrifice of larger Ω_P.
3. If $g_Y < D_I/K_P^0$, the discount rate becomes larger, but at the sacrifice of smaller valuation value.
4. However, g_Y is determined by $\frac{\pi}{\Omega_P + 1 - \pi}$ while D_I/K_P^0 is determined by g_Y: $g_Y = g_{KP} = D_I/K_P^0$. It implies that $g_Y = g_{KP}$ holds with this sufficient condition $D_I = \Delta K_P$.
5. If $g_Y \neq g_{KP}$, $D_I \neq \Delta K_P$. This is discussed in a separate paper, in the case that Ω_P varies.

Fifth, the characters of valuation values, the valuation ratio defined as the ratio of dividend valuation value to corporate capital, the growth rate of this valuation value, and the character of risk which may be defined in national economy are discussed as follows:

1. Valuation value of undistributed profit, $V_{SP}^0 = K_P^0$

$$r \equiv \frac{P}{K_P^0} \qquad r_{SP} \equiv \frac{S_P}{K_P^0} = \frac{s_{SP/P} \cdot P}{K_P^0} = \frac{s_{SP/Y} \cdot Y}{K_P^0}$$

Thus, $$V_{SP}^0 = \frac{s_{SP/P} \cdot P}{r \cdot s_{SP/P}} = \frac{P}{r} = K_P^0 \tag{44}$$

2. Valuation value of output, $V_Y^0 = Y^0$

Since $K_P^0 = K_{SP}^0 + K_{SWD}^0 = Y^0 + K_{SWD}^0 = Y^0 \cdot \Omega_P$ by Assumption 7 that $K_{SP}^0 = Y^0$, the valuation value of output, V_Y^0, is equal to Y^0. This is proved using the NPV method,

$V^0 = \sum_{t=1}^{\infty} \frac{X^e(1+g_X)^t}{(1+r)^t} = \frac{X^e}{(r-g_X)\Omega_P}$, where X^e is S_P and the denominator is the discount rate of real net national income (not of corporate capital):

The discount rate of output, $r_{SP} \cdot \Omega_P$, is equal to the growth rate of output g_Y: $r_{SP} \cdot \Omega_P = g_Y$. This is because

$r_{SP} = r - g_Y$ and

$$r_{SP}\cdot\Omega_P=\frac{S_P}{K_P^0}\cdot\frac{K_P^0}{Y^0}=g_Y. \tag{45}$$

As a result,

$$V_Y^0=\frac{S_P}{(r-g_Y)\Omega_P}=\frac{S_P}{g_Y}=Y^0 \tag{46}$$

In short, the present value of undistributed profit in infinite periods of time equals K_P^0 or Y^0 according to each discount rate. This shows a corporate supply-side in a sense.

3. Relationship between the valuation value and corporate capital

When the valuation ratio is defined as the ratio of the dividend valuation value to the initial corporate capital K_P^0, this valuation ratio, v_{DI}^0, is derived as follows:[22)]

$$v_{DI}^0\equiv\frac{V_{DI}^0}{K_P^0}=\frac{(\pi-s_{SP/Y})Y}{r_{SP}}\cdot\frac{r_{SP}}{s_{SP/Y}\cdot Y}=\frac{\pi-s_{SP/Y}}{s_{SP/Y}}=\Omega_P \tag{47}$$

or,

$$V_{DI}^0-K_P^0=v_{DI}^0\cdot K_P^0-K_P^0=(v_{DI}^0-1)K_P^0=(\Omega_P-1)\cdot K_P^0 \tag{48}$$

This shows an investors' (who own capital and receive dividends) demand-side. If there were a "valuation market" for national accounts like the stock market for individual corporations, Equation 46 shows that the valuation ratio is only determined by Ω_P.

4. Character of the growth rate of dividend valuation value

First, the growth rate of valuation value, g_{VDI}, is defined as follows:

$$g_{VDI}=\frac{V_{DI}^1-V_{DI}^0}{V_{DI}^0} \tag{49}$$

Then, using this definition and $(1+g_{VDI})=(1+g_{KP})(1+g_{\Omega P})$,

$$g_{VDI}=g_{KP}+g_{\Omega P}+g_{KP}\cdot g_{\Omega P}=g_{KP}+g_{\Omega P}(1+g_{KP}) \text{ since } V_{DI}^0=\Omega_P\cdot K_P^0 \tag{50}$$

It shows that the growth rate of valuation value is in favour of Ω_P although Ω_P

22) $v_{SP}^0=\frac{V_{SP}^0}{K_P^0}=\frac{s_{SP/P}\cdot P}{s_{SP/P}\cdot r_P}\cdot\frac{s_{SP/P}\cdot r_P}{S_P}=1$, but this is not used as a usual valuation ratio.

disturbs the sustainable growth rate of output. Capital gains come from a part of this growth rate of valuation value.

5. Character of risk in terms of sustainable growth

Finally, how is risk measured in national economy? Risk may be defined as the variance of the growth rate of output or as the coefficient of variance which is defined as the ratio of variance to the average. Then, how is this average measured? The sustainable growth rate of output g_Y equals the growth rate of labour productivity if the population growth rate is zero. This is set aside (discussed below). Then, what determines g_Y? Equation 20 was shown as;

$$g_Y = \frac{s_{SP/Y}}{1 - s_{SP/Y}} = \frac{\pi}{\Omega_P + 1 - \pi}$$

The stability of g_Y depends on the relative share of profit π and the capital-output ratio Ω_P. The larger the Ω_P the smaller the g_Y. For a sustainable growth in output, the value of Ω_P should be controlled. Nevertheless, this is not easy and the growth rate of labour productivity is usually supported by the increase in Ω_P. The ratio of valuation value is shown by Ω_P, but this is not important compared with the maintenance of g_Y. Remember the condition that Ω_P and π are constant. This was called the corporate-financed growth steady-state, where the growth rate of output g_Y equals the growth rate of corporate capital g_{KP} with some assumptions. The risk-aversion of national economy is defined as such that approaches this condition of $g_Y = g_{KP}$.

In conclusion, this section will close with the following propositions.

Proposition 7 If conditions are set under the corporate-financed growth steady-state, the discount rate of capital, $r_{SP} = r_{DI} = r - g_Y$, is commonly used for the valuation of undistributed profit, dividends, and profit, and this discount rate highly depends on the retention ratio, $s_{SP/P}$, or the undistributed propensity to save divided by π, $s_{SP/Y}/\pi$, which reflects the difference between the cost of profit and the growth rate of output, $r_{SP} = r_{DI} = r - g_Y$, if the rate of profit $\rho \equiv \frac{P}{K_P}$ is constant: $r_{SP} = r - g_Y = \rho(1 + g_Y) - g_Y = \rho - g_Y(1 - \rho)$, where $g_Y = r_{SP} \cdot \Omega_P$.

This proposition shows the relationship between undistributed profit, dividends, and profit in terms of the net present value method, and as a base for the valuation value of dividends which is a final valuation. The proposition implies that the higher the growth rate of output the higher the valuation and the higher the difference between the rate of profit ρ and the cost of profit r. The discount rate for undistributed profit and dividends is determined exactly by dividend policy. This differs from propositions set by Modigliani and Miller [1958, 1963] and Miller and Modigliani [1961] who use arbitrage in the security market. Furthermore, tax burden works differently: they assert that the cost of capital decreases by the use of external savings while the author stresses that the growth rate of output is endogenously measured as a component of the discount rate and depends only on undistributed profit and that tax burden influences the payout ratio (=1-retention ratio).

Proposition 8 If conditions are set under the corporate-financed growth steady-state, the valuation ratio of dividends v_{DI} is determined by the capital-output ratio Ω_P.

As long as the valuation **ratio** of dividends is concerned, this is not influenced by dividend policy. The higher the capital-output ratio the higher the dividend valuation. The higher the use of external savings the higher the valuation value. However, this is at the sacrifice of sustainable growth rate of output. Also, it is noted that the capital-output ratio Ω_P corresponds with the ratio of $s_{(SWD+SP)/Y}$ to $s_{SP/Y}$ as a leverage. M-M's irrelevance of leverage to dividend valuation is denied.

Proposition 9 If conditions are set under the corporate-financed growth steady-state, in the case that Tobin's q equals one and is applied to a macro economy, then corporate capital is only composed of undistributed profit, and the wage and dividend propensity to save, $s_{SWD/WD}$, equals zero.

Proposition 9 implies that Tobins q must be more than one in the case that the wage and dividend propensity to save is positive in a macro economy. Of course, any national economy does not have a market for the financial structure of products.

The stock market exists only for individual listed corporations in the corporate sector. Savings of wages and dividends supply funds for new stock and other security issues and borrowings. A leverage in the financial and stock market may be defined as the ratio of equities and other securities (or, plus borrowings) to equities in the spply-side. However, a leverage in the financial structure of products is defined as the ratio of household and corporate savings, $S_{WD}+S_P$, to corporate savings (undistributed profit), S_P, (or, K_P/K_{SP}) in the demand-side. If corporate capital equals the sum of equities, other securities, and borrowings, the two leverages become similar. However, equities are offset in national accounts and accordingly, capital gains also offset in national accounts.

Also, goods and services are produced by the corporate sector. The household and government sectors do not contribute to "positive" production although social capital is needed for social welfare. The author makes use of the ratio of corporate capital to national capital (as stated above). Proposition 9 may support the higher level of Ω_P, but it is one thing and the necessity for controlling the capital-output ratio is the other.

Proposition 10 If conditions are set under the corporate-financed growth steady-state, the capital valuation of undistributed profit, V^0_{SP}, equals the initial corporate capital, K^0_P, and accordingly the valuation ratio of undistributed profit is one, if the rate of profit ρ is constant. Likewise, the output valuation of undistributed profit, V^0_Y, equals the initial output , Y^0.

These results show that corporate capital or output can be measured using the net present value (NPV) method. A final valuation value is only one and it is the dividend valuation value. However, the above results have their own implication. They are connected with economic depreciation.[23] The accounting depreciation rate d^A_{EP} which is used for the initial capital stock is compared with a discount rate

23) Economic depreciation in national accounts has been measured as the difference between gross and net national income after subtracting indirect taxes. However, this de-↗

$r_{SP}=r-g_Y$ which is converted to an economic depreciation rate d^E_{EP}. The lower the growth rate of output, the higher the economic depreciation rate. The character of economic depreciation in terms of the NPV method was first discussed by Hotelling [1925]. Hotelling uses residual cash flow and the internal rate of return[24] and theoretically measures economic depreciation as the difference between NPV at the end of a period and that at the end of the next period. The capital stock, K^0_P, at a current point of time works for both past and future, particularly in terms of technological progress (see the next section) and human capital.

Proposition 11 If conditions are set under the corporate-financed growth steady-state and if the population growth rate is given, the objective of a national economy is to maintain sustainable growth of output and labour productivity.

This is justified under necessary and sufficient conditions since

1. The cost of profit "r" equals the rate of profit "ρ." Certainly, $r=\rho(1+g_Y)$, but r is measured at the end of a period while ρ is measured at the end of the next period. It implies that there is no room for excess profit. Profit or the cost of profit remains a condition. Investment theory in individual firms, invest if the rate of profit is larger than the cost of capital, is not applied to the national economy.
2. The dividend valuation ratio is determined by the magnitude of the capital-output ratio, but contradicts the sustainable growth rate of output. Capital market absorbs household savings, but is not related to the sustainable growth rate of output. Furthermore, corporate capital must be capital-augmenting

↘ preciation also takes into account corresponding accounting depreciation in the SNA of many countries. There has not been a right methodology for measuring economic depreciation in the past.

24) It has been argued that it is no way to measure the internal rate of return (IRR) using accounting data. $r=\rho(1+g_Y)$ in the corporate-financed growth steady-state corresponds with this IRR. Also, residual cash flow corresponds with undistributed profit. As a result, the discount rate r_{SP} corresponds with d^E_{EP}.

(capital-saving) for less risky growth of output. Note that lower the growth rate of output, the higher the economic depreciation rate, and check the increase in capital by investment. The corporate-financed growth steady-state does not diverge in discrete time.

3. The sustainable growth rate of output is most effectively attained under the corporate-financed growth steady-state, where the capital-output ratio and the relative share of profit are constant. The balanced growth steady-state may be an ideal. However, it cannot be measured and it has contradictions as discussed under the corporate-financed growth steady-state. The population growth rate is separated, but finally, the corporate-financed growth steady-state can accept this growth rate if investment is technology-oriented.
4. If the polulation growth rate is zero, the growth rate of output equals the growth rate of labour productivity. This is attained by investment using the coefficient of technological progress and its growth rate, totally supported by human capital. Without this improvement, the growth rate of labour productivity is not maintained and as a result, the national economy suffers from business cycle. This is also applicable to individual firms. Profit maximization is not directly connected with investment, and remains as a result.

5.2 Labour productivity, the population growth rate, and technological progress

This issue is discussed in more detail in another paper [1997/Oct]. However, as discussed above, this issue is closely related to the above propositions. Let the author first illustrate the functions of the capital-output ratio Ω_P in terms of the investment ratio.

The investment ratio I/Y_0 has the two aspects.

1. **The saving-side:** $s_{S/Y}=s_{SP/P}\cdot\pi\cdot\Omega_P$ (Equation 30; note $I/Y=s_{S/Y}$ and $I/Y^0=s_{S/Y}(1+g_Y)$)

2. **The investment-side:** $g_y = m^* \cdot \frac{I}{Y^0} = m^* \cdot g_Y \cdot \Omega_P$ (see below)

	Saving-side			Investment-side	
Ω_P	$s_{SP/P}$	$s_{SP/Y}$	$g_Y = g_{KP}$	$g_y = g_{Y/NE}$	Ω_P
	π		g^e_{NE}	m^*	

For the investment-side, it is summarized using the expected population (workers) growth rate g^e_{NE}, labour productivity $g_y = g_{Y/NE}$, the coefficient of technological progress m*, and the rate of technological progress g^*_m. The expected population growth rate, g^e_{NE} (where, N_E is population or workers) is a parameter and given. When the growth rate of output g_Y is measured, the growth rate of labour productivity g_y is derived under given g^e_{NE}. It implies that the corporate-financed growth steady-state holds under any value of g^e_{NE}. This relationship is expressed as an additivity, but this additivity in discrete time differs from that in continuous time:

1. Discrete time: $g_Y = g^e_{NE} + g_y + g^e_{NE} \cdot g_y$
2. Continuous time: $g_Y = g^e_{NE} + g_y$ [Solow, 1956, Pasinetti, 1962, p.276]

In the balanced growth steady-state, the growth rate of labour productivity, $g_y = g_{Y/NE} \equiv (y^1 - y^0)/y^0$, is zero. In the corporate-financed growth steady-state, if the growth rate of output is larger than the population growth rate, the growth rate of labour productivity g_y is positive, and vice versa.

The author advocates that the coefficient of technological progress, m*, is measured as the the following relationship between the investment ratio, I/Y^0, and g_y:

$$m^* = \frac{g_y}{I/Y^0} \tag{51}$$

Based on this equation, the structure of labour productivity is set up step by step.

$$\left(\frac{Y^0}{N^0_E}\right)(1+g_{Y/NE}) = \frac{Y^0(1+g_Y)}{N^0_E(1+g^e_{NE})} \tag{52}$$

$$g_Y = g_{Y/NE} + g^e_{NE} + g_{Y/NE} \cdot g^e_{NE}\text{[25]} \qquad g_y = g_{Y/NE} = \frac{g_Y - g^e_{NE}}{1+g^e_{NE}} \tag{53}$$

Enter the growth rate of output Equation 21,

$g_Y \equiv \frac{S_P}{Y^0} = s_{SP/Y}(1+g_Y) = \frac{\pi}{\Omega_P+1-\pi}$, into the above (53).

$$g_y = g_{Y/NE} = \frac{g_Y - g^e_{NE}}{1+g^e_{NE}} = \frac{\pi - g^e_{NE}(\Omega_P+1-\pi)}{(1+g^e_{NE})(\Omega_P+1-\pi)} \tag{54}$$

Since $\frac{I}{Y^0} = \frac{\Delta K}{K^0} \cdot \frac{K^0}{Y^0} = g_K \cdot \Omega_P = g_Y \cdot \Omega_P = \frac{\Omega_P \cdot \pi}{\Omega_P+1-\pi}$ (23), the value of m* is obtained using the above Equation 48:

$$m^* = \frac{\pi - g^e_{NE}(\Omega_P+1-\pi)}{(1+g^e_{NE})(\Omega_P+1-\pi)} \cdot \frac{\Omega_P+1-\pi}{\Omega_P \cdot \pi} = \frac{\pi - g^e_{NE}(\Omega_P+1-\pi)}{\Omega_P \cdot \pi(1+g^e_{NE})} \tag{55}$$

It implies that the coefficient of technological progress m* is determined by Ω_P, π, and g^e_{NE}, where human capital cooperates with factors. If $g^e_{NE}=0$, then, $m^*=1/\Omega_P=Y/K_P$. It implies that m* is determined by the initial productivity of capital, Y/K_P. The value of m* changes when Ω_P, π, and/or g^e_{NE} change with net investment.

Furthermore, the rate of technological progress, g^*_m, is defined as follows:

$$g^*_m = \frac{m^{*1} - m^{*0}}{m^{*0}} \tag{56}$$

25) When continuous time is used, the value of $g_{Y/NE} \cdot g^e_{NE}$ becomes zero by differentiation. The discrete time cannot omit this value [Tokimasa, 1997]. The difference is important and proved as follows: Suppose $Y=X \cdot Z$: $Y(t+1)=X(t+1) \cdot Z(t+1)$ and $Y(t)=X(t) \cdot Z(t)$

Then, $Y(t+1)-Y(t)=X(t+1) \cdot Z(t+1)-X(t) \cdot Z(t)$

By dividing both sides by $Y(t)=X(t) \cdot Z(t)$,

$$\frac{Y(t+1)-Y(t)}{Y(t)} = \frac{X(t+1)Z(t+1)-X(t)Z(t)}{X(t)Z(t)} = \frac{X(t+1)\{Z(t+1)-Z(t)\}+\{Z(t+1)-Z(t)\}Z(t)}{X(t)Z(t)}$$

$$= \frac{X(t+1)}{X(t)} \cdot \frac{Z(t+1)-Z(t)}{Z(t)} + \frac{X(t+1)-X(t)}{X(t)} = \frac{X(t+1)}{X(t)} \cdot g_Z + g_X$$

Using $\frac{X(t+1)}{X(t)} = \frac{X(t+1)-X(t)}{X(t)} + 1 = g_X + 1$, $g_Y=(1+g_X)g_Z+g_X=g_Z+g_X+g_X \cdot g_Z$

6. Integration of the saving-side and the investment-side: propensities to save and technological progress

6.1 Integration of the saving-side and the investment-side without technological progress

The investment ratio as I/K_P^0 differs from I/K_P which is equal to the total saving propensity to save, $s_{S/Y}$. As a result, I/K_P^0 is shown as $s_{S/Y}(1+g_Y)$. This is an expression of the saving-side. On the other hand, the investment ratio as I/K_P^0 is shown as $g_Y \cdot \Omega_P$ which is equal to g_y/m^*. This is an expression of the investment-side. There are two approaches for this integration: one is the integration without introducing technological progress, and the other is the integration with introducing technological progress. This section treats the first one. The methodology uses the investment ratio together with the fixed growth rate of output, g_Y as follows:

For the saving-side:

$$\frac{I}{Y^0}=\frac{I}{Y}(1+g_Y)=s_{S/Y}(1+g_Y)=s_{SP/Y}\cdot\Omega_P(1+g_Y)=\frac{\pi\cdot\Omega_P}{\Omega_P+1}(1+g_Y)=\frac{\pi\cdot\Omega_P}{\Omega_P+1-\pi}$$

For investment-side:

$$\frac{I}{Y^0}=g_Y\cdot\Omega_P,\ \text{and thus,}\ g_Y\cdot\Omega_P=\frac{\pi\cdot\Omega_P}{\Omega_P+1-\pi} \tag{57}$$

Both sides show the same result as above. Then, let the author fix the values of $s_{SP/P}$ and g_Y:

First, $s_{SP/P}=\dfrac{1}{\Omega_P+1}$ and accordingly,

$$\Omega_P=\frac{1-s_{SP/P}}{s_{SP/P}} \tag{58}$$

Second, $g_Y=\dfrac{\pi}{\Omega_P+1-\pi}$ and accordingly,

$$\pi=\frac{g_Y(\Omega_P+1)}{1+g_Y}\ \text{under fixed}\ g_Y. \tag{59}$$

Enter Equation 58 into Equation 59,

$$\pi = \frac{g_Y}{s_{SP/P}(1+g_Y)} \tag{60}$$

It shows that Method 1 proves that the relative share of profit, π, is determined by a given retention ratio, $s_{SP/P}$, which is another expression of the capital-output ratio, Ω_P, and a given growth rate of output, g_Y, if the population growth rate, g^e_{NE}, and technological progress, m^* or g^*_m, are not introduced. It implies that a sort of Golden Age under fixed Ω_P and π changes with given or planned $s_{SP/P}$ and g_Y, but without introducing g^e_{NE} and m^* or g^*_m. It is stressed that π and Ω_P change at the same time under this condition.

6.2 Integration of the saving-side and the investment-side with technological progress

It is now suggested that the saving-side and the investment-side should be integrated by introducing technological progress.

For the saving-side:

$$\frac{I}{Y^0} = s_{SP/Y} \cdot \Omega_P(1+g_Y) = g_Y \cdot \Omega_P$$

$$g_Y(\pi, \Omega_P) = \frac{\pi}{\Omega_P + 1 - \pi}, \text{ where no parameter exists.}$$

For investment-side with the coefficient of technological progress m*:

$$\frac{I}{Y^0} = g_Y \cdot \Omega_P = \frac{g_y}{m^*}$$

$$g_Y(\pi, \Omega_P) = \frac{m^* \cdot \pi \cdot \Omega_P(1+g^e_{NE}) + g^e_{NE} \cdot M}{M}, \tag{59}$$

where, $M \equiv \Omega_P + 1 - \pi$, and g^e_{NE} and m^* are parameters.

$$m^*(\pi, \Omega_P) = \frac{\pi - g^e_{NE} \cdot M}{\Omega_P \cdot \pi(1+g^e_{NE})} \tag{60}$$

$$\pi(\Omega_P) = \frac{g^e_{NE}(\Omega_P + 1)}{(1+g^e_{NE})(1 - m^* \cdot \Omega_P)} \tag{61}$$

$$\Omega_P(\pi)=\frac{\pi(1+g^e_{NE})-g^e_{NE}}{\pi\cdot m^*(1+g^e_{NE})+g^e_{NE}} \tag{62}$$

Using π of the saving-side which sets g_Y fixed, $\pi=\dfrac{g_Y(\Omega_P+1)}{1+g_Y}$,

$$\Omega_P(\pi)=\frac{\pi(1+g_Y)-g_Y}{g_Y} \tag{63}$$

$$\pi(\Omega_P)=\frac{g_Y(\Omega_P+1)}{1+g_Y} \tag{64}$$

There are two methods for obtaining the relationship between π and Ω_P although the results are the same:

Method 1 First, obtain π: enter Equation 63 into Equation 61, and then, obtain Ω_P using Equation 63

Method 2 First, obtain Ω_P: enter Equation 64 into Equation 62 (or use Equation 62 and 63 each on the RHS), and then, obtain π using Equation 64

By using Method 1, the following quadratic function of π is shown.

$A\cdot\pi^2+B\cdot\pi=0$,

where, $A=-m^*(1+g_Y)(1+g^e_{NE})$

$B=g_Y-g^e_{NE}+m^*\cdot g_Y(1+g^e_{NE})$

As a result, $$\pi=\frac{-B-\sqrt{B^2}}{2A}=-\frac{B}{A} \tag{65}$$

By using Method 1, the value of Ω_P is shown as

$$\Omega_P=-\frac{g^e_{NE}-g_Y}{g_Y\cdot m^*(1+g^e_{NE})} \tag{66}$$

By using Method 2, the following quadratic function of Ω_P is shown.

$A\cdot\Omega_P^2+B\cdot\Omega_P+C=0$,

where, $A=-m^*\cdot g_Y(1+g^e_{NE})$

$B=-m^*\cdot g_Y(1+g^e_{NE})+g_Y-g^e_{NE}=A+C$

$C=g_Y-g^e_{NE}$

As a result, $\Omega_P=\dfrac{-B-\sqrt{B^2-4A\cdot C}}{2A}=-\dfrac{C}{A}$

$$\text{or, } \Omega_P = -\frac{g^e_{NE} - g_Y}{g_Y \cdot m^*(1+g^e_{NE})} \tag{66}$$

$$\text{Then, } \pi = \frac{g_Y - g^e_{NE} + m^* \cdot g_Y(1+g^e_{NE})}{m^*(1+g_Y)(1+g^e_{NE})}\text{, equal to (65)}$$

Both quadratic functions hold only when the sign before each square is negative. The other values which do not hold are $\pi=0$ and $\Omega_P=-1$. The quadratic function of Ω_P becomes a linear function of Ω_P if it is divided by "Ω_P+1."[26] In short, if the values of g_Y and m* are given as parameters, then, π and Ω_P simultaneously change and are measured. Of course, it is possible to set either π or Ω_P as a parameter.

Proposition 12 If conditions under the corporate-financed growth steady-state change with a given population growth rate, then, the coefficient of technological progress, m*, is a criterion for net investment which simultaneously changes the values of Ω_P, π, and g^e_{NE}.

It implies that any prevailing rule for investment which individual corporations could take does not hold in a national economy and that Proposition 12 is also applicable to individual corporations. Proposition 12 may guarantee sustainable growth for any organization.

Proposition 13 If conditions under the corporate-financed growth steady-state changes with a given population growth rate, it is rather difficult to maintain the same level of m*.

It implies that there is no difference of g_Y or the investment ratio between countries in long periods of time. Or it implies that the "convergence controversy" [Heston and Summers, 1991; Romer, P., 1994] holds. On the contrary, the convergence controversy does not hold if the value of m* continue to increase or decrease in spite of technological progress. The corporate-financed growth steady-state corresponds with a Golden Age shown by an equality that "the natural rate of growth, $g_n=n+\lambda$, equals the warranted rate of growth, g" [Pasinetti, 1974, p.96]. The value

26) The author is obliged to Prof. Furuta's proof for Equations 65 and 66.

of g_n corresponds with "$g^e_{NE}+g_y+g^e_{NE}\cdot g_y$." In short periods of time, if the value of m* increases, then, a base for the growth rates of output and labour productivity, g_Y and g_y, is strengthened.

Proposition 14 If a national economy would have a sustainable growth rate of output, technological progress should focus on how effectively to adjust the capital-output ratio and increase the relative share of profit in terms of both saving and investment sides, by using $s_{S/Y}=s_{SP/P}\cdot\pi\cdot\Omega_P$ and $g_y=m^*\cdot\frac{I}{Y^0}=m^*\cdot g_Y\cdot\Omega_P$.

As clarified by a panel data approach (below), labour productivity $y=Y/N_E$ grows with the increase in the capital-labour ratio $\Omega_P\equiv\frac{K_P}{N_E}$, but this is not guaranteed in long periods of time. A sustainable growth of output is difficult to control unless the capital-output ratio decreases. Furthermore, the growth rate of output is replaced by the growth rate of labour productivity if $g^e_{NE}=0$. Business cycle is accelerated by the rapid increase in k and Ω_P. The value of m* checks the level of k, but it is difficult to maintain a certain level of g_Y. This comes from the structure of $\pi=\Omega_P\cdot\rho$ of the author's or the origin of endogenous growth of Romer, P. [1994, pp.3-4]. π and ρ has each much less coefficient of variation (≡variance/average) while the coefficient of variation of Ω_P is considerably high. It is suggested that an effective investment ratio supported by technological progress combined with human capital should decrease the value of Ω_P.

7. Some empirical results and implications

The observation and regression analysis using a panel data approach are considerably important in this paper. The panel data approach is well accepted in recent literature. The above importance is because that no one advocates that the growth rate of output is exclusively determined by the theoretical undistributed profit propensity to save, $s_{SP/Y}$, and that this $s_{SP/Y}$ comes from the initial value of the capital-output ratio Ω_P. Empirical works are easy in discrete time and particularly in the case of the author's model which uses the initial several values. The several initial values are: net national income Y^0, profit P^0, dividends D_I^0 which uses the actual dividend propensity to save as "expected", undistributed profit $S_P^0(=P^0-D_I^0)$ which uses the actual undistributed profit propensity to save as "expected", corporate capital K_P^0, and the number of workers N_E^0. These initial values at the beginning of a period come from those realized "nominal" values at the end of the previous period, but all set as "real" under an assumption that these values are under fixed price level for the two periods. The expected growth rate of population (workers), g_{NE}^e, can be replaced by g_{NE}^1, one period to the past. The author tried to get these data using OECD National Accounts. However, some countries do not publish capital stock by sector, and others do not publish dividends paid by the corporate sector. As a result, six countries were chosen: Japan, Sweden, UK, USA, Germany, and Australia. Number of points is 72 or 48, in 1982-1994, as panel data. For comparison of labour productivity $y \equiv Y/N_E$ and the capital-labour ratio $k \equiv K_P/N_E$ the exchange rate is required and BNZ rates on 3 March 1997 were taken for adjustment. This is discussed in labour productivity in coming paper [1997, Oct].

For a panel data approach, most important is the relationship among the actual real growth rates of net national income and corporate capital, and the actual (shown as expected) undistributed profit propensity to save, $s_{SP/Y}^e$. However, for comparison, propensities to save and their combinations, s_p^e or $s_{SP/P}^e$, s_w^e or $s_{SWD/WD}^e$, $s_{SP/Y}^e$, $s_{SWD/Y}^e$, and $s^e=s_{(SP+SWD)/Y}^e$, were also used. The methodology includes average, variance, the coefficient of variation, and t-value each by item, and the regression analysis by equation. Their results and graphs are shown by period, country, and as a whole. Some of results are shown as **Table 1**, and **Figures 4-9**.

First, the relationship between vital ratios is shown in Table 1. This is a base for statistical analysis. Interesting to say, some ratios by period and by country have much unfavorable coefficient of variation and t-value while others much favorable ones as follows:

Table 1 Average, variance, coefficient of variation, and t-value for basic ratios in four countries, 1982–1994

Test data for O'Connell's model

Observa.: 4	g_Y^{NOM}	s_W^e	$s_p^e(1-s_w^e)$	$s_p^e \cdot s_W^e$	$s_p^e - s_W^e$	r	$1/\Omega_p$	g_{KP}^{NOM}
Average: E	0.0627	0.0376	0.4174	0.0140	0.3938	0.0350	0.5699	0.0700
Variance: V	0.0010	0.0031	0.0733	0.0008	0.0857	0.0003	0.0353	0.0055
Coe. of varia	0.0158	0.0815	0.1757	0.0589	0.2176	0.0098	0.0620	0.0785
t-value	0.0091	0.0161	0.0786	0.0083	0.0850	0.0054	0.0546	0.0215

Test data for the corporate-financed growth model of the author's (1)

Observa.: 4	$s^e = s_{S/Y}^e$	$s_p^e = s_{SP/P}^e$	s^e/s_p^e	$\pi \cdot \Omega_p$	$\pi \cdot \Omega_p$	π	Ω_p	ρ
Average: E	0.0566	0.4314	0.1718	0.1146	0.1122	0.0597	1.9623	0.0350
Variance: V	0.0008	0.0781	0.2877	0.0029	0.0029	0.0004	0.4352	0.0003
Coe. of varia	0.0147	0.1810	1.6750	0.0253	0.0258	0.0072	0.2218	0.0098
t-value	0.0084	0.0811	0.1557	0.0156	0.0126	0.0060	0.1915	0.0054
					Observa.: 72			

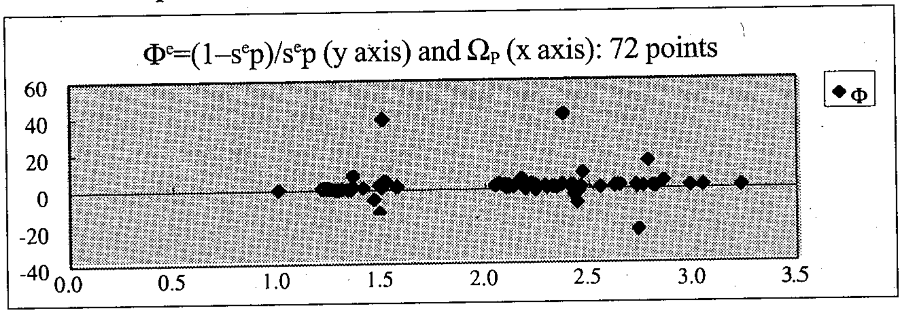
Test data for the corporate-financed growth model of the author's (2)

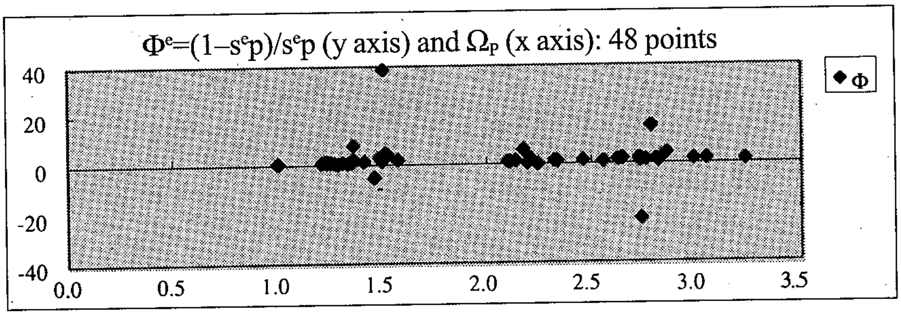
Observa.: 4	$g_Y = g_{KP}$	m^*	$\Omega_p \cdot m^*$	$\Omega_p \cdot m^*$	$\Phi^e = (1-s_p^e)/s$	$\Phi^e = (1-s_p^e)/s$	Ω_p/Φ^e	Ω_p/Φ^e
Average: E	0.0221	0.4002	0.5774	0.6162	1.6973	2.1088	1.7829	1.4771
Variance: V	0.0001	0.7332	2.4495	25.8135	52.6375	58.83	9.7159	7.1029
Coe. of varia	0.0046	1.8321	4.2423	41.8914	31.0125	27.8974	5.4495	4.8087
t-value	0.0029	0.2486	0.4545	1.1939	2.1067	1.8024	0.9051	0.0626
				Observa.: 72		**Observa.: 72**		**Obser.: 72**

Note The t-value which uses 0.025 is 2.010.
The coefficient of vairation is defined as variance divided by average.
The author uses ρ instead of r (r is used in O'Connell's model).
Saving-side: $s_{S/Y}^e = s_{SP/P}^e \cdot \pi \cdot \Omega_p$, and Ω_p/Φ^e
Investment-side: $g_y = g_Y \cdot \Omega_p \cdot m^*$, and $g_y = m^* \cdot I/Y^0$ under a given g_{NE}^e

1. The wage and dividend propensity to save, $s_w^e = s_{SWD/WD}^e$, and the undistributed profit to propensity to save, $s_{SP/Y}^e$, fluctuate by period, country and as a whole (72 or 48 points).
2. The total propensity to save, $s^e = s_{(SP+SWD)/Y}^e$, and the relative share of profit, π, are considerably stable, in contrast with the capital-output ratio Ω_P and the coefficient of technological progress m^e.

Figure 4 Relationship between Φ^e and Capital-output ratio Ω_P as a base for the growth rate of output





(1–sp)/sp=f(Ωp)

Japan, Sweden, UK, Germany: observation 48

Y: (1–sp)s=Φ

Intercept (Alpha)=2.993 t=0.8942

X:Ωp Beta=–0.6618 t=–0.4088

(1–sp)/sp=2.9936–0.6618**Ωp**

R^2=0.0036 F=0.6846

Ωp=(1–sp)/sp

Japan, Sweden, UK, Germany: observation 48

Y: Ωp The capital-output ratio

Intercept (Alpha)=1.97 t=19.9734

X: (1–sp)/sp Beta=–0.0055 t=–0.4088

Ωp=1.9716–0.0055**(1–sp)/sp**

R^2=0.0036 F=0.6846

n=**sw+sp(1–sw)r**

Japan, Sweden, UK, Germany: observation 48

Y: n The growth rate of output

Intercept (Alpha)=0.056 t=5.7396

X1: sw Beta1=–0.0015 t=–0.01677

X2: sp(1–sw) Beta2=r=0.0143 t=0.8103

n=0.0568–0.0015+0.0143**r**

R^2=0.01541 F=0.7051

n=sw+sp(1–sw)**r**

Japan, Sweden, UK, Germany: observation 48

Y: n The growth rate of output

Intercept (alpha)=sw=0.0310 t=3.7179

X1: r Beta1=sp(1–sw)=0.9059 t=4.2911

n=0.0310+0.9059**r**

R^2=0.2859 F=0.00009

n=r***(sp–sw)**+(1/Ωp)***sw**

Japan, Sweden, UK, Germany: observation 48

Y: n The growth rate of output

Intercept (Alpha)=0.058 t=6.072

X1: sp–sw Beta1=r=0.01182 t=0.7008

X2: sw Beta 2=1/Ωp=0.0022 t=0.0247

n=0.058+0.01182**(sp–sw)**+0.0022**sw**

R^2=0.01183 F=0.7668

n=(sp–sw)**r**+sw/**Ωp**

Japan, Sweden, UK, Germany: observation 48

Y: n The growth rate of output

Intercept (alpha)=0.0223 t=1.7277

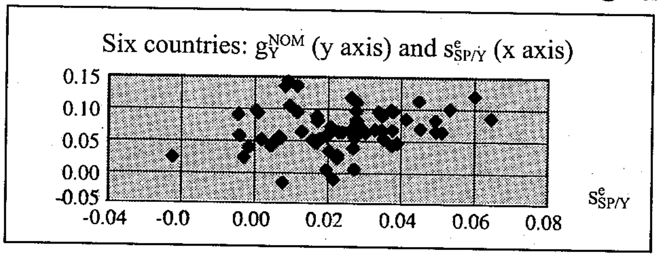
X1: r Beta1=(sp–sw)=0.6672 t=0.8738

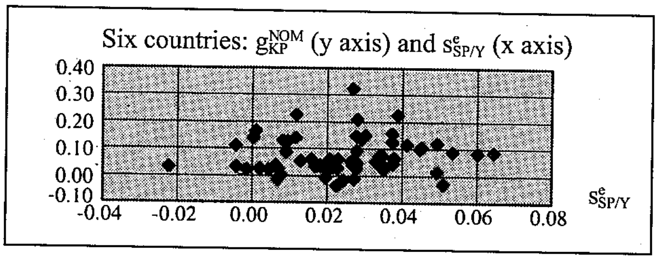
X2: 1/Ωp Beta2=sw=0.02983 t=1.9309

n=0.02238+0.6672**r**+0.02983**(1/Ωp)**

R^2=0.2978 F=0.00035

Figure 5 Regression analysis using panel data, 1982–1994: using $S^{e}_{SP/Y}$





Six countries: g^{NOM}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{Y} Observation 72
Intercept (Alpha)=g^{NOM}_{Y}=0.05638 t=9.3007
X:$s_{SP/Y}$ Beta=0.4743 t=2.2079
$g^{NOM}_{Y}=0.05638+0.4743s_{SP/Y}$
$R^2=0.01749$ F=0.0305

Six countries: g^{REAL}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{REAL}_{Y} Observation 72
Intercept (Alpha)=0.0122 t=3.1461
X: $s_{SP/Y}$ Beta=0.4961 t=3.6034
$g^{REAL}_{Y}=0.0122+0.4961s_{SP/Y}$
$R^2=0.1564$ F=0.0006

Six countries: g^{NOM}_{KP} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{KP} Observation 72
Intercept (Alpha)=0.05766 t=4.7118
X: $s_{SP/Y}$ Beta=0.4840 t=1.1163
$g^{NOM}_{KP}=0.05766+0.4840s_{SP/Y}$
$R^2=0.0175$ F=0.2682

Six countries: g^{REAL}_{KP} (y axis) and $s_{SP/Y}$ (y axis)
Y: g^{REAL}_{KP} Observation 72
Intercept (Alpha)=0.0137 t=1.2802
X: $s_{SP/Y}$ Beta=0.6140 t=1.6240
$g^{REAL}_{KP}=0.0137+0.6140s_{SP/Y}$
$R^2=0.0651$ F=0.1089

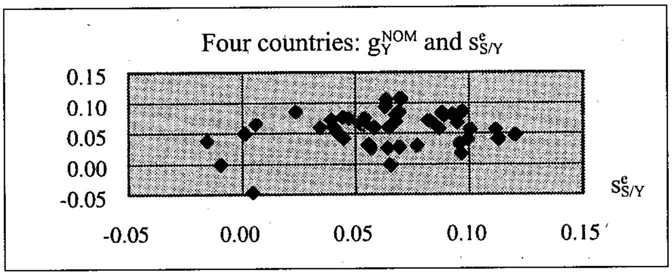
Japan: g^{NOM}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{Y} Observation 12
Intercept (Alpha)=–0.0098 t=–0.3606
X: $s_{SP/Y}$ Beta=1.8584 t=2.1657
$g^{NOM}_{Y}=-0.0098+1.8584s_{SP/Y}$
$R^2=0.3193$ F=0.0556

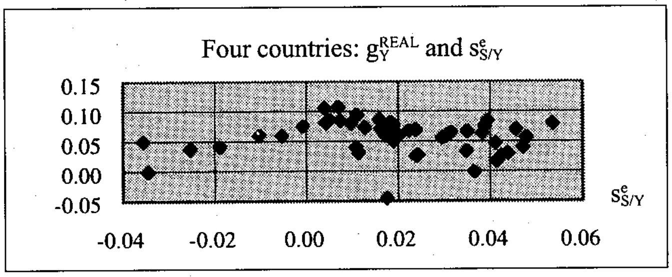
UK: g^{NOM}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{Y} Observation 12
Intercept (Alpha)=0.0630 t=6.6808
X: $s_{SP/Y}$ Beta=0.6154 t=1.8743
$g^{NOM}_{Y}=0.0630+0.6154s_{SP/Y}$
$R^2=0.2600$ F=0.0904

Sweden: g^{NOM}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{Y} Observation 12
Intercept (Alpha)=0.0342 t=1.5172
X: $s_{SP/Y}$ Beta=1.0907 t=2.0095
$g^{NOM}_{Y}=0.0342+1.0907s_{SP/Y}$
$R^2=0.2876$ F=0.0722

Germany: g^{NOM}_{Y} (y axis) and $s_{SP/Y}$ (x axis)
Y: g^{NOM}_{Y} Observation 12
Intercept (Alpha)=0.0379 t=4.8765
X: $s_{SP/Y}$ Beta=1.0922 t=2.7142
$g^{NOM}_{Y}=0.0379+1.0922s_{SP/Y}$
$R^2=0.4242$ F=0.0218

Figure 6 Regression analysis using panel data, 1982–1994: using $s^e_{S/Y}$





Japan, Sweden, UK, Germany: g^{NOM}_Y and $s_{S/Y}$
Y: g^{NOM}_Y Observation 48 X: $s_{S/Y}$
Intercept (Alpha)=g^{NOM}_Y=0.0492 t=4.9527
Beta=$g^{NOM}_Y/s_{S/Y}$=0.2394 t=1.5293
$g^{NOM}_Y=0.0492+0.2394s_{S/Y}$
R^2=0.0484 F=0.1331

Japan, Sweden, UK, Germany: g^{REAL}_Y and $s_{S/Y}$
Y: g^{REAL}_Y Observation 48 X: $s_{S/Y}$
Intercept (Alpha)=g^{REAL}_Y=0.0196 t=2.8946
Beta=$g^{REAL}_Y/s_{S/Y}$=–0.0121 t=–0.1133
$g^{REAL}_Y=0.0196-0.0121s_{S/Y}$
R^2=0.00028 F=0.9103

Japan: g^{NOM}_Y and $s_{S/Y}$
Y: g^{NOM}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{NOM}_Y=0.0492 t=4.9527
Beta=$g^{NOM}_Y/s_{S/Y}$=0.2394 t=1.5293
$g^{NOM}_Y=0.0492+0.2394s_{S/Y}$
R^2=0.0484 F=0.1331

Japan: g^{REAL}_Y and $s_{S/Y}$
Y: g^{REAL}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{REAL}_Y=0.0192 t=3.2039
Beta=$g^{REAL}_Y/s_{S/Y}$=–0.0696 t=–0.9589
$g^{REAL}_Y=0.0192-0.0696s_{S/Y}$
R^2=0.0842 F=0.3602

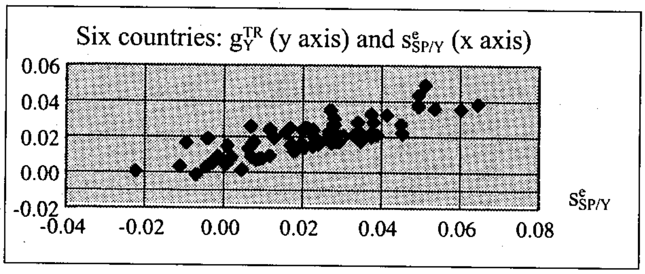
Sweden: g^{NOM}_Y and $s_{S/Y}$
Y: g^{NOM}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{NOM}_Y=0.0324 t=1.3855
Beta=$g^{NOM}_Y/s_{S/Y}$=0.8269 t=2.0045
$g^{NOM}_Y=0.0324+0.8269s_{S/Y}$
R^2=0.2866 F=0.0728

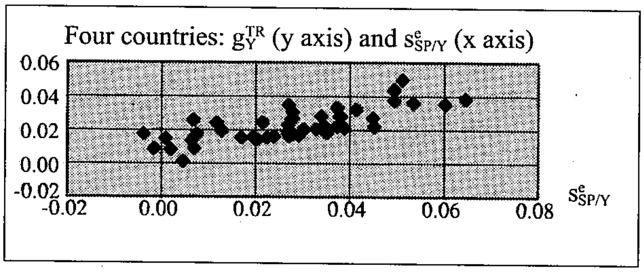
Sweden: g^{REAL}_Y and $s_{S/Y}$
Y: g^{REAL}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{REAL}_Y=0.0014 t=0.0943
Beta=$g^{REAL}_Y/s_{S/Y}$=0.2037 t=0.7672
$g^{REAL}_Y=0.0014+0.2037s_{S/Y}$
R^2=0.0556 F=0.4607

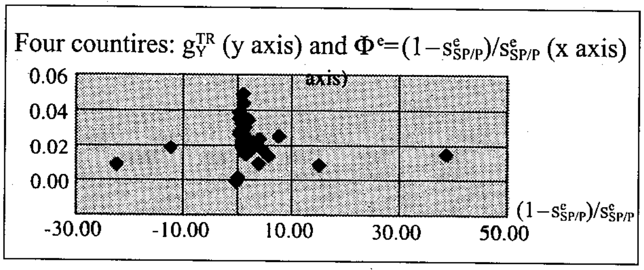
UK: g^{NOM}_Y and $s_{S/Y}$
Y: g^{NOM}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{NOM}_Y=0.0654 t=3.7414
Beta=$g^{NOM}_Y/s_{S/Y}$=0.3119 t=0.7234
$g^{NOM}_Y=0.0654+0.3119s_{S/Y}$
R^2=0.0497 F=0.4860

UK: g^{REAL}_Y and $s_{S/Y}$
Y: g^{REAL}_Y Observation 12 X: $s_{S/Y}$
Intercept (Alpha)=g^{REAL}_Y=0.0411 t=2.4335
Beta=$g^{REAL}_Y/s_{S/Y}$=–0.3933 t=–0.9443
$g^{REAL}_Y=0.0411-0.3933s_{S/Y}$
R^2=0.0819 F=0.3673

Figure 7 Regression analysis using panel data, 1982–1994: using theoretical real growth rate, g_Y^{TR} as g_Y







Six countries: g_Y^{TR} and $s_{SP/Y}$
Y: g_Y^{TR} Observation 72
Intercept (Alpha)=g_Y^{TR}=0.0094 t=8.1937
X: $s_{SP/Y}$ Beta=0.4674 t=11.5110
$g_Y^{TR}=0.0094+0.4674s_s$ $R^2=0.6543$ F=0.00001

Four countries: g_Y^{TR} and $s_{SP/Y}$
Y: g_Y^{TR} Observation 48
Intercept (Alpha)=g_Y^{TR}=0.0107 t=6.4277
X: $s_{SP/Y}$ Beta=0.4397 t=8.2536
$g_Y^{TR}=0.0107+0.4397s_s$ $R^2=0.5969$ F=0.00001

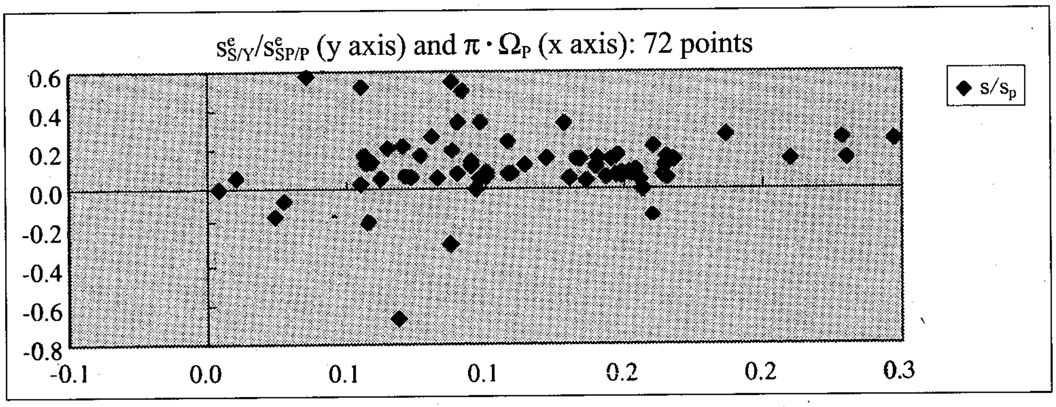
Six countries: g_Y^{NOM} and $g^{RT}Y$
Y: g_Y^{NOM} Observation 72
Intercept (Alpha)=0.055 t=6.6016
X: g_Y^{TR} Beta=0.5840 t=1.5447
$g_Y^{NOM}=0.0554+0.5840g_Y^{TR}$
$R^2=0.0323$ F=0.1269

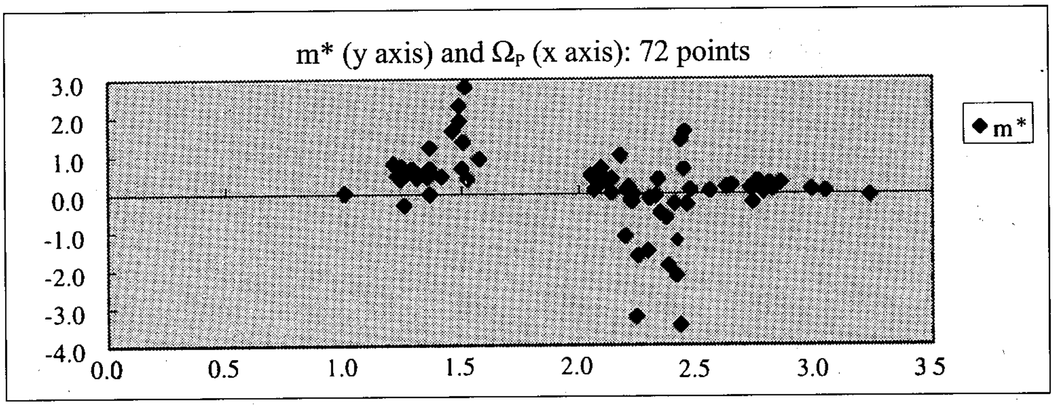
Four countries: g_Y^{NOM} and $g^{RT}Y$
Y: g_Y^{NOM} Observation 48
Intercept (Alpha)=0.027 t=2.8681
X: g_Y^{TR} Beta=1.5966 t=4.0428
$g_Y^{NOM}=0.0275+1.5966g_Y^{TR}$
$R^2=0.2622$ F=0.0002

Four countries: g_Y^{TR} (y axis) and $s_{SP/Y}/(1-s_{SP/Y})$ (x axis)
Y: g_Y^{TR} Observation 48
Intercept (Alpha)=0.010 t=6.5947
X: $s_{SP/P}/(1-s_s)$ Beta=0.4185 t=8.3201
$g_Y^{TR}=0.0108+0.4185s_{SP/P}/(1-s_{SP/Y})$
$R^2=0.6008$ F=0.00000

Four countries: g_Y^{TR} (y axis) and $(1-s_{SP/P})/s_{SP/P}$
Y: g_Y^{TR} Observation 48
Intercept (Alpha)=0.022 t=14.6904
X: $(1-s_{SP/P})$/Beta=–0.00006 t=–0.2824
$g_Y^{TR}=0.0222-0.00006(1-s_{SP/P})/s_{SP/P}$
$R^2=0.0017$ F=0.7789

Figure 8 Regression analysis using panel data, 1982–1994: $s^e_{S/Y}/s^e_{SP/P}$ and the coefficient of tech. progress m*





Six countries: observation 72

Y: $\pi \cdot \Omega_P$

	Intercept (Alpha)=0.113	t=17.0667
X: $(s_{S/Y})/(s_{SP/P})$	Beta=–0.01	t=–0.8347

$\pi \cdot \Omega_P = 0.1139 - 0.0101\,(s_{S/Y})/(s_{SP/P})$ $R^2=0.0099$

$F=0.4067$

Six countries: observation 72

Y: m*

	Intercept (Alpha)=1.206	t=1.2521
X: Ω_P	Beta=–0.409	t=–0.9104

$m^* = 1.2066 - 0.4092\Omega p$ $R^2=0.0117$

$F=0.3657$

Four countries: observation 48

Y: $\pi \cdot \Omega_P$

	Intercept (Alpha)=0.115	t=13.9879
X: $(s_{S/Y})/(s_{SP/P})$	Beta=–0.00	t=–0.3226

$\pi \cdot \Omega_P = 0.1154 - 0.0048(s_{S/Y})/(s_{SP/P})$ $R^2=0.0023$

$F=0.7485$

Four countries: observation 48

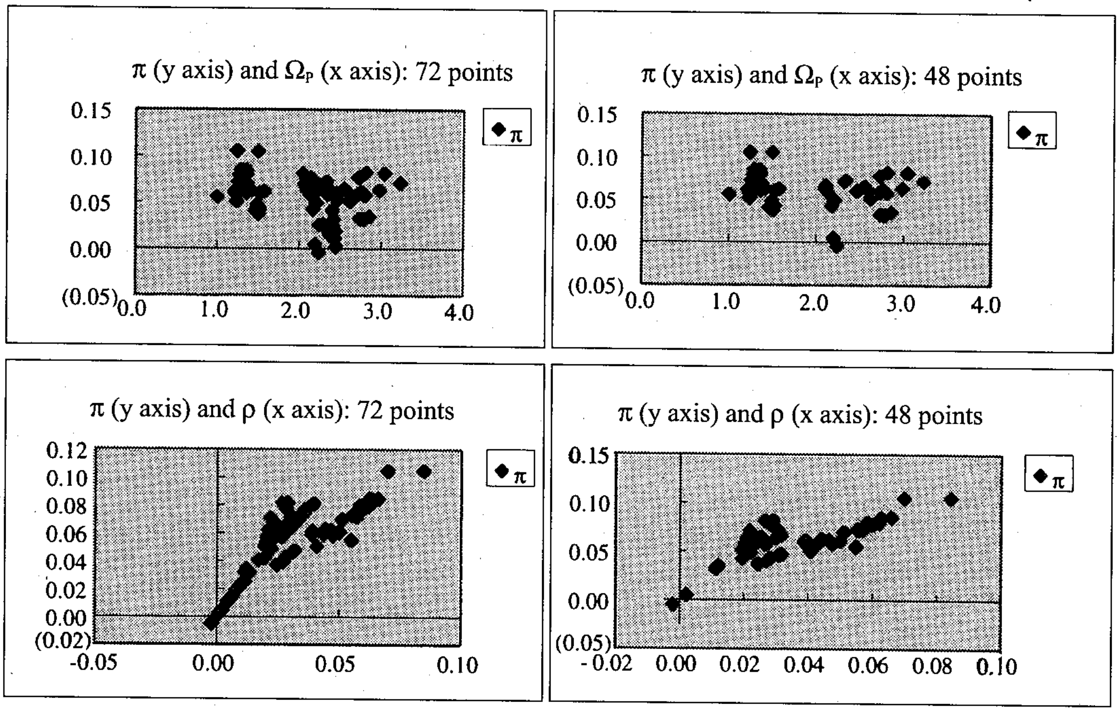
Y: m*

	Intercept (Alpha)=1.357	t=3.7024
X: Ω_P	Beta=–0.488	t=–2.7519

$m^* = 1.3578 - 0.4880\Omega p$ $R^2=0.1414$

$F=0.0084$

Figure 9 Regression analysis using panel data, 1982–1994: between π, Ω_P, and ρ



Six countries: observation 72

Y: π

Intercept (Alpha)=0.078 t=7.8527

X: Ω_P Beta=–0.0111 t=–2.3746

$\pi=0.0789-0.0111\Omega p$ $R^2=0.0746$ F=0.0203

Six countries: observation 72

Y: π

Intercept (Alpha)=0.023 t=7.5048

X: ρ Beta=1.0467 t=11.6095

$\pi=0.0239+1.0467\rho$ $R^2=0.6582$ F=0.00000

Six countries: observation 72

Y: Ω_P

Intercept (Alpha)=2.790 t=31.3092

X: ρ Beta=–23.5747 t=–9.3408

$\Omega_P=0.0789-0.0111\rho$ $R^2=0.5549$ F=0.00000

Four countries: observation 48

Y: π

Intercept (Alpha)=0.071 t=7.6647

X: Ω_P Beta=–0.0062 t=–1.3661

$\pi=0.0719-0.0062\Omega p$ $R^2=0.0390$ F=0.1785

Four countries: observation 48

Y: π

Intercept (Alpha)=0.029 t=7.1453

X: ρ Beta=0.8558 t=8.1151

$\pi=0.0298-0.8558\Omega p$ $R^2=0.5888$ F=0.00000

Four countries: observation 48

Y: Ω_P

Intercept (Alpha)=2.890 t=20.9478

X: ρ Beta=–26.5069 t=–7.5946

$\Omega_P=2.8902-0.26.5069\rho$ $R^2=0.5563$ F=0.00000

3. The growth rate of output $g_Y = g_{KP}$[27] is remarkably stable while the expected real growth rates of income and corporate capital, g^e_Y and g^e_{KP}, are unstable, where $g^e_Y \neq g^e_{KP}$ in a dynamic disequilibrium.
4. There are two ways to measure real growth rates: one uses expected nominal growth rates and the given Paasche price indexes, and the other endogenously measures real growth rates in equilibrium and disequilibrium, without using the Paasche price indexes. The former is rather rough since the Paasche price indexes are given as parameters.

These imply that a national economy recovers its equilibrium between savings and investment, using Ω_P and accordingly, $s_{SWD/Y}(=s_{SP/Y} \cdot \Omega_P)$, and particularly, m^*. They also suggest that the real growth rates, g^e_Y and g^e_{KP}, in a dynamic disequilibrium should be compared with the theoretical real growth rate $g_Y = g_{KP}$. Results of a panel data approach supports the above statements.

Turning to Figures 4 to 8, these show how the above statements are confirmed using average, variance, the coefficient of variation, t-value, and regression statistics. Particularly, Figure 4 shows how quickly the expected value of $\Phi^e = (1 - s^e_P)/s^e_P$ returns back to a theoretical stable condition in equilibrium by adjusting the values of s^e_P, $s^e_{SWD/Y}$, and Ω_P. Note that the theoretical values of $s_p = s_{SP/P}$ or $s_{SP/Y}$, and $s_{SWD/Y}$ are obtained using Ω_P and π: $s_{SP/Y} = \pi/(1+\Omega_P)$ and $s_{SWD/Y} = s_{SP/Y}(\Omega_P - 1)/(1-\pi)$. Also, Figure 8 shows how quickly the expected value of s^e/s^e_P returns back to a stable condition by adjusting the value of $\pi \cdot \Omega_P$, and how quickly the theoretical value of m^e returns back to a stable condition by adjusting the value of Ω_P.

In more detail, the relationship between the growth rate of income and the undistributed profit propensity to save in the improved two-sector model of the author's showed the highest correlation (R square) compared with the relationship between any growth rate and propensity to save (see Figures 5 and 6).

27) A dynamic equilibrium introduces technological progress and is shown as $g_Y = g_{KP}$, which is expressed also as $g^*_Y = g^*_{KP}$.

The two/one class model in literature focuses on the classification of workers and capitalists, but the relationships between the retention ratio $s_{SP/P}$ or the undistributed profit propensity to save $s_{SP/Y}$, and the wage and dividend propensity to save $s_{SWD/WD}$ is not clarified. As a result, the two/one class model in literature cannot reveal the contents of g_Y and g_{KP} if both are the same in equilibrium. The model is finally reduced to the Harrod-Domer model. However, it is interesting to know the following contrasting results (see Figure 4):

1. R^2 of $n=s_w+s_p(1-s_w)r$ and $n=(s_p-s_w)r+s_w/\Omega_P$ showed each 0.2859 and 0.2978 when r and/or Ω_P were used as variables.
2. R^2 of $n=s_w+s_p(1-s_w)r$ and $n=(s_p-s_w)r+s_w/\Omega_P$ showed each 0.0154 and 0.0118 when s_w and s_p were used as variables.

It implies that the growth rate is determined remarkably by the capital-output ratio Ω_P, the rate of profit r (ρ in the author's case), and the relative share of profit π. Also, the coefficient of variation of Ω_P is extremely high compared with π.

Underlying relationships are shown in the relationship between Ω_P and $\Phi^e=(1-s^e_p)/s^e_p$ (Figure 4), the relationship between $\pi \cdot \Omega_P$ and s/s^e_p, and the relationship between m^e and Ω_P (as already indicated). The value of Ω_P cannot be beyond a certain level, and its adjustment is expressed by Φ^e, $s^e/s^e p=s^e_{S/Y}/s^e_{SP/P}$, m^e, whose values suddenly fluctuate, but both of them at once return back within a certain level. It implies that the value of Ω_P recovers quickly after adjustment. In other words, a fundamental factor in the financial structure of products is Ω_P, and even technological progress may not positively contribute to the sustainable growth. Or, by the coefficient of technological progress, the aggravation of capital accumulation is saved. Economic depreciation may be a weapon to reduce Ω_P. An essence of the endogenous system stated by Romer, P.[1994] is clarified. The possibility of convergence [Maddison, 1982, Heston, Alan, and Summers, 1991] is justified.

8. Conclusion

Hicks, Kaldor, Pasinetti, Solow, and Harrod have approached an equality between savings and investment. There is supposed the production function behind this equality regardless of explicitly using it. However, when the production function is applied to it in terms of neutrality of technological progress, Solow has to fix the labour productivity constant, Harrod has to fix the capital-output ratio constant, and Hicks has to fix the capital-labour ratio constant. This is because the production function cannot separate capital from labour in itself. Even in the case of O'Connell [1994] who does not use the production function, she started with no change in labour productivity; i.e., the growth rate of labour productivity is zero, and accordingly, it is supposed that the growth rate of output or capital equals the population growth rate. But, it suggests that both growth rate can be separately treated.[28)]

The author's model started reviewing her model. Her model precisely clarified, by reviewing Baranzini [1991] and using the quadratic equation $n=f(r)$, the relationship between the growth rate n and three kinds of propensities to save; $n=s/\Omega$, $n=s_p/\Omega$ and $n=s_w/\Omega$. These are derived under a different conditions such as $K_c=0$ $(K=K_w)$, $s_p=0$, and $s_w=0$. $n=s/\Omega$ corresponds with Harrod's, $n=s_p/\Omega$ corresponds with Pasinetti, and $n=s_w/\Omega$ corresponds with what was shown by Samuelson and

28) The author sincerely expresses thankfulness for the Faculty of Economics and Politics, the University of Cambridge, UK, when the author could stay at there in August and September of 1996. Prof. Geoffrey C. Harcourt and Prof. Geoffrey Whittington had kindly given invaluable time whenever the author had some problems (almost everyday) in terms of macro economics and accounting. Also, Dr. O'Connell who was visiting there at that time strongly suggested the author to take the third path between the neoclassical approach and the Keynesian approach. The author's approach follows their suggestions and instructions, taking into consideration the bridge between macro and micro (accounting) framework. This opportunity started with Prof. C.A. Blyth's interest in the author's intention for my study on the relationship between macro and micro frameworks, at the University of Auckland, NZ.

Modigliani and is dual to Pasinetti's. If $s=sp=sw$, then, these three are become equal.

However, the above study did not clarified the relationship between s, s_p, and s_w. The corporate-financed growth model of the author's first sets a model under the balanced-growth steady-state which equals the above authors' assumption under the Golden Age. The result reduced to Harrod's condition, $n=s/\Omega$. The corporate-financed growth model of the author's second establishes a model under the corporate-financed growth steady-state which treats first the population growth rate separately and then, integrates this with the growth rate of output or capital. The author's model allows the growth rate of output to differ from the given population growth rate in discrete time.

As a result, the growth rate of output g_Y which equals the growth rate of capital is endogenously derived without setting any propensity to save given. The predetermined variables are the relative share of profit π, the capital-output ratio Ω_P, and the rate of profit ρ. However, the equality between savings and investment is maintained by manipulating the investment ratio, I/Y^0, which differs from the total saving propensity to save, $s=s_{(SP+SWD)/Y}=I/Y=(1+g_Y)\cdot I/Y^0$. This is possible because the capital-output ratio and labour productivity changes at the same time in the model, without introducing the marginal productivity of capital and the marginal utility theorem into the model in discrete time. This process cannot be expressed if using the production function.

The investment ratio is a vital key for solution in two ways:

1. The saving-side: $s_{(SP+SWD)/Y}=s_{SP/P}\cdot\pi\cdot\Omega_P$, the value of Ω_P is enough used for adjustment.
2. The investment-side: after introducing the population growth rate, the change in labour productivity or the growth rate of labour productivity g_y is connected with the investment ratio: $g_y=m^*\cdot I/Y^0$ and $g_y=m^*\cdot\Omega_P\cdot g_Y$.

The saving-side is explicitly determined by π and Ω_P. Samuelson and modigliani

[1966] shows the relationship between Pasinetti region and Anti-Pasinetti region on their Figure 2 in terms of $n=s_p/\Omega$ and $n=s_w/\Omega$. The boundary line shows that $s=s_p=s_w$. However, they did not clarify the relationship between s, s_p, and s_w nor the relationship between the investment ratio and these propensities to save. It was proved that **$s=s_p=s_w$ only if $\pi \cdot \Omega_P=1$.** This is not plausible since the average of $\pi \cdot \Omega_P$ shows 0.1146 (72 points) in six countries and 0.1122 in four countries (48 points) in 1982-1994 although the coefficient of variation is 0.0253 and 0.0258, and t-value is 0.0156 and 0.0126.

The saving-side is closely connected with the investment-side. The value of Ω_P is a bridge between both sides although Ω_P in the investment-side works much more positively. The coefficient of technological progress m* cooperates with the capital-output ratio and labour productivity at the same time in a period. This value of m* is expected to lower the value of Ω_P according to empirical result. When technological progress accelerates the increase in Ω_P, business cycle is inevitable. The existence of "convergence" [Romer, 1994] depends on how a national economy challenges for the control of Ω_P. Harrod [1973] indicates that the balance between the warranted growth rate and the natural growth rate is unstable. His warranted growth rate corresponds with the author's growth rate of output or capital. It is interpreted that the warranted growth rate should be less than the population growth rate. In this sense, the corporate-financed growth steady-state is another expression of a Golden Age. The Golden Age of the author's is predetermined by π and Ω_P, but is renewed by the above investment ratio together with m* and given population growth rate.

Even Solow and Harrod did not show the process how the equality between savings and investment is recovered within a period. Solow assumes that labour productivity is constant Harrod assumes that the capital-output ratio is constant. However, there is no guarantee nor proof that the warranted growth rate converges into the natural growth rate which equals the population growth rate. This is because the growth rate of labour productivity is zero if the growth rate of output equals the population growth rate. Under a condition that the growth rate of output is larger than the population growth rate, an alternative Golden Age is guaranteed in discrete time. In continuous time, an national economy approaches the Golden Age under a condition that the growth rate of output is almost equal to the population growth rate. However, it is noted in both cases that the expected population growth rate is given and the growth rate of output or capital is endogenously derived under necessary and sufficient conditions.

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