Conservation Laws of Ryuzo Sato (xv, 439, 1981) to Justify KEWT Database and, Empirical Laws Endogenously Added to Sato's Conservation Laws

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For empirical proofs of Sato's (xv, 439, 1981) Conservation Laws on Theorem 6, page 285

Common/similar key terminologies used here are:

1. Endogenous versus exogenous and externalities, market/financial
2. Purely endogenous technology versus holotheticity of a technology
3. Endogenous preferences and technology versus self-dual preferences and technology
4. Duality and symmetry
5. Speed years and transitional path or, recursive programming and turnpike
6. Inductive, deductive, and circular augment or tautology, with or without assumptions

First for Sato's Conservation Laws, the author tries to explain why author's endogenous system is justified only by the existence of Ryuzo Sato' (285, ibid., 1981) Conservation

Laws. Second the author reinforces Sato' (xv, 439, 1981) Lie Theory by adding new endogenous laws to Sato's Laws. Third, the author shows fifteen points at Ryuzo Sato's (xv, 439, 1981) Lie Theory. Finally more broadly, the author sums up the essence of the endogenous system using Appendixes A, B, and C and, connects it with geographic philosophy that does not contradict Iyonoishi (2012) developed at physics and element chemistry. As a result, the positioning of the endogenous system will be more broadly clarified, compared with the ellipse implicit universally.

1. Preliminary understanding of databases

The endogenous system has developed as a database by comparing the results of the database with the corresponding results at the transitional path expressed by recursive programming. The database is a general term of KEWT (Kamiryo Endogenous World Table) data-sets by country, sector, year, and over years, after testing the results by recursive programming. KEWT is renewed once a year soon after the publication of *International Financial Statistics Yearbook*, IMF. The current KEWT is mainly composed of 6.12 Short, 1990–2012, for 81 countries and; 6.12 Long, 1960–2010, for nine countries. The current KEWT is divided into 6.12-1 to 6.12-6 by area; i.e., Pacific/Asia, Euro, EU, Latin America, Near-East, and Africa. Some developing counties are tested at 6.12-5 for a while since data are insufficient or not so accurate compared with developed countries.

KEWT database holds under the endogenous-equilibrium, differently from several databases currently published by worldwide organizations. The current databases in the literature hold each under the price-equilibrium or based on the market principle and econometrics. There seems to be no harmony between KEWT database and the current several representative databases. But, actually, both help each other and are reinforced for overwhelming economic policies. This is because statistics aims at recording, which is not fitted for policies and because KEWT is completely policy-oriented, which is not fitted for recording. For example, the market principle tells us not causes but intuitive results like God and Nature, while KEWT tells us true causes and results simultaneously or two ways. KEWT differs from Nature so that we need a tool lying between causes and results. This is the neutrality of the financial/market assets to the real assets. The real assets behave like a god but need verification, which is composed of the financial/market assets. Actually, the verification is just for confirmation that the endogenous equilibrium holds within a moderate range of equilibrium; e.g., directly measuring the speed years by country and sector or testing results using recursive programming.

2. Why Conservation Laws and KEWT?

Now what is the necessary and sufficient condition of KEWT? KEWT reduces all the assumptions set in the literature to zero. Why is KEWT free from any assumption? This is because KEWT is based on an aggregate 'discrete' Cobb-Douglas production function, consistent with a whole economic and social system that is commonly applicable to any country if available data are accurate to some extent. There exist only one discrete Cobb-Douglas production function today and this is KEWT. Why only one? This is because of purely endogenous. Purely endogenous is defined as such that required endogenous equations are available as much as possible if and only if seven endogenous parameters are simultaneously used at that production function. Seven endogenous parameters are a key for solving endogenous equations under no assumption. On the other hand, articles in the literature use assumptions for justification independently with each other. Look into KEWT. There are several fundamental equations indispensable to erase assumptions (see below). These equations are tightly connected like an organization or a complete system. Cut one equation. Then, all the system does not work. The author, at the earlier stage, paid attention to nine assumptions carefully arranged by Meade, J. E. (1-9, 1960, no change in 1962rev.). Later the author realized that it was meaningless to reduce assumptions, one by one like a good.

For example, the literature uses the rate of interest externally and naturally, with no abuse. We set perfect competition, which is expressed by marginal productivity of capital = the rate of return, r, and marginal productivity of labor = the wage rate, w, each under endogenous equilibrium. Then, why is the use of the rate of interest wrong at KEWT? If and only if the rate of return is measured with all the other parameters and variables, KEWT could live on. A clue for finding additional Conservation Laws has been lost in the literature. When the rate of return is measured with the wage rate, the flexibility of r and w and accordingly, the marginal rate of substitution, *MRS*, and the elasticity of substitution, *sigma*, are simultaneously measure. Here, endogenous capital (stock) is one of priority steps, but it is difficult to finalize the initiation value. This is because the initial value is not given but endogenously measured and this must be consistent with all the other parameters and variables.

3. The constancy of the capital-output ratio at Sato (1981) and Samuelson (1970)

Back, why are Conservation Laws essential to KEWT? Sato's Theorem 6 on page 228 proves the constancy of the capital-output ratio by Samuelson (1970) using Lie theory. Here Theorem 6 is divided into four, where the implicit price, λ, is commonly used for national income, Y with subscript 1, and national wealth (capital), W with subscript 2. Expressing the capital-output ratio by its inverse, the output-capital ratio, Theorem 6 concludes that the two product values of $\Omega_1 = \lambda \cdot Y$ and $\Omega_2 = \lambda \cdot W$ are 'the only conservation laws globally operating in the von Neumann model of optimal growth.'

4. The constancy of the capital-output ratio at KEWT

KEWT only holds by using the constancy of the capital-output ratio, $\Omega = \Omega^* = \Omega_0 = K / Y$ and accordingly, the constancy of the rate of return, $r = r^* = r_0 = \Pi / Y$ under a constancy of the relative share of capital, $\alpha = \Omega \cdot r = \Omega^* \cdot r^* = \Omega_0 \cdot r_0$. At KEWT, if the output-capital ratio is used, the rate of return must be expressed by its inverse. The rate of return empirically equals ten year market yield of national debt, $r_{M(10yrs\ DEBT)}$, under a new concept of the neutrality of the financial/market assets to the real assets under the endogenous-equilibrium. This neutrality differs from the technical neutrality of Sato' (IV, 1981) that determines the relationship between the relative share of capital and technology; i.e., Hicks, Harrod, and Slow-types. When both $g^*_{A(FLOW)} = i(1-\beta^*)$ and the relative share of capital, $\alpha = \Pi / Y$, are unknown or externally given, the technical neutrality is a second best tool at the neo-classical school.

The author encountered the Conservation Law of Samuelson (1970) earlier and luckily. Since then, the author had treated it as a treasure but, without knowing the true foundation, just like a intuition, until the author could recently understand the true implication of Sato's Conservation Laws. And now the author, with confidence and bold courage, cites Sato's (285, 1981) Constancy of the output-capital ratio at Theorem 6 in IV, for the justification of KEWT.

5. To be modest but, an empirical discovery exists

Japanese does not usually cite, for comparison, the sentences written as the highest performance, since his/her behavior is impolite and not modest, just like against Nature. The author actually attended at Sato's study school, Tokyo, once a month, but the author, at that time, could not foresee that Sato' (1981) work surely saves economics and economic world. Now this time, the author leaves this message here. A moment comes by finding a complete set of KEWT and recursive programming. This finding is such that all the values and ratios at convergence, with superscript * (i.e., at endogenous turnpike's terminal, which corresponds with the steady state or balanced state under the price-equilibrium), numerically equals those measured by recursive programming by year.

The author never forgets a moment when the author surprisingly reconfirmed a discovery existing between FLOW and STOCK of technological progress at the discrete Cobb-Douglas production function: $A_{TFP(STOCK)}(t^*) = A_0(1 + g_{A(FLOW)}(t^*))^{1/\lambda^*}$, where $g_{A(FLOW)}(t^*) = g_{TFP(STOCK)}(t^*)$ holds with $g_{A(FLOW)}(t^*) = i(t^*) \cdot (1 - \beta(t^*))$. It is most difficult for a researcher to numerically and empirically clarify a fact or scientific discovery for the relationship lying between the current values and ratios with those at convergence. No one, including Teacher Sato, believes that KEWT expresses the discovery. This is because the literature explains topological continuous structure but without empirical results. The author did it using discrete hyperbolas reduced using endogenous equations, instead of the use of parabolas and related differential equations. The author is grateful to related scholars and researchers. This is not a statement of virtues but a will to future generations.

6. Sato' Lie theory (1981) and related assumptions

Theorem 6 at Sato' Lie theory (xv, 439, 1981) holds globally operating in the von Neumann's model of optional growth. Theorem 6 corresponds with the author's discoveries in a moderate range of the endogenous-equilibrium under no assumptions.

Von Neumann (1-9, 1945–46) KEWT (EEE) with no assumptions

Point 1-1 (1-8, 2001): Homage by Samuelson ⇔ Constant $\Omega = \Omega^* = \Omega_0$ & $r = r^* = r_0$

Point 1-2 (xi, xii, 1981): Forward by Samuelson ⇔ Lie theory common to natural & social science

Point 2 (7, II): the Solow-Stigler controversy ⇔ Discover the discrete C-D production Function

Point 3 (7, II): the MRS under weak endogenous ⇔ $MPK = r$ & $MPL = w$ under perfect competition

Point 4 (7, III): Time lag & maximum profits ⇔ Maximum $r = \Pi / K$ with minimum investment

Point 5 (9, III): the exogenous types of tech. ⇔ Correct, under indispensable assumptions

Point 6 (9, IV): the concept of "neutrality" ⇔ Constant α; $\alpha = \Omega \cdot r$, measured simultaneously

Point 7 (11-12, VI): the self duality ⇔ differently holds between real and financial assets

Point 8 (12, VII): the demand utility function ⇔ differently holds between real and financial assets

Point 9 (12, VI): the utility & demand analysis ⇔ differently holds between real and financial assets

Point 10 (12, VI): the Clark uniformity ⇔ Identify if and only if it turns to a discrete type

Point 11 (13, VII): the rate of returns to scale ⇔ Diminishing (not increasing) is normal under CRS

Point 12-1 (13-14, VII): symmetries & Laws ⇔ As shown by hyperbolas, as reduced equations

Point 12-2 (285, VII): Theorem 6 on Laws ⇔ EEE is justified only by Sato's Conservation Laws

Point 13 (14, VIII): invariance of index numbers ⇔ empirically data are not fitted for reversal test

Point 14 (15, VIII): Divisia index ⇔ theoretically but not empirically due to statistics' character

Point 15 (16, IX): group structure of the market ⇔ under the neutrality of real to market assets

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Note: The author failed to show related assumptions at neo-classical school, by point. This is because each assumption is tightly connected from the viewpoint of KEWT or EEE. The author finds several key words and definitions are identical, apart from each standpoint. It implies: Sato' Lie theory universally exists under any circumstance and aspect.

7. Figures by country, 2010, as empirical proofs, using KEWT 6.12 series

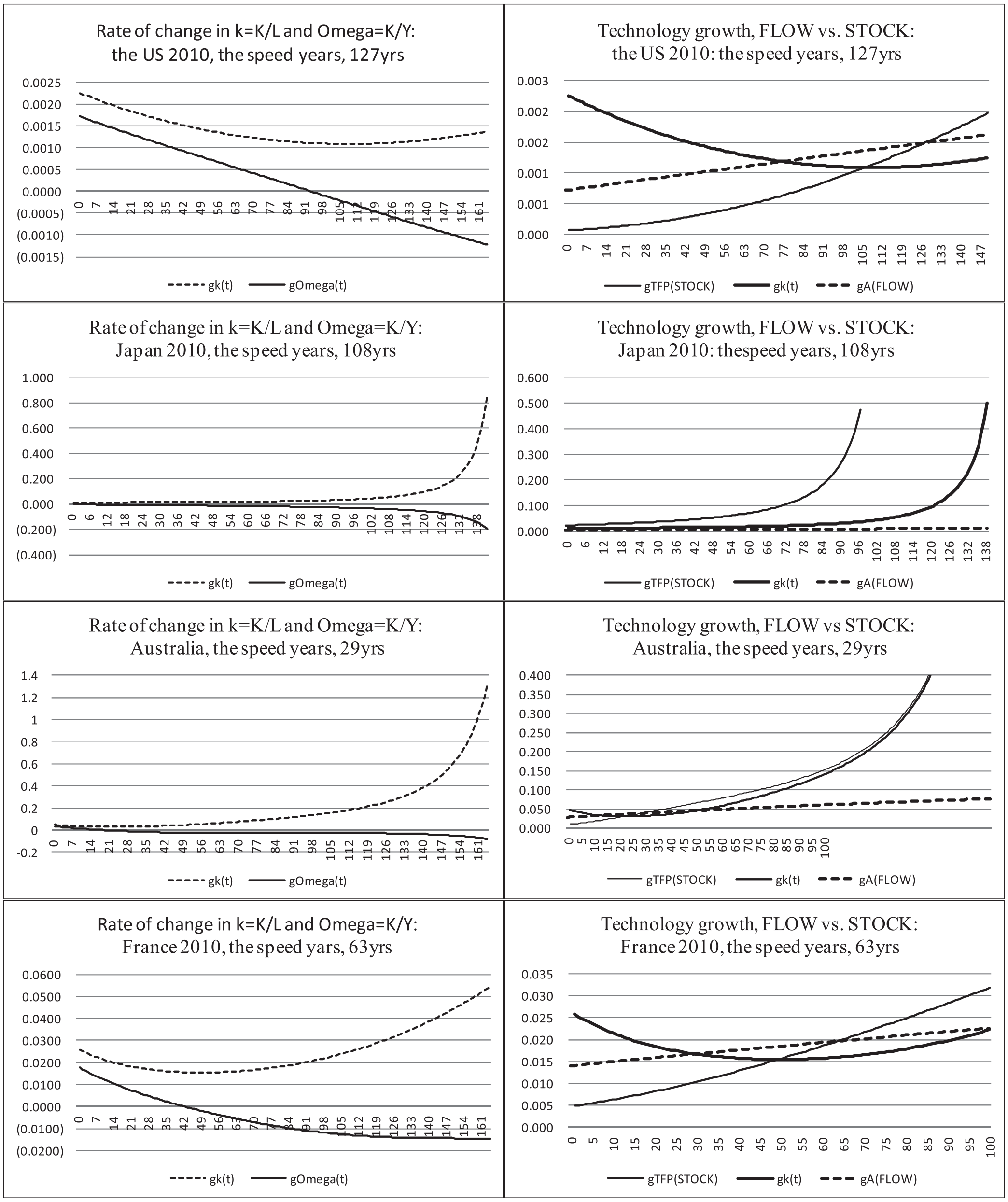


Figure 7-1 Empirical proofs lying between the capital-labor ratio and the capital-output ratio under Conservation Laws: by country, 2010

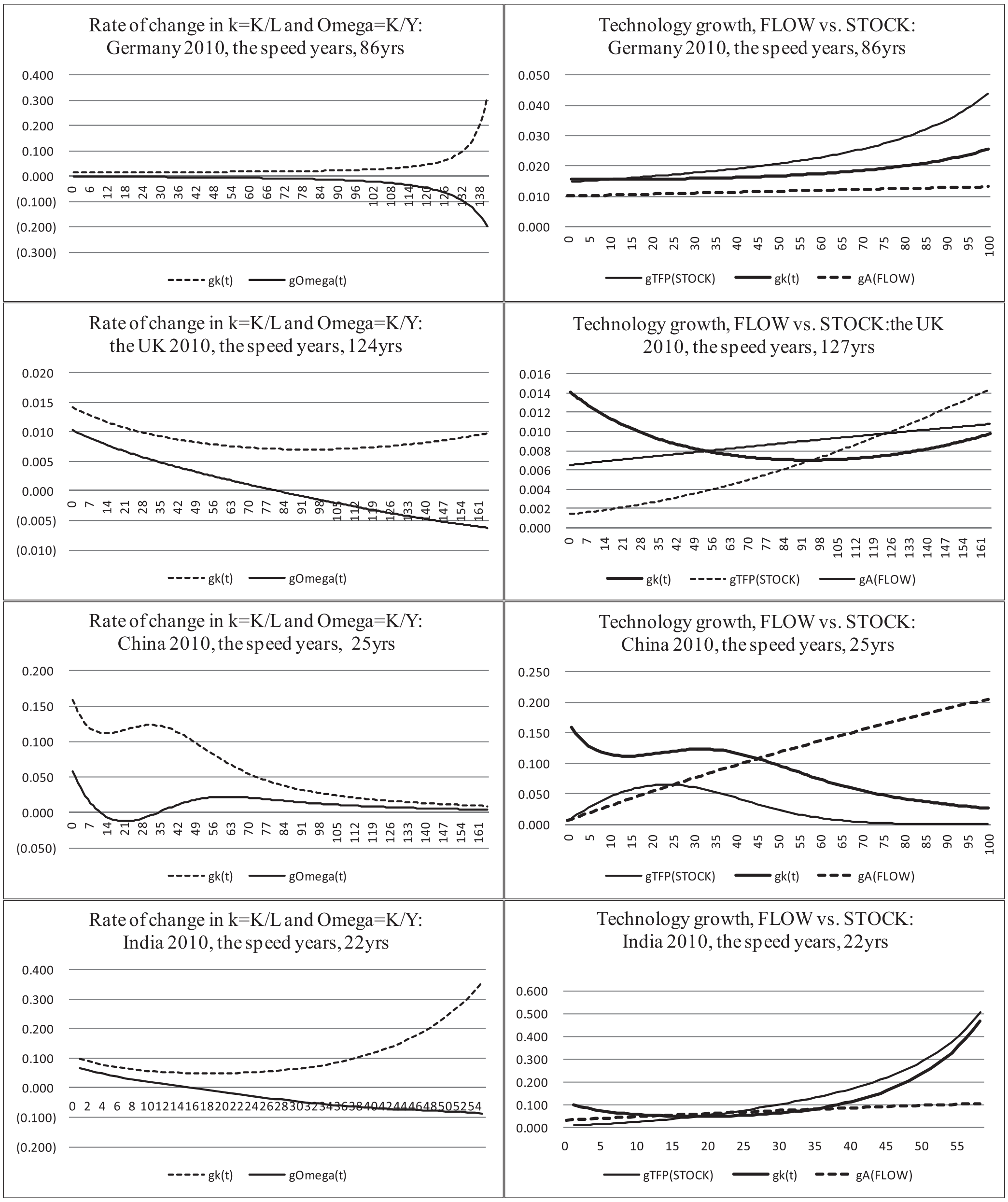


Figure 7-2 Empirical proofs lying between the capital-labor ratio and the capital-output ratio under Conservation Laws: by country, 2010

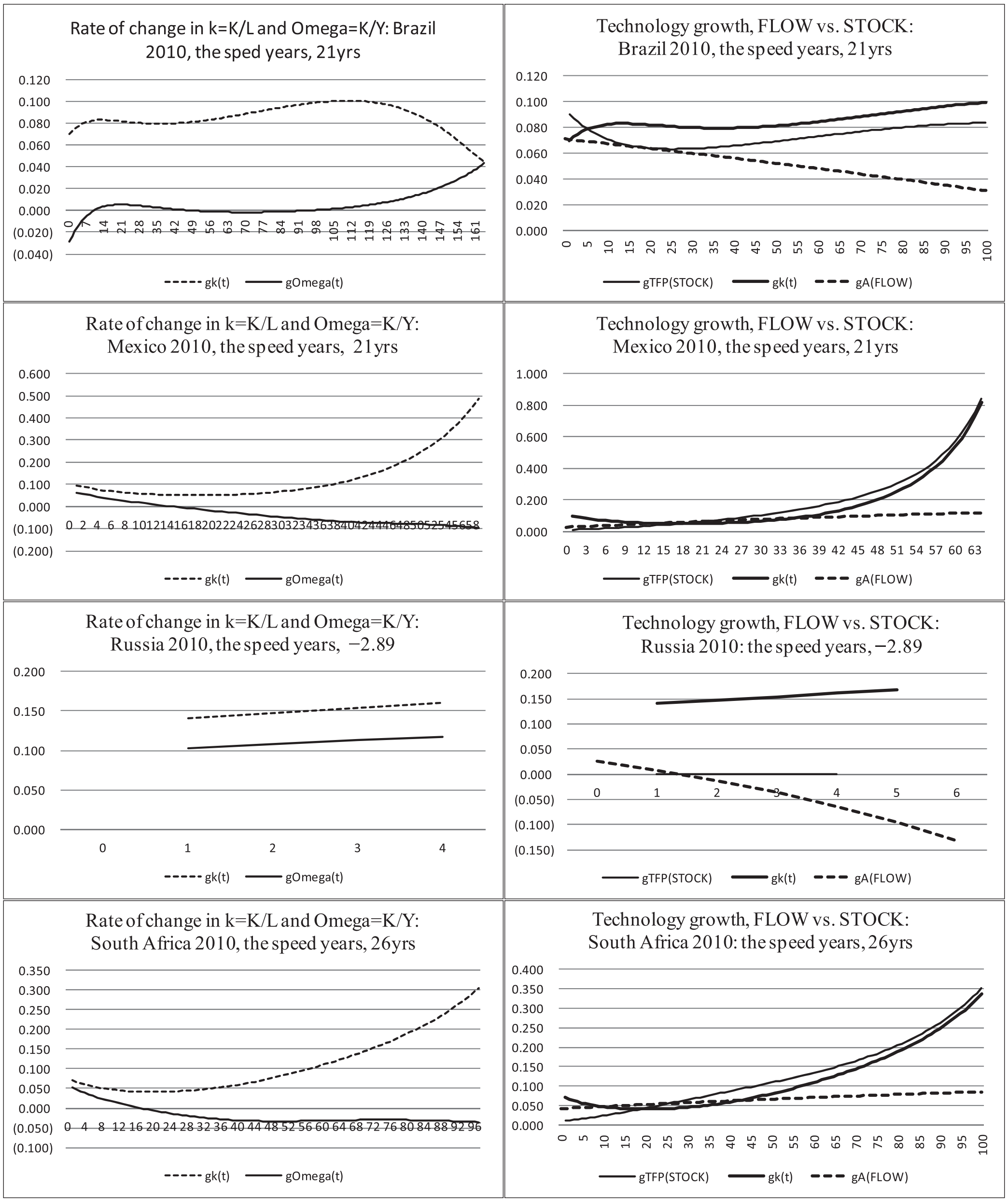


Figure 7-3 Empirical proofs lying between the capital-labor ratio and the capital-output ratio under Conservation Laws: by country, 2010

8. Fifteen points at Ryuzo Sato's (xv, 439, 1981) Lie Theory

The author selects fifteen points from Sato (xv, 439, 1981), mainly citing at his chapter 1 for simplicity and with author's interpretations using 'EEE.' To connect each point with chapters in coming Monograph, the author hereunder uses; e.g., C1 and C2 instead of Chapters 1 and 2. For equations, see Notations before Preface at the beginning of Monograph.

Point 1-1 (1-8, 2001): Personal Homage to Ryuzo Sato by Paul A. Samuelson.
Point 1-2 (xi, xii, 1981): Forward by Paul A. Samuelson.
Point 2 (page 7, II): for the Solow-Stigler controversy.
Point 3 (page 7, II): for the marginal rate of substitution (MRS).
Point 4 (page 7, III): for time lags and profit maximization problem.
Point 5 (page 9, III): for the existence of exogenous types of technical change.
Point 6 (page 9, IV): for the concept of "neutrality."
Point 7 (page 11–12, VI): for the self duality.
Point 8 (page 12, VII): for the unitary elastic demand functions and Cobb-Douglas utility function.
Point 9 (page 12, VI): for the utility and demand analysis.
Point 10 (page 12, VI): for the Clark uniformity.
Point 11 (page 13, VII): for the primal rate of returns to scale
Point 12-1 (page 13–14, VII): for dynamic symmetries and economic Conservations.
Point 12-2 (page 285, VII): for dynamic symmetries and economic Conservations.
Point 13 (page 14, VIII): for invariance of index numbers
Point 14 (page 15, VIII): for Divisia index.
Point 15 (page 16, IX): for the group structure of observable market.

Point 1-1 (1-8, 2001): **Personal Homage** to Ryuzo Sato by Paul A. Samuelson.

Samuelson states at the beginning that the Homage is his first and last go all out in personal affection and admiration for Ryuzo--as scholar and old-time friend. To author's understanding, there are two points to be cited in the Homage (2, 6-8; 2001): (1) "I knew Ryuzo even before I knew him personally. That is I knew first his early works. I still quote such classic papers as that in the 1963, Review of Economic Studies, showing how very long it takes a Solow growth path to settle down to its long-term equilibrium." (2) "Speaking for myself, our friendship goes back to before most of you in this audience were born. I cherish our published collaboration. One zealous scholar, after much research, declared that the Sato-Samuelson joint AEA paper on Money in Demand Theory has more numbered equations in it than any other in Guiness Book of Records. Non-repentant am I."

EEE (C1): For the above (1); the author pays attention to 'a Solow growth path.' EEE starts with an endogenous measure of the rate of technological progress and, simultaneously converges the price-equilibrium to the endogenous-equilibrium. As a result, 'a Solow growth path' is expressed by the speed years by country and by sector. For the above (2); the Sato-Samuelson joint AEA paper[1)] clarifies the essence of the Lie groups applied to economics under the price-equilibrium.

Point 1-2 (xi, xii, 1981): **Forward** by Paul A. Samuelson.

"Barely a century ago, the Norwegian Sophus Lie developed the theory of what have come to be called *Lie transformation groups*. Their original primary application was to the classical mechanics of Lagrange and Hamilton. Now Ryuso Sato of Brown University is making a pioneering attempt to apply the Lie theory to modern economics. ...Along an intertemporally efficient path of a closed von Neumann system, in which every good of a set of goods and its rate of growth are producible out of those same goods as inputs by a constant-returns-to-scale technology, the ratio of the total value of the capital goods to total income is a fundamental constant. Dr. Sato now shows that this 1970 finding of mine is essentially the only "energy integral" that such a dynamic system can in general process."

EEE (C6, C7, C8, C16): **Point 1-2** shows how essential it is for economics to perceive Samuelson's (1970).[2)] The author was lucky to be able to find and read Samuelson's (1970) later in the 1970s. Since then, a constant capita-output ratio has been involved in the author's discrete Cobb-Douglas production function research. The author makes use of the capital-output ratio more than the capital-labor ratio commonly used in the literature.

Point 2 (page 7, **II**): for the Solow-Stigler controversy.

"In response to the Solow-Stigler controversy, we are now able to conclude that the effects of a given type of technical progress and scale effects are independently identifiable if and only

1) Samuelson, Paul, A., and Sato, Ryuzo. (1984). Unattainability of Integrability and Definiteness Conditions in the General Case of Demand for Money and Goods. *American Economic Review* 74 (Sep, 4): 588–604.
2) Samuelson Paul A. (1970). Law of the Conservation of the Capital-output ratio, Proceedings of the National Academy of Sciences, *Applied Mathematical Science* 67: 1477–79.

if the production function is not holothetic under that particular type of technical progress.”

EEE (C1, C6): The rate of technological progress is endogenously measured first and, simultaneously with *seven* endogenous parameters in the ‘discrete’ Cobb-Douglas (C-D) production function and under the endogenous-equilibrium. Capital and labor in the discrete C-D production function are not homogenous, incidentally similar to G. Stigler’s. Heterogeneous is justified if and only if the production function is discrete C-D with no assumption.

Point 3 (page 7, **II**): for the marginal rate of substitution (MRS).

“As is well known, the optimal behavior of a cost-minimizing firm can be observed in the marketplace by studying the differential equation for the marginal rate of substitution (MRS). Given the Lie “representation” of the MRS, it is possible to test if a particular production function technology is holothetic under a given Lie type of technical progress by the compatibility condition of the infinitesimal generators.”

EEE (C1, C7, C16): The database of KEWT holds without any assumption. The marginal rate of substitution (MRS) is measured at database by year and, its transitional path using recursive programming by year. The MRS is measured if and only if the rate of return and the growth rate of output are simultaneously measured with other parameters and variables in the endogenous-equilibrium (for MRS, see **Point 8**).

Point 4 (page 7, **III**): for time lags and profit maximization problem.

“But there are time lags between inputs and outputs of the research activity. Formally writing cumulative effects by the *integro-differential equation system*, we justify the existence of endogenous technical progress as the optimal solution of the long-run profit maximization problem with a continuous time lag.”

EEE (C5, C6, C7): Time lag is indispensable commonly to discrete and continuous. Profit maximization holds under an assumption of perfect competition. Both are solved either by the Lie groups staying at the literature or, by EEE as the hidden side of the literature with no assumption.

Point 5 (page 9, **III**): for the existence of exogenous types of technical change.

“However, one *basic and new feature* of the present analysis is that the model of optimal technical progress with *continuous lags* is reduced to a model with no lags, but with the appropriate weights to the corresponding model with no lags, but with the appropriate weights to the corresponding differential equations. This enables one to study not only the steady state, but also the intertemporal paths even under more general conditions than the standard optimal model. This chapter is intended to justify the *existence of exogenous types of technical change* used throughout this book."

EEE (C2, C5, C9): The difference between the book and EEE is traced back to the character of the rate of technological progress; whether it is exogenous or endogenous. This difference is replaced by the difference between the price-equilibrium and the endogenous-equilibrium. Nevertheless, the difference is absorbed into the Lie group solution and disappears thoroughly. Again, one from theoretically and the other from empirically.

Point 6 (page 9, **IV**): for the concept of "neutrality."

"The concept of "neutrality" is basically the same as the group concept of invariance. Specifically we extend the concept of "neutrality" used in the earlier works to "neutrality in the sense of transformation groups"—"G (group) neutrality type of technical change."

EEE (C5): It is impossible to estimate the relative share of capital or labor accurately in the literature. The book finally goes through a solved problem of the relative share of capital or labor in the literature by setting "neutrality." EEE measures the relative share of capital/labor under the endogenous-equilibrium. The relative share of capital changes at EEE by year, while it remains unchanged at the transitional path.

Point 7 (page 11–12, **VI**): for the self duality.

"The versality of applications of the Lie theory does not end here. In Chapter 6 it is demonstrated that we can further apply this theory to the study of the duality and self-duality of preferences and technologies. After presenting the necessary and sufficient conditions for the self-duality of preferences as the set-symmetric conditions on the implicit functions of price and quantity vectors, we present a specific method of deriving such implicit functions."

Point 8 (page 12, **VII**): for the unitary elastic demand functions and Cobb-Douglas utility function.

"…It turns out that the self-dual demand functions that in fact satisfy these fundamental restrictions are simply the continuous transformations of the unitary elastic demand functions associated with a Cobb-Douglas preference ordering. In other words, we can make use of the fact that the system of demand functions arising from a Cobb-Douglas utility function with equal exponents can be used as the basis for the identical transformation."

EEE (C2, C5, C9, C12, C13) to **Points 7** and **8**: The self duality is indispensable under the price-equilibrium. The self duality is replaced by the relationship at EEE existing between the real assets and financial/market assets in national accounts. The real assets act positively but, the financial/market assets are another expression of the real assets. The author proves the neutrality of financial/market assets to real assets. The neutrality exists everywhere in the endogenous system. The author, for global comparison by country, sets up a new concept of 'the relative discount rate function at the macro level, $(rho/r)(c)$,'' as a surrogate for consumption, individual utility, and taste/preferences inherited in the literature. The elasticity of substitution, *sigma*, is 1.00 by year in the transitional path.[3)]

Point 9 (page 12, **VI**): for the utility and demand analysis.

"Although the basic duality principle of the utility and demand analysis will carry over to production and cost analysis, the self-duality of production and cost functions is usually different. The main reason for this is that here we are not comparing the direct and indirect production functions. Using the normal cost function, we first state the necessary condition for the self-duality of the production and cost function: The production function must be *implicitly holothelic* under the uniform factor-augmenting type of technical progress. …Hense we must deal with the problem of implicit self-duality. It is shown that such an implicitly homothetic production function has cost function of a particular form. If and only if the production function is implicitly homothetic can the cost function be written in the form $C = q(C_1(Y),\ldots,C_{n-1}(Y);p)$ $C_n(Y)$."

Point 10 (page 12, **VI**): for the Clark uniformity.

"The uniformity of factor demand functions is defined as the similarity of the functional forms of all demand functions. J. B. Clark's treatment of the marginal productivities of capital and labor clearly assumes this special property for the factor input demand functions (Clark uniformity). A theorem in Chapter 6 is designed to find the necessary and sufficient conditions for the Clark uniformity condition (228, Theorem 6)."

Point 11 (page 13, **VII**): for the primal rate of returns to scale

"Alternatively, given a production function, usually implicit, the effect of technical progress

3) In detail, see graphical explanation at the end of this Notes: $\sigma = -\dfrac{\Delta k / k}{MRS / (r / w)}$ using consecutive average; e.g., $\Delta k_t = (k_t - k_{t-1}) / ((k_t + k_{t-1}) / 2)$.

is completely transformed to the dual rate of return to scale of the cost function if and only if the type of technical progress is of uniform factor-augmenting type. Of special interest is the case of implicitly homothetic technology. Here the primal rate of returns to scale of the production function is in fact identical to the dual rate of returns to scale of the cost function."

EEE (C2, C5, C8, C9, C12, C13, C14) to **Points 9. 10** and **11**: Economics produces production and cost functions inevitably and more fundamentally, aggregate demand and supply under the price-equilibrium. Further, the literature uses 'final' distribution of national income. The treatments to the two functions are one of most difficult issues in economics, as treated above using the Lie theory. Contrarily EEE treats the above issues differently based on endogenous real assets and uses endogenous data just before redistribution of national income. This is because if final national income is used, an endogenous rate of return at the government sector reduces to zero. Since the rate of technological progress is measured, factor-augmenting disappears. Technology is independent of taste/preferences, although both are settled simultaneously and wholly as a system. However, EEE is never justified without the following conservation laws based on the Lie theory.

Point 12-1 (page 13–14, **VII**): for dynamic symmetries and economic conservations.

"Economic conservation laws and turnpikes are closely related. Thus, the two additional local laws are the turnpike constants of the system. The weighted difference between investment and capital multiplied by the inverse of the negative turnpike exponent, and the weighted sum of investment and capital multiplied by the inverse of the positive turnpike exponent, are always constant. …Finally, the Samuelson conservation laws in a von Neumann growth model is derived through the application of the Noether theorem. It is shown that the Samuelson laws are the only laws globally operating for that system, although there are several local laws. Again the turnpike constants are shown to be closely related with the local conservation laws."

Point 12-2 (page 285, **VII**): for dynamic symmetries and economic conservations.

EEE (C2, C5, C8, C9, C12, C13, C14, C16) to **Points 12-1** and **12-2**: The above Theorem 6 presents a wonderful base commonly to the book and EEE. von Neumann (1945–46)[4] stays at the price-equilibrium. The rate of interest has to be given externally. EEE measures the rate of return simultaneously with the capital-output ratio. Therefore, the author adds the second

4) von Neumann, J. (1945–46). A Model of General Economic Equilibrium. *Review of Economic Studies* 13: 1–9.

conservation laws in the case of purely endogenous as a system under the endogenous-equilibrium.

"*Theorem 6.* (*Two Conservation Laws*) (*i*) *For the optimal control problem defined by* (80)[5], $\Omega_1 = \lambda Y = const$. *That is to say, the product of the implicit price λ and national income Y is always constant.*

(*ii*) $\Omega_2 = \lambda W = const$. *That is to say, the product of the implicit price λ and national wealth W is always constant.*

(*iii*) $\Omega_1 / \Omega_2 = Y / W = const$. *That is to say, the aggregate output-capital (wealth) ratio is always constant.*

(*iv*) Ω_1 *and* Ω_2 *are the only conservation laws globally operating in the von Neumann model of optimal growth.*

Using endogenous turnpike, $\Omega = \Omega^* = \Omega_0$ and $r = r^* = r_0$. Also, endogenous turnpike in the transitional path produces two laws each as a surrogate for the price level: (1) the relative price level, $p = 1.0000$, and (2) the elasticity of substitution, $\sigma = 1.0000$. These are empirically proved using 81 countries, 1990–2010 by sector, as shown in C16 for recursive programming.

Point 13 (page 14, **VIII**): for invariance of index numbers

"In some cases, these index numbers satisfy the criterion of the factor reversal test. Furthermore, the standard quantity index is not completely symmetric with the price index. However, using our theory, it is possible to drive a 'dual" quantity index which is symmetric with the corresponding price level."

Point 14 (page 15, **VIII**): for Divisia index.

"The concept of "negenal (or quasi)" Divisia index is introduced to study a dynamic index number related to *nonhomothetic behavior*. The general Divisia index number is the weighted sum of percentage changes, the weights resulting from the relative shares and appropriate coefficients. Thus the almost homothetic Divisia index is a special case when the underlying utility (production) function is of the almost homothetic type. The dynamic invariance condition is again applied to study the holotheticity property of the general index."

5) (279, Eq. 80, VII): The optimal-control problem is defined as $\max(K_t)\int_0^T \dot{K}_t\, dt$ subject to $F[K_1, K_2; \dot{K}_1, \dot{K}_2] = 0$.

EEE (C2, C5, C8, C9, C12, C13, C14, C16) to **Points 13** and **14**:

Point 15 (page 16, IX): for the group structure of observable market.

"Most optimal economic systems possess some forms of symmetries. The group theoretic approach is the basic apparatus to uncover hidden and unhidden symmetries (static or dynamic) of optimal behavior. Here we study the relationship between the Hamilton-Jacobi transformation theory and the theory of technical and/or taste change. Instead of regarding the Hamilton-Jacobi theory as the practical computational method, we use it to study the invariant properties of a dynamic economic system, invariant under the transformations due to technical and taste change."

EEE (C2, C5, C8, C9, C12, C13, C14, C16) to **Points 15**: Point 15 shows Sato's intention to the book. In EEE, symmetry is another expression of hyperbola, whose dialogue is 45^{0}. The target of EEE is a maximum rate of returns on the y axis to minimum net investment expressed by the range of net investment to output on the x axis. Maximum returns or profits hold using parabola and, with no dimensional expansion to the 4^{th} quadrant.

> The author hereunder presents three Appendixes. Endogenous equations selected at Appendix A are indispensable to empirical proof of Sato's Conservation Laws. A core of endogenous equations is productivity (per capital) analysis, where the growth rate of technology stocks equals the rate of technological progress. The here indicates a common contradiction in the literature, particularly at neoclassical: If one aims at maximal profits, it contradicts minimization of economic rent of Vernon L. Smith (24–31, 1963). Maximal profits leads to maximal growth, which in turn maximize the rate of return. Then, the relationship between interest rate and the rate of return becomes contradictory. This contradiction is traced to an external interest rate plus economic depreciation prevailing commonly to the macro and micro levels: total cost versus productivity.

Appendix A Equations at the transitional path using recursive programming: towards endogenous turnpike: with References specified to Ryuzo Sato' Lie groups

Appendix B From five to six dimensions in the real world: From Pythagoras, Gauss to Fermat, Wiles and Iyonoishi (2012)

Appendix C Why do we remain circle and hyperbola plane at the endogenous system?

Appendix A Equations at the transitional path using recursive programming: towards Endogenous Turnpike

For endogenous proofs to Sato's (xv, 439, 1981) Conservation Laws on Theorem 6, page 285

1. Fundamental values and ratios at KEWT that correspond with Sato's (1981):

$Y(t) = y(t)\cdot L(t)$. $\quad y(t) = A(t)\cdot k(t)^{\alpha}$. $\quad A(0) = \dfrac{k(0)^{1-\alpha}}{\Omega(0)}$. $\quad A(t) = A(t-1) + i_A(t)$.

$i_A(t) = \dfrac{i(t)(1-\beta(t))}{k(t)^{\delta(t)}}$. $\quad i(t) = y(t)\cdot i$. $\quad L(t) = 1(1+L_0)^t$.

$P(t)\cdot Y(t) = r(t)\cdot K(t) + w(t)\cdot L(t)$. $\quad r(t) = \alpha / \Omega(t)$. $\quad w(t) = (1-\alpha)\cdot y(t)$.

$K(t) = k(t)\cdot L(t)$. For $K(0)$, see C6 to measure 'K & r' in Monograph, 2012.

2. Simultaneously, $C(t) = W(t)\cdot\left(\dfrac{rho}{r}\right)$. $\quad C(t) = T(t)\cdot c(t)$.

$Y(t) = C(t) + S(t) = W(t) + \Pi(t)$.

$i_K(t) = i(t)\cdot\beta(t)$. $\quad i(t) \neq i_K(t) + i_A(t)$.

$\Omega(t) = k(t) / y(t)$. $\quad k(t) = \dfrac{k(t) + i_K(t)}{1+n}$. $\quad rho(t) = \left(\dfrac{rho}{r}\right) r(t)$.

For ratios, simply set $L(0)$ = 1.0000 and $L(t) = 1\cdot(1+n)^t$. For values, $L(0)$ = population (STOCK).

3. Two proofs hidden at Sato's two Conservation Laws (see figures at the end):

1). The elasticity of substitution = 1.000: $\sigma = -\dfrac{\Delta k / k}{MRS / (r/w)}$, using consecutive average; e.i.,

$\Delta k_t = (k_t - k_{t-1}) / ((k_t + k_{t-1})/2)$. $\quad MRS_t = \Delta r_t / \Delta w_t$.

2). The relative price level = 1.000: $p(t) = P(t)\cdot Y(t) / Y(t)$, newly defined,

to justify the endogenous-equilibrium.

4. Proofs to add to Sato's two Conservation Laws (see figures at the end):

$g_k(t) = g_{K/L}(t)$. $\quad g_{Omega}(t) = g_{K/Y}(t)$.

(both are derived from the above **2**).

5. Proof of productivity growth, at convergence in the transitional path, using FLOW and STOCK:

$g_{A(FLOW)}(t^*) = g_{TFP(STOCK)}(t^*)$, where A = total factor productivity (TFP) as STOCK.

$g_{A(FLOW)}(t^*) = i(t^*)\cdot(1-\beta(t^*))$. $\quad A_{TFP(STOCK)}(t^*) = A_0(1 + g_{A(FLOW)}(t^*))^{1/\lambda^*}$.

6. Proof at KEWT database, differently from the above $g_{TFP(STOCK)}(t^*) = g_{A(FLOW)}(t^*)$:

Starting with endogenous Conservation Laws, $\Omega = \Omega^* = \Omega_0$ and $r = r^* = r_0$,

under $\alpha = const.$: $\alpha = r^*\cdot\Omega^*$, 1). $A^* = A_0(1+g_A^*)^{1/\lambda^*} = k^{*1-\alpha} / \Omega^*$. $k^* = (A^* \cdot \Omega^*)^{1/1-\alpha}$. $y^* = A^* k^{*\alpha}$.

2). $L^* = L_0(1+n)^{1/\lambda^*}$. $K^* = k^* L^*$. $Y^* = A^* K^{*\alpha} L^{*(1-\alpha)}$. Or, $Y^* = y^* L^*$.

3). Equations prevailing commonly to KEWT and its recursive programming,

$A(t) = \dfrac{k(t)^{1-\alpha}}{\Omega(t)}$. (See Note 11 on page25, PhD thesis, 2003/Nov).

$1/\lambda^* = 1/((1-\alpha)n + (1-\delta_0)g_A^*)$.

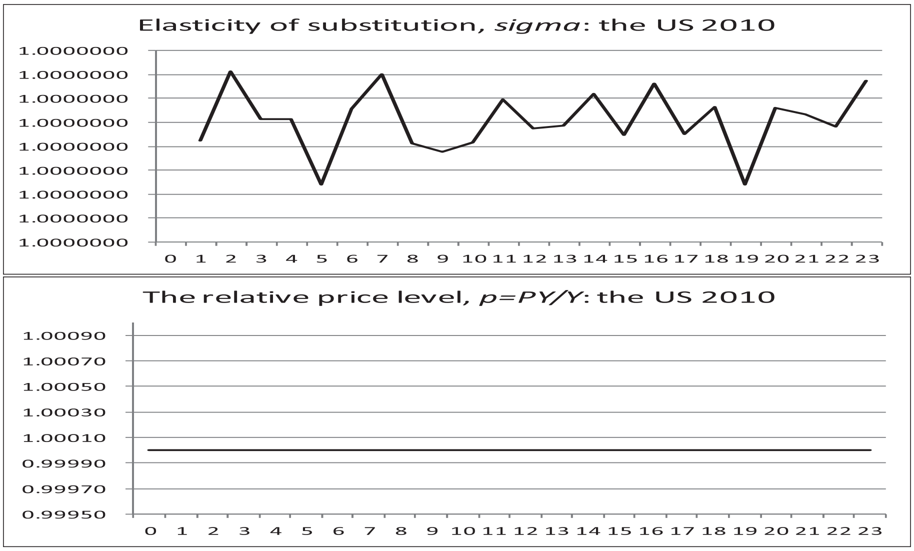
$\Omega^* = \dfrac{\beta^* \cdot i(1-\alpha)}{i(1-\beta^*)(1+n) + n(1-\alpha)}$. $\beta^* = \dfrac{\Omega^*(n(1-\alpha) + i(1+n))}{i(1-\alpha) + \Omega^* \cdot i(1+n)}$.

$\delta_0 = 1 + \dfrac{LN(\Omega^*)}{LN(B^*)}$ and $B^* = (1-\beta^*)/\beta^*$.

$r_{\beta_0 - \beta^*} = \dfrac{LN(\beta_0) - LN(\beta^*)}{speed\ years}$.

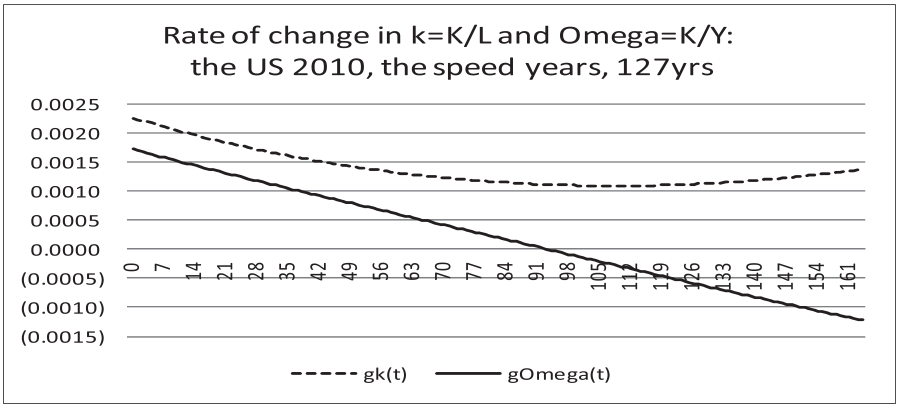
$r_{\delta_0 - \delta^*} = \dfrac{LN(\delta_0) - LN(\delta^*)}{speed\ years}$. (Limit of errors to each discount rate is negligible).

7. Figure to 1) and 2), at **3**. (see above)



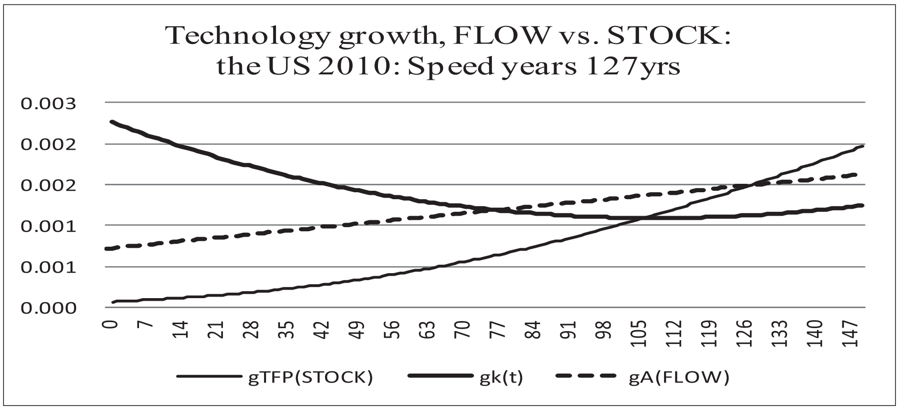
Data source: KEWT 6.12. Note: time, *t*, on the x axis, at the transitional path.
81 countries each show the same results by country.

8. Two Figures to 4. (see above)



Data source: KEWT 6.12-1.

The above figure indicates the relationship between the capital-labor ratio and the capital-output ratio by using each growth rate or the rate of change, under Sato's (285, 1981) Conservation Laws. This proves that the assumption of $k = K / L$ and $k^* = K^* / L^*$ is perfect.



Data source: KEWT 6.12-1. Note: The rate of technological progress as FLOW, $g^*_{A(FLOW)}$, equals the growth rate of technology, A=TFP as STOCK, $g_{TFP(STOCK)}(t^*)$, at convergence or at the endogenous turnpike point; $g_{A(FLOW)}(t^*) = g_{TFP(STOCK)}(t^*)$.

The cross point of the growth rate of capital-labor ratio, $g_k(t)$, and $g_{TFP(STOCK)}(t^*)$ differs from that of $g^*_{A(FLOW)}$ and $g_{TFP(STOCK)}(t^*)$. This fact is true by testing 65 countries available at KEWT and recursive programming. There empirically exists no contradiction between the literature under the price-equilibrium and KEWT and recursive programming under the endogenous-equilibrium. This fact is another proof of Sato's Lie theory, since both are proved theoretically and empirically and both match cooperatively.

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Appendix B Five to six dimensional at the real world: From Pythagoras, Gauss to Fermat, Wiles and Iyonoishi (2012)

1. Iyonoishi (xxvii-xviii, 2012)[6] proves, for the first time in history, a common mechanism that nine problems unsolved at the current physics are wholly solved by only one equation prevailing in Supersymmetric Grand Unified Theory and, this equation is $x^n + y^n = z^n$ $(n \geqq 3)$.

   $x^n + y^n = z^n$ remains the same as that of Pierre de Fermat's (1601–1665) Grand Theorem and also that of Pythagoras (572–492 B. C.) theorem. Iyonoishi indicates that $x^n + y^n = z^n$ is an equation that has mass by the breakthrough of natural symmetry and changes to mol-amount of substance.

   $x^n + y^n = z^n$ is an equation that produces pentagram form from the breakthrough of natural symmetry. $x^n + y^n = z^n$ is an equation that produces a balanced feel beauty ratio i.e., the golden ratio of 1:1.16 and shows the law of beauty (goodness, truth, and beauty) hidden in all things formation. Iyonoishi earlier finds: $x^n + y^n = z^n$ is an equation required for the beginning of human body DNA. This fact was shown by 'Kanon (body's ideal ratio)' drawn by Leonard da Vinci (1452–1519). Iyonoishi exclusively finds that its mathematical geometry is another expression of elementary particle.

6) Iyonoishi. (2012). Solve the Universe by Japanese Language: with an Article, 'To Solve Neutrino's Puzzle Why Neutrino is Faster Than Rays (written in Japanese). Tokyo: KonnichinoWadaisha. xxvii, 355p.

Pierre de Fermat's Grand Theorem (1601–1665) had not been proved for 350 years but, in 1994 Andrew John Wiles (1953–) discovered a proof of Fermat's Grand Theorem equation that except for $n = 2$, there is no (rational) integer n to satisfy $x^n + y^n = z^n$.

Iyonoishi interprets Wiles' (1994) chance to proof as follows: Wiles could prove Fermat's equation when he realized that all the elliptic curves were composed of modular forms, whose final path was given by 'Taniyama, Shimura, and Iwasawa forecast.'

According to Iyonoishi (though Kamiryo is responsible for translation[7)]), compiled module format stays at the upper half of complex plane (whose x axis is 'real axis,' and y axis is 'imaginary axis') and is characterized by non-Euclidean geometry. Non-Euclidean geometry is shown by $Rij - 1/2gijR = KTij$. The LHS of non-Euclid geometry shows bent space and time and, the RHS mass and energy; space and time is warped by mass. Originally Carl Friedrich Gauss (1777–1855) hit this module format. Gauss discovered that natural number was composed of three triangles, which was equal to Iyonoishi's mass root form. Finally Wiles proves Grand Theorem by using exponent 5. This '5' is the same as $n = 5$ at $3^2 + 4^2 = 5^2$, which Direchlet Peter Gastav Lejeune (1805–1859) proved. Iyonoishi stresses that this '5' is the origin that produces 'warped' five dimensional universes (i.e., six dimensional). Therefore five produces six dimensional in the real world).

2. Iyonoishi proves, for the first time in history, 'Higgs Boson' by expressing its substance using her own spiral-movement equation that shifts from five dimensional in spirituality to six dimensional in reality based on currently existing Gauss's Plane. It implies that Higgs Boson is a boson that shapes geometrical super symmetric particle, plus and minus in this real world.

   On 5 Aug 2012, the European Organization for Nuclear Research (CERN) discovered a new particle that seemed to be Higgs Boson, it was reported by newspapers. Contrarily, another report says that it was not discovered dated in the same August. Currently common consensus in the literature is that Higgs Boson is difficult to catch or trace back since it disappears at a moment when it appeared. Kamiryo confirmed her proof by a reply letter to Kamiryo dated 15 Sep 2012.

3. Iyonoishi (i-xxvii, with 18 figures, 2012) theoretically proves, for the first time in history, the mechanism that μ neutrino is faster than the speed of light, by using imaginary numbers. She mentions that without imaginary numbers natural science no more expresses any explanatory fact and its proof. And, she proves, using 18 figures, that by using imaginary

7) The author confirms related terminologies using http://en.wikipedia.org/wiki.

numbers the above mechanism does not contradict Einstein's theory at all.

4. Further Kamiryo pays attention to Iyonoishi's great discovery (beyond scientific discovery) that the real world simultaneously expresses every phenomenon at the spiritual world (see Iyonoishi (Figure 17 at 17, 2012).

Appendix C Why do we remain circle and hyperbola plane at the endogenous system ?

1. When spiral-parametric equation in physics is expressed at x-y plane, rotation does not appear but circle appears. Add time axis to plane then, three dimensional appears with the shape of spring and circle disappears, where the values of length and area differ from those at plane. The author takes advantage of this spiral-parametric equation and dare to remains at x-y plane. Further, the endogenous system does not need parametric equations since all the parameters and variables are endogenously, precisely, and simultaneously measured under the endogenous-equilibrium instead of the price-equilibrium.
2. Circle exists at plane and does not exist at any higher planes. KEWT database stays at plane and consistently follows circle as a base for Kamiryo's right-hyperbola to reinforce all the endogenous equations. Circle is directly related to Hicks' use of 'sin' that expresses business cycle. Circle is also related to the exponential, e^x, along with real and imaginary numbers when its complex plane exchanges equations.[8] Here the exponential, e^x, broadly connects Iyonoishi's with Furuta's proofs.
3. Plane in the real world implicitly includes space and time, from two dimensions to four dimensions. This is because the real world simultaneously expresses every phenomenon at the spiritual world or the five dimensional zone, as discovered and theoretically and empirically proved by Iyonoishi (2012, since 1998). The author explains its outline only at Chapter 10. The author stays at 'Cross-Roads Scientific Discovery Diagram' fixed by certain level of spirituality, as stressed in Chapter 1, and follows Samuelson's (1970) and Sato's (1981) discoveries based on the Lie Theory (see Notes at the beginning of Monograph).
4. Topology at plane remains explanations by researchers in the literature. Topology at the endogenous system is always measured precisely by county, sector, year, and over years. This is because all the endogenous equations are respectively reduced to hyperbola. This result is due to the circle existing behind each circle. For example, an econometrics model uses

8) The author is grateful to Yoshiomi Furuta's confirmations/explanations with graphs to the above relationships by email dated on 15 Sep 2012. Yoshiomi is my teacher and Emprof. Dr. of mathematics to integer, Kanazawa University.

CES production function whose values of elasticity is fixed, instead of using a discrete Cobb-Douglas production function under constant returns to scale, as involved in the endogenous system. The econometrics model is similar to the parametric equation in physics. The endogenous system does not need supposed elasticity values, which are endogenously measured. The endogenous system does not need module format, which is only applicable to natural science.

5. Topology in the literature shows not circle but ellipse. The literature is based on the price-equilibrium and aims at maximum profits/returns, which is shown by parabola. The endogenous system has maximum rate of return to minimum ratio of net investment to output. No one takes advantage of hyperbola unless he/she succeeds in formulating equations of all the parameters and variables at a whole system, similarly to the endogenous system. The origin is not required for parabola: anywhere parabola exists. Similarly hyperbola exists, whose origin exists anywhere although the origin of plane is fixed. Parabola is symmetric. In the case of hyperbola, the circle drawn behind the hyperbola is symmetric to the 45^0 diagonal. Hyperbola empirically proves conservation laws of Sato's (1981), which in turn justifies the endogenous system itslef.
6. Minkowsky, Hermann (1918 and many…) shifted space and time to four dimensions from two dimension plane and showed a line. The line exists at plane and also his four dimensions. The line holds anywhere simultaneously and with no contradiction, from the viewpoint of Iyonoishi's discoveries in Physics and element chemistry (see above 3). The author agrees to Iyonoishi's conclusive reply by letter dated on 15 Sep 2012. Social science eventually follows discoveries at natural science. And, this is a correct road.