«**Note**»

An expanded version: Structure of endogenous growth

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Abstract

My model [2001 under review at the University of Auckland], as an endogenous growth model, has a few problems to be solved. My model introduced three financial parameters into corporate and household savings, by means of which qualitative and quantitative investments are distinguished. This note (working paper) presents the following improvements: (1) The ratio of quantitative investment to total (quantitative and qualitative) investment, *beta*, controls more effectively the relationship between the three financial parameters. (2) The rate of technological progress is measured as a constant under constant returns to capital (CRC). This rate is constant when the elasticity of quality improvement with respect to effective labour units per capital (or the capital-labour ratio), *delta*, equals the relative share of profit, *alpha*, and is shown only using the three financial parameters and the ratios of corporate and household saving ratios, each as a parameter. (3) The rate of technological progress is shown to change over time under decreasing or increasing returns to capital (DRC or IRC), where *delta* differs from *alpha*. (4) The relationship between *beta* and *delta* is clarified in recursive programming, after fixing *delta* = *alpha*, by setting either the root mean square error (RMSE(g_Y(300), r(300)) = 0) or $r(0) = r(300)$. This comes from an assumption for the qualitative improvement in investment. Under DRC, *delta* is greater than *alpha* and the rate of technological progress is weak with the same rate of saving. On the other hand, under IRC *delta* is less than alpha or less than zero, and the rate of technological progress is high with the same rate of saving. Furthermore, if the rate of saving decreases when other conditions remain unchanged, rate of technological progress (and, accordingly, the growth rate of output or per capita output) decreases and vice versa. These results, being different from Solow's [1956], imply that the rate of technological progress changes with the changes in the rate of saving, *delta*, and *beta*, and, accordingly the three financial parameters. However, changes in *delta* have the most influence.

1. Outline and preliminary discussions

1.1 Outline

The purpose of this note (working paper) is to describe an endogenous growth model starting with the Cobb-Douglas production function, $Y = A(t)K(t)^{\alpha} L(t)^{1-\alpha}$, under constant returns to scale. This is the generalized model in my thesis.[1)] First, I must deal with this question: How can I measure the rate of technological progress? If I can measure this rate under constant returns to capital (CRC), I can measure the growth rate of output since the growth rate of output equals the growth rate of capital under CRC.

This note expands my endogenous growth model, particularly with regard to the method of measuring the rate of technological progress which is directly connected with qualitative and quantitative investment. For this purpose, I add two new parameters to my model, *delta* and *beta*, and determine the relationship between these two parameters under increasing, constant, and diminishing returns to capital.

The first inevitable hurdle for the above purpose is how to represent investment *as a coefficient*. I use net saving which is equal to net investment after capital consumption (hereafter investment), where it is shown *as an amount*. I intend to divide investment into qualitative and quantitative parts and to use qualitative investment for the level of technology and the rate of technological

1) I thank Dr. Bryce Hool, Head, and Dr. Debasis Bandyopadhyay, the University of Auckland, for their extensive and warm-hearted advice since 1995. In my previous manuscript [2001 at the U of A], I could not clarify the relationship between *delta*, *beta*, and the three calibrated financial parameters. Also, thanks to Dr. Hiroshi Noma, Dr. Yoshiomi Furuta, and Dr. Wang Jianxiong, my student, for their advice: I could replace θ_2 by *beta* as an essential parameter. The most difficult hurdle was how to measure *delta* in recursive programming to which I finally hit an idea to add another root mean square root (RMSE) method that uses the rate of technological progress.

progress expressed as coefficients. How can I convert qualitative investment from an amount to a coefficient in the Cobb-Douglas production function? If I succeed in resolving this issue, I will be able to measure the rate of technological progress.

The second inevitable hurdle for the above purpose is how to properly divide investment into qualitative and quantitative parts? Also, how can I express the qualitative or quantitative parts as ratios to investment? I will assume that the ratio of qualitative investment to total investment is related to the rate of technological progress and that the ratio of quantitative investment to total investment is related to the growth rate of capital. Also I need to divide saving into two parts, corporate saving and household saving, and introduce the derive channels into my model.

The third inevitable hurdle for the above purpose is how to distinguish the condition of constant returns to capital from the other conditions of increasing and decreasing returns to capital (IRC and DRC[2]). The growth rate of output or capital as well as the rate of technological progress are measured with certainty only under the CRC condition, where these rates are constant in the long run (at $t=\infty$). It is traditionally possible to derive the growth rate under IRC with increasing returns to scale. Is it possible to derive the growth rates under IRC as well as under DRC with the same constant returns to scale?

It is to be noted that the above hurdles are closely related each other. This note will attempt to clarify the above issues in the generalized model using equations. Simulation and empirical results will follow in a separate paper/thesis.

2) For constant returns to scale, there is the proof done by Inada [1963]. The Cobb-Douglas production function well expresses DRC under constant returns to scale. This note intends to express increasing returns to capital (IRC) even under constant returns to scale.

1.2 Framework of my model and the introduction of the financial channels

In this section I describe analytically how my model extends Solow's [1956] model of growth. In my model, I divide the saving (of Solow's) into two parts: corporate saving and household saving. Some parts of corporate and household savings are used for qualitative investment while the remaining parts are used for quantitative investment. For this dividing up, I introduce financial channels.

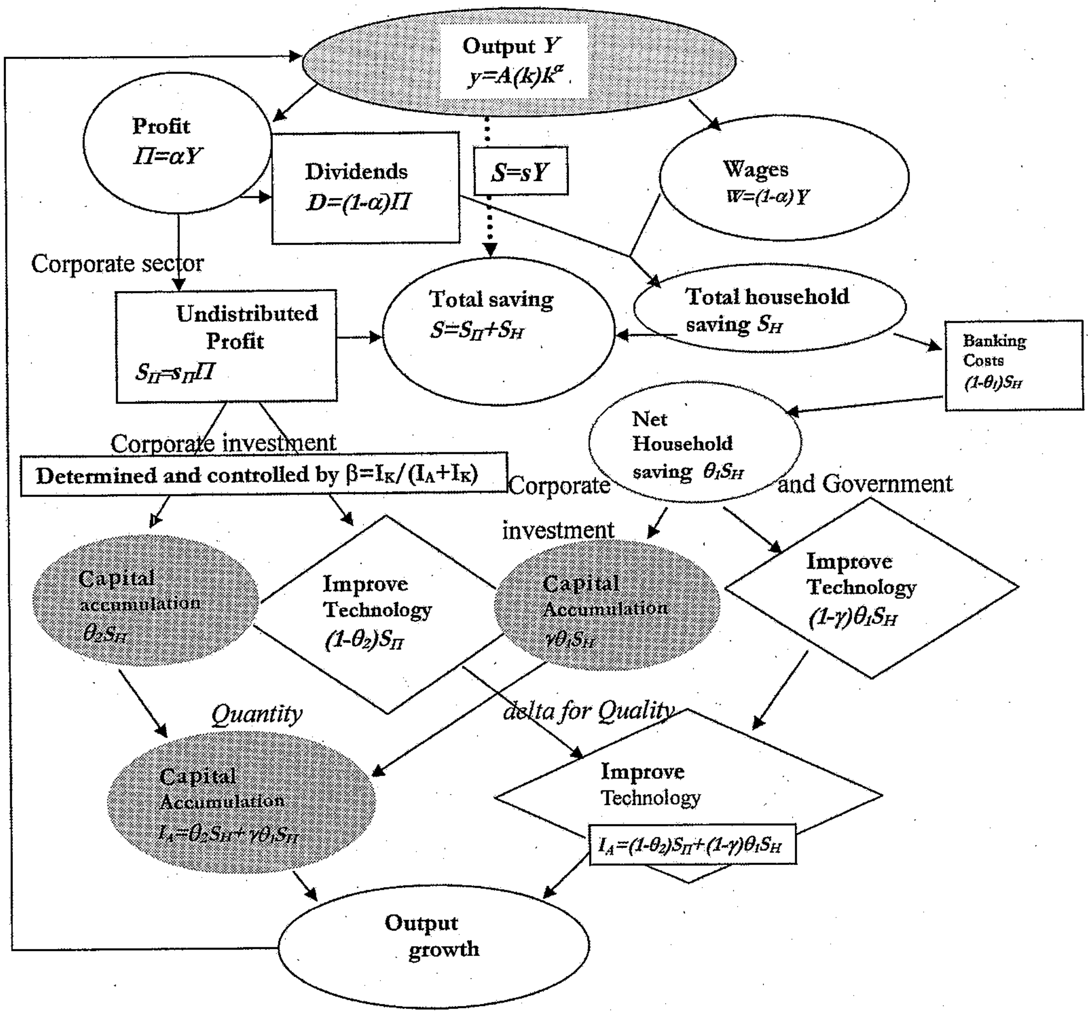
First, I have drawn a diagram of the financial channels through which savings/funds go towards investment in capital accumulation and improvement of technology in both the corporate and government sectors. Data in this section is expressed as amounts and the related ratios are calculated.[3] The conversion from an amount to a coefficient is discussed separately in following sections.

Using **Figures 1** and **2**, I will explain the framework of the generalized model. In Figure 1, I compare the Solow [1956] model and the generalized model as follows:

In these diagrams, savings, after deducting banking costs, equal net investment (for the basic ideas, see Figure 1). Saving is composed of corporate saving/undistributed profit and household saving less banking costs. Investment less capital consumption or depreciation is net investment (hereafter simply investment), and is composed of undistributed profit and household saving less banking costs. Investment is composed of both qualitative and quantitative components. It is created by the corporate sector and the government sector and assumes no investment in the household sector. Solow's model assumes that investment augments the total quantity of capital stock. However, in my model a part of the national investment is allocated to improving the *quality* of the

3) I express the coefficient of conversion using the elasticity of quality improvement with respect to effective labour units per capital, *delta*. I assume that capital, *K*, and output or income, *Y*, are expressed by amounts, although labour, *L*, is expressed using employed persons. If the above coefficient is 1.0 as a very special case, then the value expressed using an amount is equal to the value expressed using a coefficient.

Figure 1 Saving and investment diagram: comparing the generalized model with Solow's model



Note 1: This diagram shows $S = I$ under an assumption that exports, *EX*, equal imports, *IM*, and the household sector does not invest. When exports differ from imports, "$S + IM\text{-}EX$" equals net investment, *I*. Also, the change in inventories is first deducted from saving and capital transfers, receivable and payable, are added to saving. Note that the *EX-IM* is included in investment while capital transfers receivable and payable are included in saving in statistics.

Note 2: The Solow [1956] model is shown by the shaded areas in the diagram, where saving is all used for capital accumulation: $g_A = 0$ and *beta* = 1.0.

future capital stock.

Three financial parameters determine the amount of investment. These are:

1. θ_1: a financial intermediary parameter defined as the fraction of household saving that goes to the corporate and government sectors.

Figure 2 The variables and parameters used in the generalized model

Initial values

Y: output≡ $\Pi+W$	S_{total}: total saving
Π: profit≡ $S_{\Pi}+D$	S_H: household saving
W: compensation	IM: Imports
S_{Π}: corporate saving	EX: exports
D: dividends	I_{NV}: inventories
K: capital	S: net saving for investment
L: workforce	I: Investment

$I=S=S_{total}-I_{NV}+(IM-EX)$,
where $S-I=(1-\theta_1)S_H$ holds.

The generalized model using recursive programming

The initial ratios

Using the initial variables:

$\Omega(0)$: the capital-output ratio≡K/Y, where t=0 is the initial year.

$k(0)$: the capital-labour ratio≡K/L, where t=0 is the initial year.

As derived $r(0)$: the rate of profit≡Π/K

$A(0)$: the level of technology $=k(0)^{1-\alpha}/\Omega(0)$

$y(0)$: per worker output $=A(0)k(0)^{\alpha}$

beta for I_K *delta* for I_A

$I=I_A+I_K$

9 key parameters in the generalized model

θ_1: the financial intermediary	n: the growth rate of workers (for the whole period)
β: the ratio of $I_K(t)$ to $I(t)$	α: the relative share of profit≡Π/Y
θ_2: the decision-making of managers	s_{Π}: the retention ratio≡ S_{Π}/Π
γ: barriers to tech/environ/structural reform	s: the rate of saving≡S/Y
	δ: elasticity (see below)

Values always satisfy $Y = AK^{\alpha}L^{1-\alpha}$ or $y = Ak^{\alpha}$.

Endogenous variables using recursive programming, where *t=infinity*

The growth rates of output and capital, $g_Y(\infty)$, $g_K(\infty)$

The rate of technological progress, $g_A(\infty)$

The capital-output ratio, $\Omega(\infty)$; as an implicit variable.

The rate of profit, $r(\infty)$, where $r\equiv\alpha/\Omega$ and $r(\infty) = \alpha/\Omega(\infty)$.

s_H: the household saving ratio≡$S_H/(W+D)=(s-s_{\Pi}\cdot\alpha)/(1-s_{\Pi}\cdot\alpha)$, using the retention ratio and the rate of saving.

In the above programming, if the elasticity of quality improvement with respect to effective labour units per capital, *delta*, is equal to *alpha*, then the condition is always under constant returns to capital (CRC).

2. θ_2: a decision-making parameter that shows the relationship between the qualitative and quantitative investment as decided on by corporate managers. It is defined as the fraction of undistributed profit that the managers decide to invest in (quantitative) capital.

3. γ: a parameter for barriers to technological/structural reform that shows the

balance between private and public investment in terms of efficiency. It is defined as the fraction of total household saving net of banking cost that is invested in (quantitative) capital. If the government of a country competes for household saving by running a deficit or requiring a business to pay for legal/bureaucratic costs, then a part of the household saving would go to the government and not to the corporate sector. If government uses up that portion of household saving without using any of it for qualitative investment purposes then it acts as a barrier to growth. If the government uses a high proportion of household saving to cover bureaucratic costs, and to run inefficient public companies, then the corporate sector will also be affected. For example, if inefficiencies cause banking costs to be high, then private companies will be forced to absorb these costs as well, and this lowers the proportion of household saving that can be efficiently invested.

Figure 1 illustrates how much of total investment is used for qualitative and quantitative investment, I_A and I_K, respectively. The relationship between θ_2 and γ is determined using the ratio of quantitative investment to the sum of qualitative and quantitative investment, β, expressed as $I_K/(I_K + I_A)$. The fundamental equations are presented in the next section using the parameters defined above: β, θ_1, θ_2, and γ.

The characteristics of the three parameters, θ_1, θ_2, and γ, are as follows:

1. θ_1: The higher the better. When costs are low, the value of θ_1 is closer to one.
2. θ_2 and γ: The lower the better. When regulations are removed and structural reform is improved, these ratios are lowered, resulting in technological progress. Or alternatively, when public investment is less effective than private investment, the value of γ is higher than the value of θ_2. These parameters are determined using calibration.

I can now show that Solow's model is a special case of the generalized model where qualitative investment is zero: $I_A(t) = 0$ or $\gamma = 1$ and/or $\theta_1 = 0$ and $\theta_2 = 1$.

In the generalized model, investment is composed of $I_A(t)$ and $I_K(t)$ as well as investment in the corporate/private sector and in the government/public sector. This note assumes that household saving after deducting banking costs is used for both the corporate and government sectors. At the end of this note I have included an idea for the optimum division of household saving between the corporate and government sectors based on the above equations for $I_A(t)$ and $I_K(t)$.

2. Formulation of the generalized model

2.1 Preliminary step for the measurement of the growth rates of capital and output

I use several basic values and ratios for the measurement of the rate of technological progress and the growth rates of capital and output. These values and ratios with their basic equations are as follows:

1. $L_t = L_0 (1 + n)^t$, (3.1)

   where n is the growth rate of employed persons.[4)]

2. $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. (3-2)
3. $\Pi_t = \alpha Y_t$. (3-3)
4. $W_t = (1-\alpha) Y_t$. (3-4)
5. $D_t = (1 - s_{\Pi}) \Pi_t$. (3-5)

4) I use usually the terminology of "employed persons" for population or workers in my thesis. My model is applicable to either population or workers, depending on data availability. For my data of national accounts, I use the number of "employed persons" as workers by country, where the number of "employed persons" is larger than the number of actual "employees." The literature sometimes use population (e.g., Jones [1998, p. 180] and OECD National Accounts I by year).

For simplicity, assume that $L(0) = 1$. When the growth rate of population/workers is not zero, "$K_t = k_t$ ($\equiv K_t/L_t$) and $Y_t = y_t$ ($\equiv Y_t/L_t$) under $L_t = 1$" do not hold. For simplicity in my recursive programming, I use $y_t = Ak^{\alpha}$, instead of $Y_t = A_t K_t^{\alpha} L^{1-\alpha}$, where $g_{Kt} = g_{kt} + n$ and $g_{Yt} = g_{yt} + n$ are applied.

6. $S_{Ht} \equiv s_H(W_t + D_t)$, where s_H is the household saving ratio defined as the ratio of household saving to wages and dividends. (3-6)

7. $S_{\Pi t} = s_\Pi \Pi_t$, where s_Π is the retention ratio defined as undistributed profit (corporate saving) to total profit. (3-7)

8. $S_t = S_{Ht} + S_{\Pi t}$. (3-8)

9. $(A_{t+1} - A_t) = \Delta A = I_{At}$ (qualitative investment). (3-9)

10. $(K_{t+1} - K_t) = \Delta K = I_{Kt}$ (quantitative investment). (3-10)

11. $I_t = I_{At} + I_{Kt}$. (3-11)

12. $S_t = I_t$, if banking costs are zero. (3-12)

In short, saving is divided into two parts: household saving and corporate saving. This saving as a whole equals net investment if banking costs equal zero. When banking costs exist, saving is larger than net investment by the amount of the banking costs. Net investment does not include capital consumption or economic depreciation. I measure the growth rate of capital in the long-run, which equals the rate of depreciation.

How can net investment (hereafter investment) be divided into qualitative and quantitative investment? If a method for doing can be found, the rate of technological progress in the long-run can be measured endogenously. The next section will discuss this measurement of the rate of technological progress.

2.2 Measurement of the rate of technological progress in the long-run

The purpose of this section is to explain how to measure the rate of technological progress under constant returns to capital (CRC). Then, the growth rate of output can be derived since the growth rate of capital is equal to the growth rate of capital under constant returns to capital. How can I connect qualitative and quantitative investment with the Cobb-Douglas production function (where qualitative investment is related to the level of technology and the rate of technological progress)? For this, I use additional two parameters, δ and β. When these two parameters are introduced, problems lying behind the use of the Cobb-Douglas production function will be solved. By using δ and δ, I can measure a constant rate of technological progress; however, sufficient and necessary conditions must be clarified.

2.2.1 Introduction of the elasticity, δ, into qualitative investment

To finalize my model, I need a new parameter, δ (*delta*), in the generalized model which is defined as the elasticity of quality improvement with respect to the capital-labour ratio. Once *beta* was determined, *delta* is determined in calibration. When *delta* is equal to zero, qualitative investment as an amount, $I_A(t)$, equals qualitative investment as a coefficient, $I'_A(t)$. This section proves (1) that qualitative investment, $I_A(t)$, must be shifted to per capita qualitative investment. $I_A(t)/L(t)$, as a sufficient condition under CRC and (2) that *delta* must be equal to *alpha* as a necessary condition under CRC. This is because the rate of technological progress, $g_A(t)$, is measured under CRC and as a constant under CRC. After these two proofs, the essence of *delta* will be formulated more clearly.

As a preparation for the above two proofs, I distinguish the difference between qualitative investment as an amount, $I_A(t)$, and qualitative investment as a coefficient, $I'_A(t)$. This is because investment is shown by an amount, but the level of technology, $A(t)$, and the rate of technological progress, $g_A(t)$, are expressed as coefficients in the Cobb-Douglas production function. In the case of quantitative investment, there is no problem because quantitative investment, $I_K(t)$, and capital stock, $K(t)$, in the Cobb-Douglas production function, are both shown as amounts. Accordingly, I establish a relationship between qualitative investment as an amount and qualitative investment as a coefficient by using "the conversion coefficient," $b(t)$:

$$b(t) \equiv k(t)^{\delta}, \tag{3-13}$$

where $I'_A(t)=\frac{I_A(t)}{b(t)}=\frac{I_A(t)}{k(t)^{\delta}}$. (3-14)

The above elasticity, δ, basically adjusts the relationship between qualitative investment as an amount and that as a coefficient under the following assumption:

Assumption 1 The effectiveness of an investment in improving the quality of a machine increases with the total units of effective labour attached to that machine. In other words, we get a bigger bang for the buck spent in improving the quality of a machine, if there is a larger number or a better quality of employees (or both) per capital invested.

Assumption 1 does not directly treat the level of technology. However, note that the level of technology, $A(t)$, is the sum of $A(t-1)$ and qualitative investment as a flow. Also, $A(t)$ is connected with $k(t)$ (since $A(t)=k(t)^{1-\alpha}/\Omega(t)$) and thus, labour is by nature effective labour.[5)]

Now back to the two proofs, *first*, why qualitative investment should be expressed as per capita qualitative investment (as a necessary condition under CRC)?

1. Determine qualitative investment as an amount:

$$I_A(t)=(1-\gamma)\theta_1 S_H(t)+(1-\theta_2)S_{\Pi}(t). \tag{3-15}$$

2. Determine qualitative investment as a coefficient:

$$I'_A(t)=\frac{(1-\gamma)\theta_1 S_H(t)+(1-\theta_2)S_{\Pi}(t)}{k(t)^{\delta}}. \tag{3-16}$$

3. Determine the rate of technological progress: $g_{A_t}(t+1)\equiv\frac{\Delta A(t)}{A(t)}=\frac{I'_A(t)}{A(t)}$.
(3-17)

5) This assumption starts with Dr. Debasis' idea. I express his idea differently in my modelling (see after Equation 3-24). Effective labour (shown as AL/K) is included in our definition and equations, but when I measure the rate of technological progress, the A of the numerator in my equation and the A of the denominator in g_A are offset.

Under what conditions (both sufficient and necessary) can I prove that the value of $I'_A(\infty)$ divided by $A(\infty)$ in Equation 3-16 becomes a constant? If the relationship between $S_H(\infty)$, $S_\Pi(\infty)$, $k(\infty)^\delta$, and $A(\infty)$ reduces to a constant, then the *sufficient* and *necessary* conditions for $g_A(\infty)$ will be satisfied.

4. Now, household saving, $S_H(t)$, can be shown as $S_H(t) = s_{SH/Y}Y(t)$, where

$$s_H = S_H(t) / (W(t) + \Pi(t)) \text{ and } s_{SH/Y} = s_H(1 - s_\Pi \cdot \alpha). \tag{3-18}$$

5. And corporate saving (undistributed profit), $S_\Pi(t)$, can be shown as

$$S_\Pi(t) = s_{S\Pi/Y}Y(t), \text{ where } s_\Pi = S_\Pi(t) / \Pi(t) \text{ and } s_{S\Pi/Y} = s_\Pi \cdot \alpha. \tag{3-19}$$

It is clear that $S_H(\infty)$ and $S_\Pi(\infty)$ are expressed using $Y(\infty)$. Then, it is suggested that *a sufficient condition* for the conversion of qualitative investment from an amount to a coefficient is derived from the relationship between $A(\infty)$, $Y(\infty)$, and $k(\infty)^\delta$ shown in Equation 3-16 as follows:

For the sufficient condition, let me tentatively suppose that $Y(\infty)$ is replaced by $y(\infty)$ as per capita output or $S_H(t)$ and $S_\Pi(t)$ are each divided by labour, $L(t) = L(1 + n)^t$. Then, Equation 3-16, qualitative investment, is shown on a per employed person basis as:

$$I'_A(t) / L(t) = \frac{(1-\gamma)\theta_1 \cdot s_{SH/Y} \cdot y(t) + (1-\theta_2)s_{S\Pi/Y} \cdot y(t)}{k(t)^\delta}. \tag{3-20}$$

$$I'_A(t) / L(t) = \frac{(1-\gamma)\theta_1 \cdot s_{SH/Y} \cdot A(t)k(t)^\alpha + (1-\theta_2)s_{S\Pi/Y} \cdot A(t)k(t)^\alpha}{k(t)^\delta}. \tag{3-21}$$

Thus, the sufficient condition may be that investment is expressed using "per capita" values.

Second, why *delta* should be equal to *alpha* as a necessary condition under CRC? In Equation 3-21, nevertheless, the value of $I'_A(t) / L(t)$ is still not a constant, yet it suggests that the difference between a variable and a constant comes only from the difference between *delta* and the relative share of profit, *alpha*. For a *necessary condition*, let me now assume that d*elta* = *alpha*, $\delta = \alpha$, in Equation 3-21. Then,

$$I'_A(t)/L(t) = \left((1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2)\cdot s_{S\Pi/Y}\right)A(t). \qquad (3\text{-}22)$$

As a result,

$$g_A(t+1) = \frac{I'_A(t)/L(t)}{A(t)} = (1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2)\cdot s_{S\Pi/Y}.^{6)}$$

Define that $i_A \equiv (1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2)\cdot s_{S\Pi/Y}$.

Then, $g_A(\infty) = i_A$ (3-23)

Therefore, the rate of technological progress now becomes "a constant." If $\gamma = 1$ and $\theta_2 = 1$ as in Solow's model, this rate becomes zero. If $\gamma = 1$ and $\theta_2 = 0$, this rate is equal to $s_{S\Pi/Y}$.

In short, it was proved that $\delta = \alpha$ in per capita qualitative investment was a necessary condition to the measurement of rate of technological progress under CRC. As a result, the rate of technological progress becomes a constant: i_A. I state this result as a proposition:

Finally after proving the sufficient and necessary conditions as above, let me formulate the *delta* as a final form and describe the relationship between the conditions of DRC, CRC, and IRC.

delta is generally expressed using equations as follows:

1. Write down the full expression of

$$k(t)^\wedge\delta = \frac{I_A(t)/L(t)}{i_A(t)}, \text{ where } i_A(t) \equiv ((1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2)\cdot s_{S\Pi/Y})y(t).^{7)} \qquad (3\text{-}24)$$

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6) The value of $A(0)$ is the same, regardless of using $Y = AK^\alpha L^{1-\alpha}$ or $y = Ak^\alpha$. In both cases, the increase in A, ΔA, is expressed using per capita ΔA.

7) $Y(0)$ *or* $y(0)$ is not related to *delta, where* $A(0) = k(0)^{1-\alpha}/\Omega(0)$. However, $A(1) = A(0) + I_A(1)/k^\delta$. Therefore, $A(1) = \dfrac{k(0)^\delta \cdot A(0)}{k(0)^\delta} + \dfrac{I_A(1)}{k(0)^\delta}$ and, accordingly, $Y(1) = A(1)k(1)^\alpha = \left(\dfrac{k(0)^\delta \cdot A(0) + I_A(1)}{k(0)^\delta}\right)k(1)^\alpha$.

Thus, *delta* is involved in $A(t)$ and $Y(t)$ except for $A(0)$ and $Y(0)$. These equations do not contradict Assumption 1 above. Thanks for Dr. Wang's advice.

2. Denote $\frac{I_A(t)/L(t)}{i_A(t)}$ by $M(t)$.

   $M(t)$ = $k(t)$^*delta*.

3. Take the natural log of both sides to get: $LnM(t) = delta \cdot Lnk(t)$.

4. The partial derivative of Ln$M(t)$ with respect to $k(t)$ equals approximately the elasticity of "$M(t)$" with respect to "$k(t)$."

In the generalized model, $M(t)$ is expressed as the ratio of per capita qualitative investment as an amount, $I_A(t)/L(t)$, to qualitative investment as an coefficient (after introducing *delta*), $i_A(t)$, and, thus, the elasticity value is defined as $\delta = \frac{LnM(t)}{Lnk(t)}$, where *delta* is given: $\delta = \frac{Ln(I_A(t)/L(t)) - Ln(i_A(t))}{Lnk(t)}$.

In the generalized model, delta determines the conditions of DRC, CRC, and IRC as follows:

1. If *delta* is close to zero or a little less than zero under IRC, investment is significantly oriented to qualitative investment.
2. If *delta* equals *alpha*, the condition is under CRC.
3. If *delta* is more than *alpha* under DRC, investment is definitely oriented to qualitative investment.

Proposition 1: If qualitative investment is measured by the per capita value with $\delta = \alpha$, the rate of technological progress under CRC is shown as a constant value of $i_A \equiv (1-\gamma)\theta_1 \cdot s_{SH/Y} + (1-\theta_2) \cdot s_{S\Pi/Y}$.

It is suggested that both saving ratios, corporate and household, must be followed by corresponding improvements in the financial parameters, particularly, θ_2 and γ.

2.3 Introduction of the ratio of quantitative investment to qualitative and quantitative investment, β

I need another new parameter, β (*beta*), which is defined as the ratio of quantitative investment to qualitative and quantitative investment. This is because the two values of θ_2 and γ are determined using calibration, but must be determined at the same time. To solve this issue, I use β. The relationship between β, θ_2, and γ is shown as follows:

$$I_K(t) = \gamma \cdot \theta_1 \cdot S_H(t) + \theta_2 \cdot S_\Pi(t) \tag{3-25}$$

Using Equations 3-16 and 3-24,

$$\beta = \frac{I_K(t)}{I(t)} = \frac{i_K \cdot Y(t)}{i_K \cdot Y(t) + i_A \cdot Y(t)} = \frac{i_K}{i_K + i_A},$$

$$\text{or } \beta = \theta_2\left(\frac{s_{S\Pi/Y}}{s}\right) + \gamma\left(\frac{s_{SH/Y}}{s}\right), \text{ which is constant.} \tag{3-26}$$

Using Equation 3-26, the value of γ is derived as follows:

$$\gamma = \beta\left(\frac{s_{S\Pi/Y}}{\theta_1 \cdot s_{SH/Y}} + 1\right) - \frac{\theta_2 \cdot s_{S\Pi/Y}}{\theta_1 \cdot s_{SH/Y}} \tag{3-27}$$

Therefore, assume that θ_1 and θ_2 are given, then if β is given, γ is derived and if γ is given, β is derived. Recursive programming can show both ways, but it is recommended that β should be first given as a basic parameter.

Finally, let me clarify the relationship between β and the rate of technological progress, $i_A = g_A$.

$$\beta = \frac{i_K}{i_K + i_A} = \frac{i_K}{i_K + g_A} \text{ or } 1 - \beta = \frac{i_A}{i_K + i_A} = \frac{g_A}{i_K + g_A}$$

$$g_A = (\theta_1 \cdot s_{SH/Y} + s_{S\Pi/Y})(1 - \beta) \text{ or } g_A = \frac{1 - \beta}{\beta} \cdot i_K \tag{3-28}$$

For a sustainable rate of technological progress, it is important that:

1. The ratio of qualitative investment to the sum of qualitative and quantitative investment, β, be lowered.
2. Corporate and household savings be larger, but with a lower θ_2 and γ.

Proposition 2: If corporate and household savings increase with a lower θ_2 and γ, the rate of technological progress will be robust.

2.4 Measurement of the growth rates of capital and output in the long-run

The growth rates of capital and output in the short-run, $g_K(t)$ and $g_Y(t)$, remain variables and are measured using recursive programming. The growth rates of capital and output in the long-run are obtained by setting $g_K(t) = g_Y(t)$.

The growth rate of capital in the short-run is expressed as follows:

Define that $I_k(t) \equiv I_K(t) / L(t)$ and $i_K \equiv \gamma \cdot \theta_1 \cdot s_{SH/Y} + \theta_2 \cdot s_{S\Pi/Y}$ (corresponding with Equation 3-24).

$$I_k(t) = I_K(t) / L(t) = (\gamma \cdot \theta_1 \cdot s_{SH/Y} + \theta_2 \cdot s_{S\Pi/Y}) y(t) = i_K \cdot y(t)$$

$$g_k(t+1) = \frac{\gamma \cdot \theta_1 \cdot s_{SH/Y} + \theta_2 \cdot s_{S\Pi/Y}}{k(t)} \cdot y(t) = \frac{i_k \cdot y(t)}{k(t)} = i_k \cdot A(t) k(t)^{\alpha-1}. \text{[8)]} \qquad (3\text{-}29)$$

It is suggested that it is better to measure the growth rate of per capita capital using $y(t)$ and i_K, instead of the growth rate of capital using $Y(t)$ and i_K. Both growth rates, $g_K(t)$ and $g_Y(t)$ (or $g_k(t)$ and $g_y(t)$), are the same under CRC. I can now measure $g_K(t)$ and $g_Y(t)$ under CRC (or $g_Y(\infty)$ and $g_K(\infty)$), using the following equations derived from the Cobb-Douglas production function:

$g_Y(t) = g_A(t) + \alpha \cdot g_K(t) + (1-a)n$ and $g_K(t+1) = g_k(t+1) + n$.

$$g_Y(\infty) = g_K(\infty) = \frac{g_A(\infty)}{1-\alpha} + n = \frac{i_A}{1-\alpha} + n \text{ since } g_A(\infty) = i_A. \qquad (3\text{-}30)$$

Therefore, the growth rate of output or capital at t = infinity becomes a constant.

In short, in the generalized model, the rate of technological progress and, accordingly, the growth rate of output or capital are measured endogenously using the Cobb-Douglas production function, $Y(t) = A(t)K(t)^{\alpha} L(t)^{1-\alpha}$ or $y(t)$ $A(t)k(t)^{\alpha}$, where the level of technology, $A(t)$, must be a coefficient, not an amount.

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8) When K, instead of k, is used, $g_K(t+1) = \frac{r \cdot \theta_1 \cdot s_{SH/Y} + \theta_2 \cdot s_{S\Pi/Y}}{K(t)} \cdot Y(t)$.

3. The relationship between IRC, CRC, and DRC

3.1 The relationship between *delta* and *alpha*

Let me now clarify the relationship between IRC, CRC, and DRC, using the relationship between *delta* and *alpha*. I can distinguish the condition of constant returns to capital (CRC) from conditions of increasing returns to capital (IRC) and decreasing returns to capital (DRC). This is possible by using the relationship between *alpha* and *delta* as follows:

Following again the above Assumption 1:

1. If *delta* < *alpha*, the condition is under IRC, where $g_A(t)$ is a variable of time.
2. If *delta* = *alpha*, the condition is under CRC, where $g_A(t)$ is a constant.
3. If *delta* > *alpha*, the condition is under DRC, where $g_A(t)$ is a variable of time.

The rate of technological progress under CRC can take on any value which is consistent with changes in the three financial parameters. In other words, the CRC condition holds if and only if *delta* is equal to *alpha*, regardless of the values of the three financial parameters. And, the growth rate of output under CRC is exactly shown as the rate of technological progress divided by "1-*alpha*." I stress here that the form, $g_y(\infty)=\frac{g_A(\infty)}{1-\alpha}=\frac{g_A}{1-\alpha}$, is consistent with the Solow's [1956] model.[9] The value of g_A, however, is $g_{A_t}(\infty)=g_A=i_A=(1-\gamma)\theta_1\cdot s_{SH/Y}+(1-\theta_2)\cdot s_{S\Pi/Y}$ in my model (see Equation 3-24).

3.2 The growth rates of capital, output, and technology under IRC or DRC

Then, how can the equations for the rate of technological progress be expressed under IRC or DRC, where *delta* ≠ *alpha*? If the growth rate of capital is measured under IRC or DRC, then and the rate of technological progress

9) In Solow's model, however, the value of β is always 1.0.

and, accordingly, the growth rate of output will also be measured under IRC or DRC, using recursive programming.

3.2.1 The growth rate of capital under IRC or DRC

The growth rate of capital under IRC or DRC, $g_K(t)$, was already shown using $k(t)$, instead of $K(t)$, where $g_K(t) = g_k(t) + n$ (see Equation 3-29).

$$g_k(t+1) = \frac{i_k \cdot y(t)}{k(t)} = i_k \cdot A(t)k(t)^{\alpha-1}.$$

$$g_K(t+1) = i_k \cdot A(t)k(t)^{\alpha-1} + n, \; under \; n \neq 0.$$

3.2.2 The rate of technological progress under IRC or DRC

Next, let me show the rate of technological progress, $g_A(t)$ under IRC or DRC. The conversion coefficient, $b(t) = k(t)^\delta$, significantly influences the level of technology and the rate of technological progress, $A(t)$ and $g_A(t)$. Under IRC or DRC, the conversion coefficient, δ, is not equal to α. Therefore, related equations are expressed as in generalized forms, regardless of IRC, CRC, or DRC, using the elasticity of quality improvement with respect to effective labour units attached to capital, *delta*.

$$I'_A(t)/L(t) = \frac{(1-\gamma)\theta_1 \cdot s_{SH/Y} \cdot A(t)k(t)^\alpha + (1-\theta_2)s_{S\Pi/Y} \cdot A(t)k(t)^\alpha}{k(t)^\delta}.$$

$$I'_A(t)/L(t) = \frac{i_A \cdot A(t)k(t)^\alpha}{k(t)^\delta}.$$

$$I'_A(t)/L(t) = A(t) \cdot i_A \cdot k(t)^{\alpha-\delta}. \quad (3\text{-}31)$$

When *alpha=delta*, then $k(t)^{\alpha-\delta} = 1$ and Equations 3-23 and 3-24 hold, where the rate of technological progress is a constant. When *alpha* $\neq$ *delta*, then $k(t)^{\alpha-\delta} \neq 1$, and the following equations hold:

$$g_A(t+1) \equiv \frac{I'_A(t)/L(t)}{A(t)} = i_A(t) \cdot k(t)^{\alpha-\delta}. \quad (3\text{-}32)$$

[10]

10) Note that $i_A(t) \cdot k(t)^{\alpha-\alpha} = i_A$. When θ_2 and γ are 1.0, the rate of technological progress is constant: this rate is shown simply as $g_A = \alpha \cdot s_\Pi$. The rate of technological progress in Solow's model is also constant, but as a given parameter.

3.2.3 The growth rate of output under IRC or DRC

The generalized forms under *delta* ≠ 0, where $b(t)$ = k(t)^*delta* ≠ 1 are:

$$y_{t+1} = A_0\left(\prod_{i=0}^{t}(1+i_A\cdot k_{t-i}^{\alpha-\delta})\right)k_{t+1}^{\alpha}, \text{ where A}(t) \text{ is a variable:} \tag{3-33}$$

$$A_{t+1} = A_t(1+i_A\cdot k_t^{\alpha-\delta}).$$

$$= A_{t-1}(1+i_A\cdot k_{t-1}^{\alpha-\delta})(1+i_A\cdot k_t^{\alpha-\delta}).$$

$$\Rightarrow A_{t+1} = A_0\prod_{i=0}^{t}(1+i_A\cdot k_{t-i}^{\alpha-\delta})$$

Using the above basic equation,

$$g_y(t) = \alpha g_k(t) + \alpha g_k(t-1) + \cdots + \alpha g_k(0)^{11)} \tag{3-34}$$

$$= \alpha\sum_{i=0}^{t} g_k(t-i). \qquad \text{(Denote } j \equiv t\text{-}i\text{).}$$

$$= (\alpha\cdot i_A)\sum_{j=0}^{t} A_j k_j^{\alpha-1}. \quad \text{(Recall } A_j = \prod_{i=0}^{j-1}(1+i_A\cdot k_{j-i}^{\alpha-\delta})\text{).}$$

$$= (\alpha\cdot i_A)A_0\sum_{j=0}^{t}\left(\prod_{i=0}^{j-1}(1+i_A\cdot k_{j-i}^{\alpha-\delta})\right)k_j^{\alpha-1}.$$

$$= (\alpha\cdot i_A)(A_0k_0^{\alpha-1} + A_1k_1^{\alpha-1} + \cdots + A_tk_t^{\alpha-1}). \quad \text{Thus,}^{12)}$$

$$\lim_{t\to\infty} A_t k_t^{\alpha-1} = A_0\left(\prod_{j=0}^{t-1}(1+i_A\cdot k_{t-j}^{\alpha-\delta})\right)k_t^{\alpha-1} \tag{3-35}$$

$$= A_0\left[(1+i_A\cdot k_t^{\alpha-\delta})(1+i_A\cdot k_{t-1}^{\alpha-\delta})(1+i_A\cdot k_{t-2}^{\alpha-\delta})\cdots(1+i_A\cdot k_{t-(t-1)}^{\alpha-\delta})(1+i_A\cdot k_0^{\alpha-\delta})\right]k_{t+1}^{\alpha-1}.$$

= ∞ if $\delta < \alpha$ (as a situation under increasing returns to capital, IRC).

= constant if $\delta = \alpha$ (as a situation under constant returns to capital, CRC).

= 0 if $\delta > \alpha$ (as a situation under decreasing returns to capital, DRC).

These results are shown typically using recursive programming. I state these results, as a proposition:

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11) $t = t-i$, where $i = 0$. $t-1 = t-i$, where $i = 1$. $0 = t-i$, where $i = t$.

12) It is necessary for $A_tK_t^{\alpha-1}$ to be zero under a special condition for this equation to hold.

Proposition 3: If the elasticity of quality improvement with respect to effective labour units per capital, δ, is smaller than the relative share of profit, α, the condition is under increasing returns to capital (IRC). If $\delta = \alpha$, the condition is under constant returns to capital (CRC), and if $\delta > \alpha$, the condition is under decreasing returns to capital (DRC).

Thus, my model shows IRC, CRC, and DRC, using constant returns to scale (or $y = Ak^{\alpha}$). I proved that both IRC and DRC are shown in the same expanded Cobb-Douglas production function, dividing investment into qualitative and quantitative investment.

4. Other issues to be resolved

4.1 Speeds of convergence

Finally, let me show how speeds of convergence can be measured. Under CRC, the growth rate of capital or per capita capital equals the growth rate of output or per capita output. The rate of technological progress is not equal to the growth rate of per capita output, but both rates are parallel over time, satisfying the Slow's finding, $g_y(t) = g_A(t)/(1 - \alpha)$. Then, speeds of convergence are realized when $g_y(t) = g_k(t)$. Or, alternatively, speeds of convergence are realized when $g_y(t)$ or $g_k(t)$ becomes parallel to $g_A(t) = i_A$. For this measurement, I prefer to take the relationship between $g_k(t)$ and $g_A(t) = i_A$. This is because $g_y(t)$ is realized as a result of using capital and labour and because the growth rates have the following characteristics.

1. Under IRC, both $g_A(t)$ and $g_y(t)$ increase over time while $g_k(t)$ slightly decreases over time.
2. Under DRC, both $g_A(t)$ and $g_y(t)$ decrease over time while $g_k(t)$ decreases over time.
3. Under CRC, $g_A(t)$ is flat over time and most reliable.

Speeds of convergence depend on the values of parameters involved in $gk(t)$

and $g_A(t) = i_A$ *under CRC*, where $g_k(t+1) = i_k \cdot A(t)k(t)^{\alpha-1}$.

Speeds of convergence are significantly related with the elasticity of quality improvement with respect to the capital-labour ratio (see Appendix). When the above level of technology is replaced by "qualitative investment," Assumption 1 holds as a basis for *delta*.

A theoretical and empirical study will be done more in a separate paper/thesis. I am testing speeds of convergence using the two different values of *delta*.[13)]

4.2 Calibration of *delta*, *beta*, and the three financial parameters

In this section, I show how to calibrate the value of *beta* and *delta*, and the three financial parameters, θ_1, θ_2, and γ. This calibration is done at the same time using the method of root mean square error (the RMSE method). The value of *beta* is the weighted average of θ_2 and γ which use corporate and household saving ratios and calibrates the relationship between quantitative and qualitative investment. The value of *delta* only determines the improvement in qualitative investment. Without fixing the value of *delta*, the value of *beta* cannot be calibrated under the CRC situation. For the RMSE, if I calibrate the value of *beta* first using the growth rate of output and the rate of profit, then the value of *beta* remains under the DRC or IRC situation, where the corresponding *delta* is also calibrated under DRC or IRC.

In details, for the two sets of variable, $g_Y(t)$ and $r(t)$, and $g_Y(t)$ and $g_A(t)$, in the root mean square error (RMSE), I set the difference of each actual value at $t = 0$[14)] and the model's value at $t = 300$ at zero. Recursive programming usually proves that the results of the RMSE method approach zero except for some

13) I repeat how difficult it was to measure the two different values of *delta* under CRC and DRC or IRC. I am really thankful to the tolerance shown by Dr. Bryce Hool, Head, and Dr. Debasis Bandyopadhyay,

14) The actual rate of technological progress is calculated using the equation of Cobb-Douglas production function: $g_{A(actual)} = g_{Y(actual)} - \alpha \cdot g_{K(actual)} - (1-\alpha)n$.

extreme cases. For the model's values, I prefer the values at $t = 300$ to the values at $t = 100$ since the values at $t = 300$ are more stable than those at $t = 100$. Furthermore, I can use a planned growth rate of output and a planned rate of technological progress, instead of the actual growth rate of output and the actual rate of technological progress.

The final equations of the RMSE are:

For *beta*:

$$RMSE_{MIN} \equiv \sqrt{((g_{Y(model)}(300) - g_{Y(actual)}) / g_{Y(actual)})^2 + ((r(300) - r_{(actual)}) / r_{(actual)})^2} \quad (3\text{-}36)$$

For *delta*:

$$RMSE_{MIN} \equiv \sqrt{((g_{Y(model)}(300) - g_{Y(actual)}) / g_{Y(actual)})^2 + ((g_{A(model)}(300) - g_{A(actual)}) / g_{A(actual)})^2} \quad (3\text{-}37)$$

I sometimes simply show the above equations as $RMSE_{MIN}$ ($g_Y(300)$, $r(300)$) and $RMSE_{MIN}$ ($g_Y(300)$, $g_A(300)$), where minimum (MIN) approaches zero.

I state the following necessary notes for calibration:

1. Theoretically, the longer the time (e.g., $t = 300$) the more accurate the RMSE, although the growth rate of output and the rate of profit are lower. Financial parameters move more slightly.
2. For proposing economic policies, the shorter the time (e.g., $t = 100$) financial parameters move more sharply, where the growth rate of output and the rate of technological progress are higher and financial parameters change more quickly.
3. The growth rate of output and the rate of profit move a little bit differently in the real world. $RMSE_{MIN}$ ($g_Y(300)$, $r(300)$) always approaches zero but the resultant *beta* is g_Y-oriented rather than r-maximum-oriented.
4. It is noted that under constant returns to capital (CRC) the capital-output ratio must be flat over time, which takes much more time to approach flat

than other variables such as the rates of technological progress, capital, and output. Therefore, we must confirm whether the capital-output is already flat over time or not under constant returns to capital (CRC). This is important when we measure *beta* or γ under CRC.

5. There are two alternatives for calibrating parameters: (1) using the value of *beta* that was calibrated with *delta* = 0 and (2) using a new value of *beta* that is calibrated by replacing the value of the rate of profit under CRC by the initial/current value of the rate of profit. In the latter case, the capital-output ratio under CRC exactly equals the initial value of the capital-output ratio. I affirm the latter case. This is because by using *beta* under CRC, where the capital-output ratio is the same as the initial one, we can propose true investment policies (see Samuelson's [1970] law of the conservation). The necessary policies are suggested by comparing different elasticity-values between *beta* and *delta*.[15)]

6. Previously (before introducing *beta* and *delta*), I had to calibrate both θ_2 and γ at the same time, but separately,[16)] where the growth rates of output under CRC using θ_2 and γ must be the same. To fix the value of θ_2, it is possible to introduce the following equation: $\theta_2 = c_{\theta 2}(1 - s_\Pi)$. This comes from an assumption that the payout ratio is closely related to the value of θ_2. The coefficient of θ_2, $c_{\theta 2}$, shows stable values between 2.0 and 3.0 in Japanese national accounts in 1992 to 2000 (see empirical results at the end of this

15) For example, $\eta\ \delta/\beta$, $\eta\ \gamma/\beta$, $_{gY}/\ \delta$, and $\eta\ _{gY}/\beta$, where η indicates the value of elasticity, the numerator indicates the numerator of the elasticity, and the denominator indicates "with respect to the denominator."

16) In this case, first γ is gradually decreased until the RMSE value no longer decreases, then θ_2 is decreased to further decrease the RMSE value (albeit only slightly) until it again no longer decreases. At this point I have found the minimum value of the RMSE. This is because θ_1 only very slightly influences the growth rate of output as opposed to θ_2 and γ.

note).

Thus, I state Assumption 2 below. The payout ratio or "one less the retention ratio" is closely related to the value of $\theta_2 = \theta_2 = c_{\theta 2}(1 - s_{\Pi})$.[17] (3-38)

Assumption 2 The higher the retention ratio the lower is the value of θ_2. When the coefficient of θ_2, $c_{\theta 2}$, becomes higher under a certain value of θ_2, this movement implies that corporate saving is used less effectively.

In short, it is most important for calibration to determine the values of *beta* and *delta*. The value of *beta* directly controls both θ_2 and γ, and each of θ_2 and γ influences the growth rate of output significantly. If we intend to improve the value of *beta*, θ_2, and γ, each of these parameters must improve.

4.3 Introduction of the government sector

Finally, it is possible for the generalized model to separately introduce the government sector. For simplicity and lack of data, I do not use this extension of the model for simulation and calibration. However, an outline of it follows:

The government receives taxes from both the household and corporate sectors. A part of these taxes is spent and the rest is invested in public capital. In this case, the government also uses investment for capital accumulation and improvement of technology.

$\varepsilon \equiv$ fraction of total tax revenues that is used up by "useless" government expenditures.

$(1 - \varepsilon) \equiv$ fraction of total tax revenues that is used to build up public capital stock and to improve technology.

$\tau \equiv$ average tax rate for undistributed profit, dividends, and wages.

$\mu \equiv$ fraction of "useful" government spending that is allocated in improving

17) This idea was suggested by Dr. Debasis Bandyopadhyay before introducing *beta* into my model. However, this assumption still holds even after introducing *beta* into my model. *Bata* is calibrated after fixing θ_2.

technology or the quality of capital (technology) and labour (e.g., education, healthcare, R & D, etc.).

$$I_K(t) = \gamma\theta_1 S_H(t) + \theta_2(1-\tau)S_\Pi(t) + \tau S_H(t)(1-\varepsilon)$$

$$I_K(t) = \gamma\theta_1 S_H(t) + \theta_2(1-\tau)S_\Pi(t) + \tau(1-\mu)(1-\varepsilon)(S_\Pi(t) + W_t + D_t) \quad (3\text{-}39)$$

$$I_A(t) = ((1-\gamma)\theta_1 S_H(t) + (1-\theta_2)(1-\tau)S_\Pi(t) + \tau\mu(1-\varepsilon)(S_\Pi(t) + W_t + D_t)) / k^\delta \quad (3\text{-}40)$$

Since $g_A(t) = \dfrac{I_A(t)}{A_t}$ and $S_\Pi(t) = s_\Pi A_t k_t^\alpha$,

$$\begin{aligned} g_A(t) &= (1-\gamma)\theta_1 s_H k_t^{\alpha-\delta} + (1-\theta_2)(1-\tau)s_\Pi k_t^{\alpha-\delta} + \tau\mu(1-\varepsilon)k_t^{\alpha-\delta} \\ &= ((1-\gamma)\theta_1 s_H + (1-\theta_2)(1-\tau)s_\Pi + \tau\mu(1-\varepsilon))k_t^{\alpha-\delta} \end{aligned} \quad (3\text{-}41)$$

Budget deficit, defined as the difference between government revenue and expenditures, is offset finally in domestic output.[18) However, budget deficit is directly connected with the rate of technological progress in the government sector. Assume that this rate is much lower than in the corporate sector. In this case, a budget deficit is justified since the rate of technological progress, as a whole, will be lower. In this respect, the introduction of the government sector is important to the growth rate of output of a country. Also, assume that the surplus of the nation, defined as the difference between exports and imports, is fixed. High or low financial leverage is justified when the rate of technological progress in the government (and household sectors) differs from that in the corporate sector. When the government sector is not introduced into

18) This is because the sum of the difference between saving and investment in the private sectors plus the difference between government revenue and expenditure is equal to the sum of the difference between domestic and external fund deficit/surplus. The fund deficit or surplus is defined as the difference between flows of financial assets and liabilities (in financial accounts). The difference between the flows of domestic financial assets and liabilities are wholly offset. Thus, the surplus of the nation (in national accounts) or the external fund deficit/surplus (in financial accounts) remains as a residual. Finally, capital transfers (net; receivable and payable) are added to saving. These relationships are important and tested using the calibration of my model.

the generalized model, financial leverage[19] is wholly offset in qualitative and quantitative investments.

5. Conclusions

In this note, I continued to improve/expand the generalized model as an endogenous growth model based on the Cobb-Douglas production function. For this, I succeeded in:

1. Dividing total net investment (which equals net saving) into qualitative and quantitative investments, using corporate and household savings and the three financial parameters. For this, the use of a new parameter, *beta*, is essential.
2. Introducing a new parameter, *delta* into my model. This parameter definitely determines the level of technology and the rate of technological progress. Also this parameter shifts qualitative investment as an amount to one as a coefficient.
3. The value of *delta* is defined as the elasticity of quality improvement with respect to effective labour units per capital (or the capital-labour ratio) (see

19) Regardless of whether I introduce financial leverage or not, Equations for $I_K(t)$ and $I_A(t)$ hold. Let me, for example, define three kinds of leverages in terms of the corporate and household sectors, F and H, each as a parameter: $L_{EV} \equiv \dfrac{S_H / Y}{S_\Pi / Y} = \dfrac{S_{HF} / Y + S_{HH} / Y}{S_\Pi / Y}$, $L_{EVF} \equiv \dfrac{S_{HF} / Y}{S_\Pi / Y}$, and $L_{EVH} \equiv \dfrac{S_{HH} / Y}{S_\Pi / Y}$, where $S_H = S_{HF} + S_{HH}$. I can express the relationship between public and private investments, using $\omega = \dfrac{S_{HF} / Y}{S_H / Y}$. $S_H / Y(1-\omega)$ is used for public investment and $S_H / Y \cdot \omega + S_\Pi / Y$ is used for private investment. In this case, private investment can take into consideration all three financial parameters. However, Equations for $I_K(t)$ and $I_A(t)$ still hold totally. Note that the values of various savings change over time while the ratio of each value to output remain unchanged over time. Also note that for leverage we can use household saving as one after deducting the banking costs, $(1-\theta_1)S_H$.

Assumption 1). How to determine the value of *delta* was unresolved until appropriately considered in calibration, where I need the two RMSEs that separate *delta* from *beta*.

4. Finding that under constant returns to capital (CRC) the above *delta* equals *alpha*, regardless of the values of *beta* and the three financial parameters (see Proposition 1). Note that growth rates are measured under CRC only.
5. Proving that the rate of technological progress is endogenously measured as: $g_A(\infty)=(1-\gamma)\theta_1 \cdot s_{SH/Y}+(1-\theta_2)\cdot s_{S\Pi/Y}=i_A$ (see Equation 3-24), where the growth rate of per capita output is expressed as $i_A/(1-\alpha)$ as an endogenous constant (compared with the Solow's [1956] exogenous constant: $g_y(\infty)=\dfrac{g_A(\infty)}{1-\alpha}=\dfrac{g_A}{1-\alpha}$).
6. Under the same levels of corporate and household saving ratios, $s_{S\Pi/Y}$ and $s_{SH/Y}$, the growth rate of per capita output under CRC increases, only if *delta* and *beta* (or θ_2 and γ) improve/decrease (see Proposition 2). Different levels of the rate of saving, and accordingly, $s_{S\Pi/Y}$ and $s_{SH/Y}$, also change the growth rate of per capita output under CRC. These results differ from conventional ones.
7. Finally, my model shows both the theoretical situation under constant returns to capital and the current/actual situation under decreasing returns to capital (DRC) or increasing returns to capital (IRC) using calibrated *beta* and *delta*. This is an important result (see Proposition 3).

Supplement 1: the relationship between saving and consumption

In the generalized model, I directly use the retention ratio, the household ratio, and the rate of saving. However, these ratios are simultaneously related to the ratio of consumed dividends to output and the consumed wages to output. This is because saving is not equal to profit. I stress here that optimal consumption has been discussed since Ramsey [1928] and Robinson [1966],[20] but not under an endogenous growth model as in my thesis. Therefore, I need to explain optimal consumption, using my saving and consumption structure. This is the purpose of this supplement.

For this, *first,* I discuss the relationship between saving and consumption in terms of profit and wages whose sum constitutes output/income. I need a concept of consumed dividends. Dividends are divided into two: consumed and saved dividends that can be replaced by saved wages. *Second,* I define optimal consumption as the consumption which maximizes the growth rate of output in the long-run. For this, I need a concept of time preference. I define the coefficient of time preference as consumed dividends to undistributed profit. Let me explain the details as follows:

First, the relationship between saving and consumption is classified into three cases:

20) (1) Assume a one-class model: saving in the household sector, S_H, is composed of saved wages and dividends: $S_H = S_W + S_D$. However, in any model, if $S = \Pi$ in the steady-state, $D = S_H$ and $C_H = W$ since $Y = S + C = (S\Pi + S_H) + W$, where dividends are all saved as an agent of saved wages (see Kamiryo [2000, p.19]). Robinson's [1966] expression is $\Pi = I + (1 - s\Pi)\Pi - s_W W$. If investment, I, is smaller than profit, Π, dividends are larger than saved wages. A part of dividends is consumed marginally and additionally under $S \neq \Pi$. Robinson states that wages equal consumption. This is true if $S = \Pi$, but total consumption (including marginal) does not equal wages if $S \neq \Pi$.

(2) My model is explained using the rate of saving, s, and the relative share of profit, α, each as a parameter. However, the difference between s and α constitutes consumed dividends. Consumed dividends are discussed separately in the Supplement for this note using the coefficient of time preference.

(3) In the model using $S_H = S_W + S_D$ (as shown traditionally), household saving, S_H, is always divided equally between saved wages, S_W, and saved dividends, S_D. However, if either S_W or S_D is equal to zero, the other will become equivalent to the full amount of household savings, S_H. This is explained using Figure 3. The basic idea comes from Kamiryo [2000].

Case 1. Saving is smaller than profit: $s < alpha$.

Case 2. Saving is equal to profit: $s = alpha$.

Case 3. Saving is larger than profit: $s > alpha$.

In Case 2, as a basis, wages are all consumed and dividends and undistributed profit are all saved. Therefore, the differences between saving and alpha depend on whether a part of dividends are consumed or a part of wages are saved. In other words,

Case 1. When a part of dividends are consumed, consumption is larger than wages and, as a result, saving is smaller than profit: $s < alpha$. The difference between saving and alpha is consumed dividends: *s-alpha* = consumed dividends.

Case 2. When dividends are all saved, consumption is equal to wages and, as a result, saving is equal to profit: $s = alpha$. The difference between saving and *alpha* is zero: $s\text{-}alpha = 0$.

Case 3. When saving is more than the sum of undistributed profit and dividends, consumption is less than wages and, as a result, saving is larger than profit: $s > alpha$. The difference between saving and *alpha* is saved wages: *s-alpha* = saved wages.

Below is an illustration of the relationship between saved wages and saved dividends, using **Figure 3**. This relationship holds, regardless of whether the rate of saving is equal to the relative share of profit, *alpha*, or not.

In Figure 3, wages are equal to average consumption, and marginal consumption is equal to consumed dividends. The proportion of dividends that are saved is denoted as S_D and makes up the whole of saving less the sum of undistributed profit and dividends. Assume that saved dividends are zero, then household saves the same amount and total consumption remains unchanged.

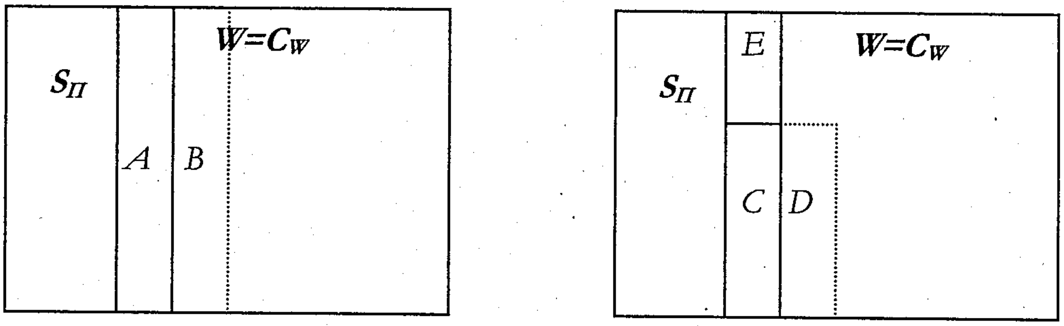
This relationship between consumption and saving is expressed using symbols and equations as follows:

The relative share of profit, $\alpha \equiv \Pi/Y$. (S3-1)

Figure 3 Why are saved wages always replaced by saved dividends?

If $S = \Pi$, then

If $S < \Pi$, where E is consumed dividends, then



Where, A is dividends.
If A is consumed, B shows saving.
If B is saved, A shows consumption.
Thus, $A = B$ holds when saving equals profit.

Where, C + E is dividends.
If C is consumed, D shows saving.
If D is saved, C shows consumption.
Thus, $C = D$ holds when saving is less than profit.

The retention ratio, $s_\Pi \equiv \dfrac{S_\Pi}{P}$,

$$s_{S\Pi/Y} \equiv \frac{S_\Pi}{\Pi} \cdot \frac{\Pi}{Y} = s_\Pi \cdot \alpha \qquad \text{(S3-2)}$$

The household saving ratio, $s_H \equiv \dfrac{S_H}{W+D}$,

$$s_{SH/Y} \equiv \frac{S_H}{W+D} \cdot \frac{W+D}{Y} = s_H(1 - s_\Pi \cdot \alpha). \qquad \text{(S3-3)}$$

$$\text{The rate of saving, } s \equiv \frac{S}{Y} = s_{S\Pi/Y} + s_{SH/Y} \qquad \text{(S3-4)}$$

In Cases 1 and 2, $S_H \leq D$ and $C_H \geq W$, where consumed dividends are added to wages that are all consumed. In Case 3, $S_H > D$ and $C_H < W$, where saving is the sum of undistributed profit, saved dividends and saved wages.

Second, let me discuss optimal consumption and its related equations. I use the coefficient of time preference as consumed dividends divided by undistributed profit. Why do we need this definition instead of consumed dividends or the ratio of consumed dividends to output? Assume that the larger the consumed dividends are the higher the growth rate of output in the long-run is, as examined in my simulation in this note. In this case, consumed dividends are not stable since if the retention ratio is higher, then consumed dividends are smaller. Thus, consumed dividends or the ratio of consumed dividends to output is not an appropriate criterion and I must establish a new definition of time preference as the coefficient of time preference.

Now, I repeat here the result of my simulation in this note where I found the relation between the rate of saving, alpha, and the growth rate of output in the long-run.

Supplement 2: Introduction of time preference into my model

This supplement discusses how to introduce time preference into the generalized model. My model sets the rate of saving as a parameter so that the growth rate of output equals the growth rate of consumption. However, growth models usually introduce time preference into their models. In this respect, this supplement clarifies how to measure time preference in my model.

In this note, I have defined the rate of saving; as the sum of the ratio of undistributed profit to output and the ratio of household saving to output, where the three financial parameters are involved. I assume that $Y = S + C$, where S is saving and C is consumption. Then S or C are shown as $S = Y - C$ or $C = Y - S$. Thus, the rate of saving is also explained using the contents of consumption such as time preference and consumed dividends.

The literature I reviewed separately almost always introduced time preference into each model. I relate time preference to the retention ratio. The growth rate of output in the long run is equal to the growth rate of consumption assuming that the relative share of labour is fixed. Time preference between present and future consumption is measured (as the coefficient of time preference) using the same undistributed profit and household saving. Furthermore, the question of how to obtain IRC from a situation under DRC is more important and answered by improving the three financial parameters and financial leverage. These ideas are consistent with the concept of the coefficient of time preference and a level of maximized consumption.

How can I measure the coefficient of time preference in the generalized model? I have done this by connecting it with the contents of saving and without using a utility function. When this coefficient is zero, time preference is in equilibrium, where the rate of saving, s, equals the relative share of profit, α. How can I measure time preference? For this, I need to develop a new method of measuring time preference in the generalized model. This method differs from Kamiryo's [2000, pp. 50–77], as that model is not based on the Cobb-Douglas production function; however, the definition of the coefficient of time preference is the same as Kamiryo's [ibid.]: the coefficient of time preference is the ratio of consumed dividends to undistributed profit.

Solow [1992, pp. 15–16] suggests that under the situation of $s \neq \alpha$ (under the modified golden rule) there is a range of maximized consumption for changes in the rate of saving, however, he does not present his empirical work. I interpret his statement to mean that maximized consumption holds if the decrease in consumed dividends equals the increase in output/consumption. However, I found the growth rate of output does not increase with an increase in the rate of saving, particularly when the situation is under DRC. Assume that the situation is under IRC, then the growth rate of output increases for any increase in the rate of saving. It is then possible for maximized consumption to remain unchanged.[21] Again, whether this maximized situation holds or not depends principally on the values of the three financial parameters and financial leverage.

21) Even in this case, I need to set an appropriate range between $s < \alpha$ and $s > \alpha$. For this I can use the coincident value of the relative share of profit, α^* (see Section 4.3.3 in Kamiryo [2001]). Recall that if the value of α/α^* equals 1.0, then the situation is at $s = \alpha$. For example, I set the appropriate range of maximized consumption between 0.8 and 1.2, using the value of α/α^*.

Now let me describe how to measure the coefficient of time preference. This is defined as the ratio of consumed dividends to output divided by the ratio of undistributed profit to output. In this case, the coefficient remains a parameter. First, what is implied by this coefficient? Time preference between present and future consumption is expressed using this coefficient. The numerator shows present consumption and the denominator shows future savings. When this value is high the situation is consumption-oriented while when this value is low the situation is saving-oriented.

Let me formulate related equations with their definitions.[22)]

1. The consumed dividends to output is measured as $\chi_Y \equiv C_D / Y$, (S3-5)
since the ratio of undistributed profit to output is the product of the relative share of profit and the retention ratio, $s_{S\Pi/Y} \equiv S_\Pi / Y$.

2. The coefficient of time preference is defined as: $\chi_{S\Pi} \equiv \dfrac{C_D / Y}{S_\Pi / Y}$. Accordingly, $\chi_{S\Pi} = \dfrac{\chi_Y}{\alpha \cdot s_\Pi}$ or $\chi_Y = \chi_{S\Pi} \cdot \alpha \cdot s_\Pi$, where $\chi_Y = \alpha\text{-}s$. (S3-6)

3. The ratio of consumed dividends to dividends is derived using the ratio of consumed dividends to output: $\chi_D \equiv C_D / D$ and $\chi_D = \dfrac{\chi_Y}{\alpha(1-s_\Pi)}$. (S3-7)

4. When the value of χ_D is 1.0, where $C_D = D$, $\chi_Y = \alpha(1-s_\Pi) = \chi_{S\Pi} \cdot \alpha \cdot s_\Pi$, and
$$\chi_{S\Pi} = \frac{\alpha(1-s_\Pi)}{\alpha \cdot s_\Pi} = \frac{1-s_\Pi}{s_\Pi}. \quad \text{(S3-8)}$$
In the case of $C_D = D$, if the retention ratio is 0.5, the coefficient of time preference is 1.0.

5. Under $s < \alpha$, when the decrease in consumed dividends (along with the

22) Under the same definition of the coefficient of time preference, $\chi_{S\Pi}$, the generalized model measures $\chi_{S\Pi} \equiv (C_D / Y)/(S_\Pi / Y) = \chi_{CD/Y} / \alpha \cdot s_\Pi$. The Kamiryo model [Growth Accounting, 2000], without using the Cobb-Douglas production function, measured this coefficient, $\chi_{S\Pi}$, as $\chi_{S\Pi}(t) = (1 - s_{S\Pi}(t) \cdot \Omega(t)) / s_{S\Pi}(t)$ by using the equation, $s_\Pi(t) = 1/(\Omega(t) + \chi_{S\Pi}(t))$, where the retention ratio, s_Π, is set as a function of time.

increase in the rate of saving) is offset by the increase in output (along with the increase in the rate of saving), the following equation holds, where consumption equals wages ($C = W$), and remains unchanged:.

$\alpha - s = g_{Y(s=\alpha)} - g_{Y(s<\alpha)}$, where $a > s$ and $g_{Y(s=\alpha)} > g_{Y(s<\alpha)}$. (S3-9)

Under $s > \alpha$, when the increase in negative consumed dividends (along with the increase in the rate of saving) is offset by the increase in output (along with the increase in the rate of saving), consumption equals wages ($C = W$) and remains unchanged. This case is limited to the situation under IRC. An appropriate range of $C = W$ (that corresponds with maximized consumption in the modified golden rule) can be estimated using the coincident value of the relative share of profit: e.g., when the value of α / α^* is between 0.7 to 1.3.

6. The value of the product of the relative share of profit and the retention ratio is exactly the rate of technological progress in the Solow case of the generalized model (see Table 4-5), where $\theta_1 = 1$, $\theta_2 = 1$, and $\gamma = 1$ and $g_A(t) = \alpha \cdot s_{\Pi}$. In this case, the coefficient of time preference is directly related to the rate of technological progress: $\chi_{s\Pi} = (\alpha - s) / g_A$. (S3-10)

Time preference can be expressed more straightforwardly as the ratio of consumed dividends to dividends. Its value is highest in the USA (0.7 to 1.4), lowest in Japan (0.3 to 0.6), and moderate in the UK (0.5 to 1.0). When this value is too high, a situation of over-consumption occurs, and when it is too low, a situation of over-saving occurs. In the case of the UK, the rate of saving is low, but the ratio of consumed dividends to dividends is also low so that the UK enjoys the highest growth rate of output. The observation about the above values by country do not contradict the overall interpretations of economic growth.

Finally, let me compare the coefficient of time preference with financial leverage. The denominator of both the coefficient and financial leverage is the ratio of undistributed profit. The difference in the coefficient and financial leverage comes from each numerator: the ratio of consumed dividends to output or the ratio of household saving to output. Then, the difference between consumed dividends and household saving determines the relationship between the coefficient of time preference and financial leverage assuming that the ratio of undistributed profit to output, $\alpha \cdot s_{\Pi}$, remains unchanged. In this respect, Equation S3-11 is derived using $\chi_Y = \alpha - s$.

$$\chi_{s\Pi} = \frac{\alpha - s}{\alpha \cdot s_{\Pi}}. \quad \text{(S3-11)}$$

From Equation S3-11, the following statements are valid with respect to other relationships among parameters.

1. The higher the ratio of consumed dividends to output, the higher the value of the coefficient of time preference. As a result, the ratio of household saving to output, $s_{SH/Y}$, or the household saving ratio, s_H, will be lower.
2. The higher the retention ratio the higher the value of the coefficient of time preference. As a result, the ratio of household saving to output, $s_{SH/Y}$, or the household saving ratio, s_H, will be lower.
3. Thus, the coefficient of time preference and financial leverage change conversely. The higher the value of the coefficient the higher the financial leverage.[23)]

The value of coefficient of time preference in the US and UK is relatively high while for the Japanese economy it shows a negative value. This corresponds with the comparisons of financial leverage.

Finally, let me explain **Figure 4**. This figure indicates the relationship between the rate of saving and the coefficient of time preference. The higher

23) $\alpha - s = \chi_Y = \alpha - s_{S\Pi/Y} - s_{SH/Y}$. Assume that the relative share of profit, α, and the ratio of undistributed profit to output, $s_{S\Pi/Y}$, are fixed. Then, the higher the ratio of consumed dividends to output, $\alpha - s$, (and, accordingly, the coefficient of time preference, $\chi_{S\Pi}$), the lower the ratio of household saving to output, $s_{SH/Y}$, (and, accordingly, financial leverage) .

Figure 4 Time preference, $\chi_S\Pi$, and the rate of saving: $\chi_{S\Pi} = C_D/S_\Pi = (\alpha - s)/s_{S\Pi/Y}$

$C_D/Y=\alpha\text{-}s$ $\quad \Pi=S_\Pi+D \quad S=S_\Pi+S_H \quad \Pi\text{-}S=D\text{-}S_H=C_D$

alpha	0.08				
s	**0**	**0.05**	**0.1**	**0.15**	**0.2**
s_Π	**0.3**	**0.3**	**0.3**	**0.3**	**0.3**
$s_{SH/Y}$	**-0.024**	**0.026**	**0.076**	**0.126**	**0.176**
$s_{S\Pi/Y}$	**0.0240**	**0.0240**	**0.0240**	**0.0240**	**0.0240**
$\chi_{S\Pi}$	**3.33**	**1.25**	**-0.83**	**-2.92**	**-5.00**

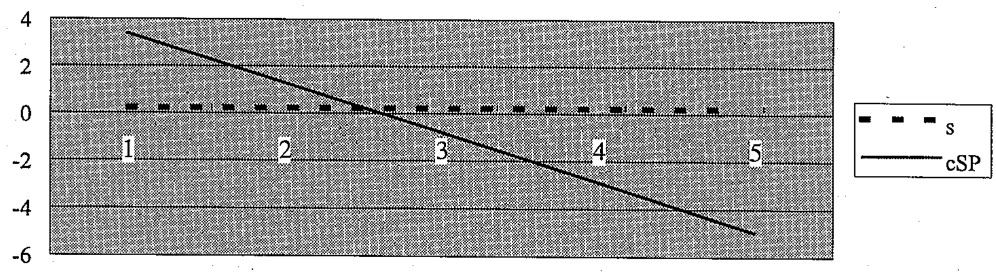
The lower s_Π the higher $\chi_{S\Pi}$ under $s<alpha$.

s	**0**	**0.05**	**0.1**	**0.15**	**0.2**
s_Π	**0.45**	**0.45**	**0.45**	**0.45**	**0.45**
$s_{SH/Y}$	**-0.036**	**0.014**	**0.064**	**0.114**	**0.164**
$s_{S\Pi/Y}$	**0.0360**	**0.0360**	**0.0360**	**0.0360**	**0.0360**
$\chi_{S\Pi}$	**2.22**	**0.83**	**-0.56**	**-1.94**	**-3.33**

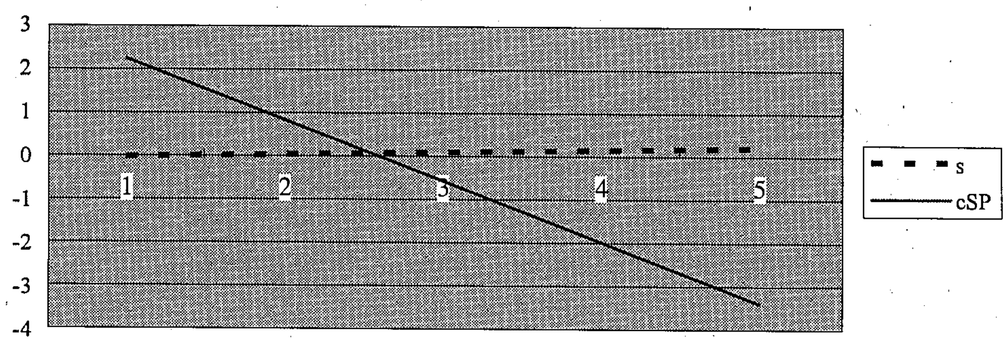
s	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
s_Π	**0.6**	**0.6**	**0.6**	**0.6**	**0.6**
$s_{SH/Y}$	**0.052**	**0.052**	**0.052**	**0.052**	**0.052**
$s_{S\Pi/Y}$	**0.0480**	**0.0480**	**0.0480**	**0.0480**	**0.0480**
$\chi_{S\Pi}$	**-0.42**	**-0.42**	**-0.42**	**-0.42**	**-0.42**

The higher the *alpha* the lower the $\chi_{S\Pi}$ under $s<alpha$.

Coeff. of time preference (Y axis) and the rate of saving (X axis): s_Π=0.3



Coeff. of time preference (Y axis) and the rate of saving (X axis): s_Π=0.45



the rate of saving the lower linearly the coefficient of time preference. At $s = \alpha$, this coefficient is zero. From the viewpoint of consumption, the rate of saving should be lower than *alpha* or the coefficient of time preference should be positive. For empirical values of the coefficient of time preference by case and country are shown in Appendixes C and E (omitted here).

In short, time preference works with saving and consumption. The coefficient of time preference is consistent with financial leverage. The propositions can be supplemented using time preference. Under $s < \alpha$, the situation is more saving-oriented, where investment is more important than consumption, as in the US. Under $s > \alpha$, the situation is more consumption-oriented, where the value of the coefficient of time preference must be watched carefully, as in the Japanese case.

Supplement 3: Growth rates, the capital-output ratio, and speeds of convergence in Japan 1990s

This appendix shows an interpretation of the Japanese economy in the 1990s. In the 1990s the Japanese economy lost the economic growth it had maintained in the 1980s. I found the following facts (see figures):

1. When an economy is robust, its condition shows powerful increasing returns to capital (IRC). For example, in the Chinese economy, its condition has been under a significant IRC since 1993. The level of robustness is shown by the value of *delta*: if *delta* is less than *alpha*, it is under IRC and if *delta* is more than *alpha*, it is under decreasing returns to capital (DRC). In the 1990s the Japanese economy was under IRC for the years 1992, 1993, 1995, 1998, and was under IRC for the years 1994, 1996, 1999, and 2000. However, the condition of IRC was weak compared with other countries since the value of *delta* is close to *alpha* (except for that in 1996).
2. The current condition, whether under DRC or IRC, is converted to a condi-

tion of constant returns to capital (CRC) by replacing *delta* with *alpha*, when we measure the growth rate of output/capital and the rate of technological progress. In this case, the lower the value of *delta* the lower the capital-output ratio. Also, the lower the value of *delta* the faster the speed of convergence: in 1996 convergence time, t_{conv}, is less than 300 while in 2000 t_{conv} is 800. When the condition is under a significant DRC, convergence does not occur even when $t = 1000$.

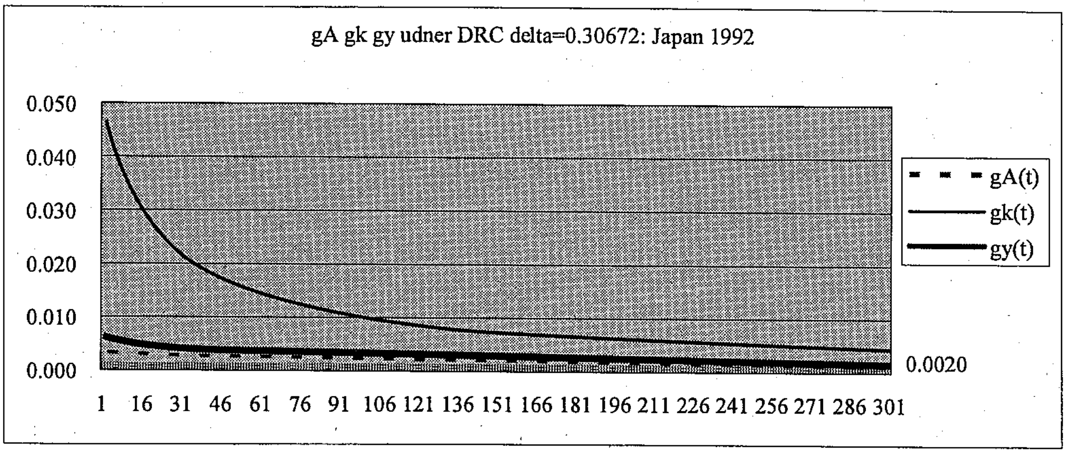
3. The rate of technological progress is very low compared with other countries. In particular, the values of *beta* and γ are usually much higher than other countries. This implies that the level of structural reform is significantly retarded. The difference between *beta* and γ comes not only from the lower rates of saving after 1998, but also from a higher *delta*. How can we improve *beta* and *delta*? We need to more aggressively shift investment from quantity to quality (R & D and eco-oriented) together with improvements in regularization and structural reform.
4. The corporate sector in the latter half of the 1990s has lost efficient management. This implies that corporate saving/investment is not used effectively, but the situation is even worse as far as investment in the public sector. The public sector needs to understand how significantly the Japanese economy has lost growing power owing to the often wasteful use of people's savings in the public sector.

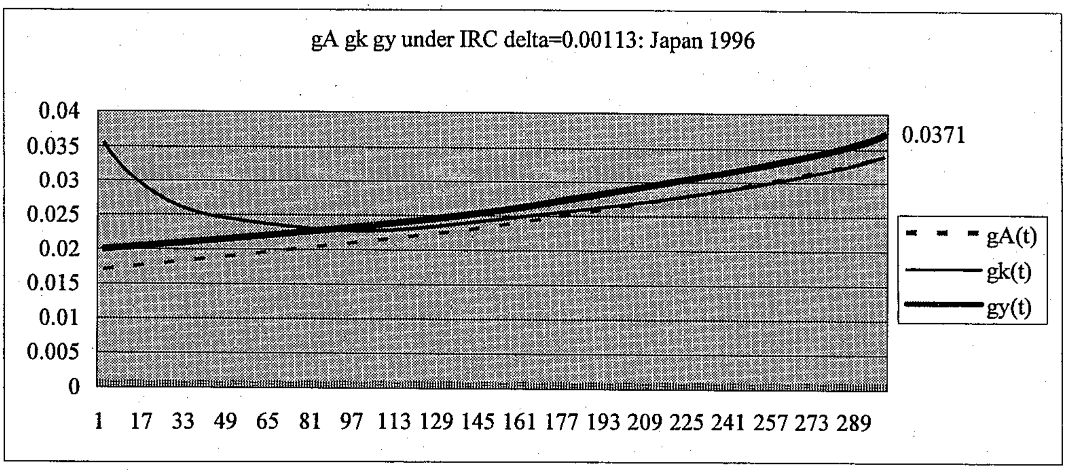
Appendix 1 relationships among parameters and variables in Japan 1991–2000

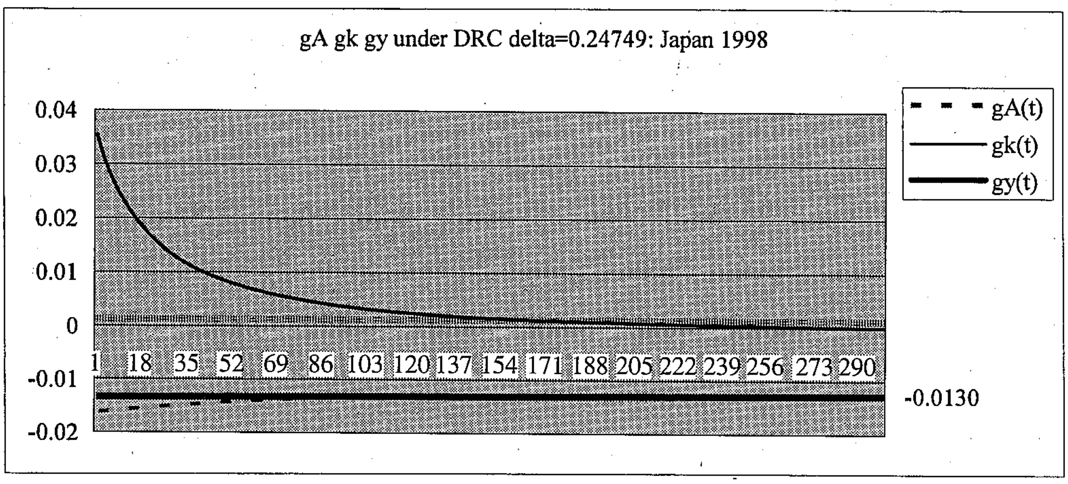
	1992	1993	1994	1995	1996	1997	1998	1999	2000
δ CRC$\delta=\alpha$	0.05620	0.03770	0.05962	0.06710	0.09042	0.08966	0.08244	0.10094	0.10416
δ RMSE=0	0.30672	1.00000	(0.00647)	0.00723	0.00113	0.05141	0.24749	0.02324	0.06369
$g_Y(300)$	0.01001	0.00423	0.04282	0.02491	0.04527	0.02072	(0.02275)	0.00701	0.00790
$g_Y(actual)$	0.01181	0.00372	0.04215	0.02469	0.04469	0.02175	(0.02234)	0.00675	0.00803
$g_{YCRC}(\delta=\alpha)$	0.01624	0.00638	0.01982	0.01480	0.02135	0.01706	(0.03567)	0.00155	0.00625
$g_{yCRC}(\delta=\alpha)$	0.00821	0.00228	0.01940	0.01390	0.01494	0.01065	(0.02589)	0.00753	0.00552
alpha	0.05620	0.03770	0.05962	0.06710	0.09042	0.08967	0.08244	0.10094	0.09894
beta	0.95913	0.98635	0.87659	0.90738	0.91161	0.93067	1.22540	0.93798	0.95494
$beta_{CRC}$	0.7938	0.7939	0.7986	0.8076	0.8113	0.8055	0.8115	0.8088	0.8163
gamma	1.01832	1.02308	0.94259	1.01359	1.1093	1.1720	1.87438	1.66997	1.6407
$gamma_{CRC}$	0.8152	0.8060	0.8354	0.8627	0.9153	0.9159	0.9492	1.1435	1.1291
i_K	0.18185	0.15764	0.12955	0.12699	0.13462	0.13009	0.14075	0.10228	0.10465
i_A	0.00775	0.00218	0.01824	0.01296	0.01305	0.00969	(0.02589)	0.00676	0.00494
$i=i_K+i_A$	0.18960	0.15983	0.14778	0.13996	0.14767	0.13978	0.11486	0.10904	0.10958
i_{KCRC}	0.1505	0.1269	0.1180	0.1130	0.1198	0.1126	0.0932	0.0882	0.0895
i_{ACRC}	0.0391	0.0329	0.0298	0.0269	0.0279	0.0272	0.0217	0.0208	0.0201
leverage	4.3773	7.7961	2.6757	1.9525	1.0704	0.9559	0.8096	0.3251	0.3718
saving rate	0.2282	0.1952	0.1747	0.1631	0.1668	0.1569	0.1277	0.1157	0.1170
s_Π	0.6274	0.4819	0.6744	0.7064	0.7888	0.7971	0.7699	0.8153	0.7669
$c_{\theta 2(CRC)}$	1.8789	1.3512	2.1499	2.3844	3.3150	3.4492	3.0426	3.7891	3.0032
$c_{\gamma(CRC)}$	4.0759	4.0299	4.1770	4.3133	4.5767	4.5793	4.7458	5.7173	5.6455
$\eta\ \delta/\beta$	4.74	4.93	114.71	(75.26)	(717.59)	(5.53)	1.97	(24.28)	(4.38)
$\eta\ \gamma/\beta$	1.16	1.09	1.28	1.35	1.59	1.62	1.46	2.29	2.15
$\eta\ g_Y/\delta$	(3.91)	(9.67)	0.02	0.02	(0.00)	0.90	2.43	0.46	2.98
$\eta\ g_Y/\beta$	(18.51)	(47.72)	2.68	(1.88)	1.18	(4.98)	4.79	(11.28)	(13.04)

Note: Set $\theta_1 = 0.8$ and $\theta_2 = 0.7$ while γ is derived using *beta*. *Beta* is calibrated at *delta* = 0.
Also, *delta* under CRC is calibrated sy setting $r(0) = r_{CRC}$.
$c_{\theta 2(XPX)} = \theta_2/(1 - s_\Pi)$ and $c_{\gamma(\mathrm{CRC})} = \gamma_{\mathrm{CRC}}/(1-\theta_1)$.

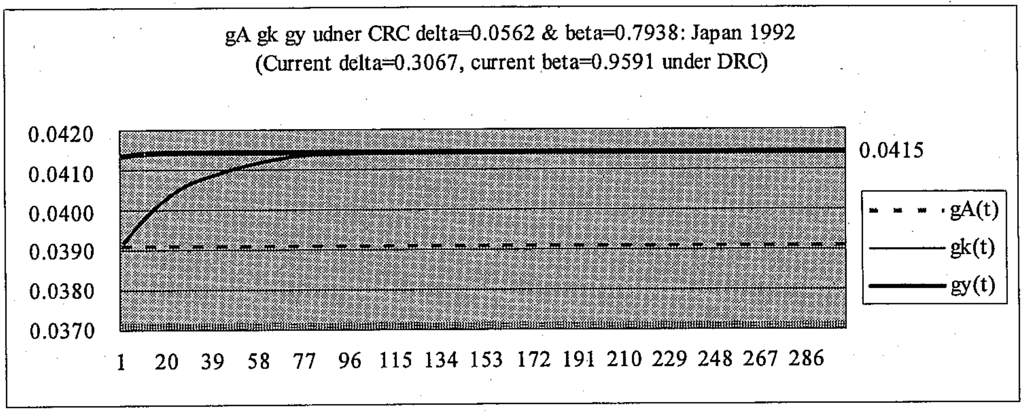
Appendix 2 Growth rates and the capital-output ratio under current DRC or IRC: Japan 1992, 1996, 1998, and 2000 (omitted: similar to 1992)

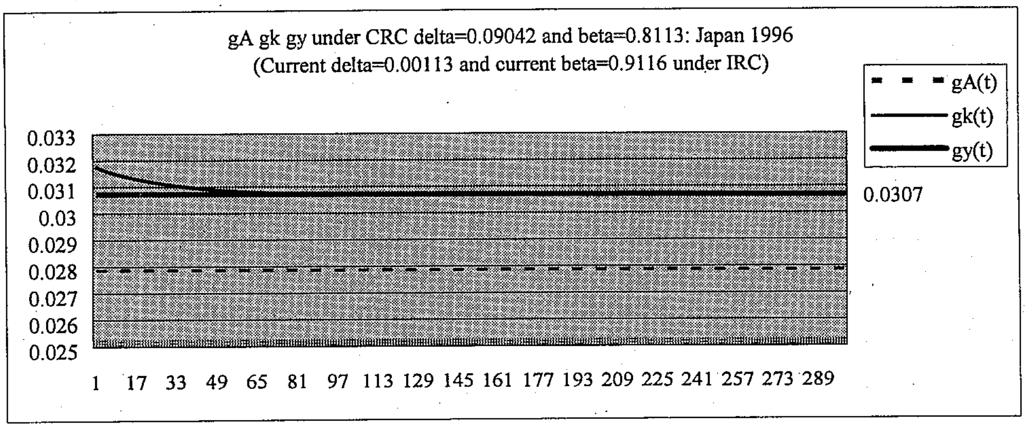


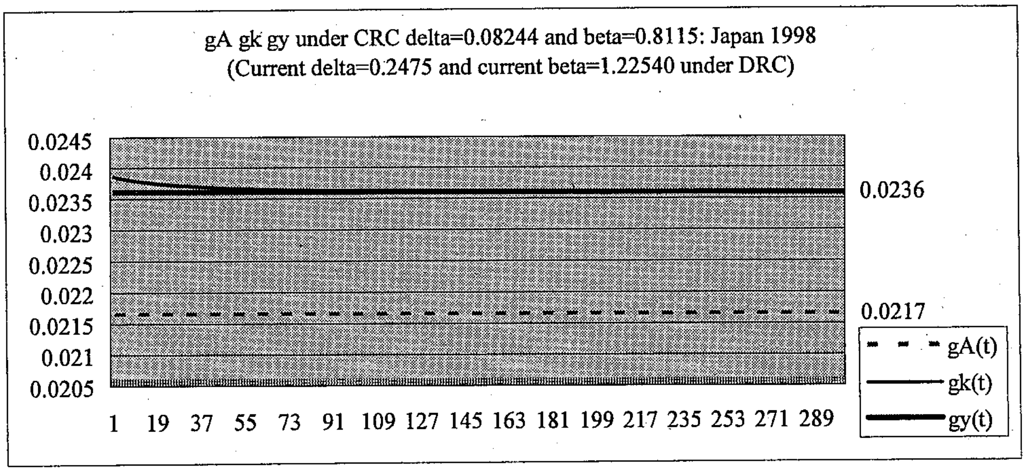




Appendix 3 Growth rates and the capital-output ratio under CRC: Japan 1992, 1996, 1998, and 2000 (omitted: similar to 1992)







Note: The lower the *delta*, *beta*, and, accordingly, *gamma*, the higher g_A and g_y.