

«Note»

Empirics of A Function of Consumption Consistent with A Technology-Golden Rule: Using Three Dimensional Graphs

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1. Introduction

This paper, first, intends to express the relationship between saving and consumption by connecting three kinds of ratios in saving with a utility function designed from the idea of Ramsey [1928] and second, to connect saving and consumption with wages and returns/rental by using the utility function of consumption. In the current statistics of national accounts, compensation of employees (hereafter wages) is included in GDP while final consumption is included in national disposable income. I indicate that there is no way to connect wages with consumption as far as I investigated.

Why do I need to search for an integrated relationship between saving or consumption and wages or rental? First, I am curious about confirming the relationship between the structure of saving (for the retention ratio, the ratio of saved dividends to dividends, and the saved wages to wages) and the utility function of consumption that uses a discount rate for consumers, where saving and consumption are dual to national disposable income, NDI. Second, I need new data for wages and returns/rental in my endogenous growth model, whose base is still “production” that guarantees an optimum/maximum rate of rental. I need new

1) I am much obliged to the advice of Dr. Toshimi Fujimoto for the understanding of the utility function using the calculus of variations. I am also thankful to Dr. Itsuo Sakuma (on 11 June 2005) for the discussion about the character of wages and rental in this paper.

definitions for wages and rental so that wages and rental in production are compatible with optimum consumption in income. Assume that NDI happens to be equal to GDP, and still NDI is for income and GDP is for production.

My previous model [PhD, 2003] is based on corporate profit. The model uses output as the sum of corporate profit (calculated as the sum of corporate saving and dividends) and total compensation or wages shown in GDP, where corporate profit is much less than the operating surplus as a residual of GDP. My current model (after 2004) researches optimum each for consumption and production.

In this paper, I started with the OECD National Accounts Statistics data that disclose both wages and consumption (see Appendix) to establish my own method to connect consumption with compensation. Then, I totally clarified the essence of my method by using Table 5-1. This table presents a basis for the ratio of a discount rate, ρ , to the optimum rate of rental under convergence, r , (ρ/r), by the level of a technology-golden rule/age, where the rate of saving, s , equals the relative share of rental, α : $s=\alpha$. Any country has its own level of $s=\alpha$, where $\rho=r$ holds. For empirical work, I use IMF data (available in IFSY and GFSY) and apply Table 5-1 to the data.

How can consumption be converted to wages or how can wages to consumption? Theoretically, if we assume that GDP is equal to NDI, we can directly use both consumption and wages in statistics. When I compare wages shown in GDP with estimated wages in production using my method, I use the word, “estimated” still based on production. For estimating wages, I need the help of the literature, in particular the utility function of consumption originated in Frank Plumpton Ramsey [1928]. In this case, first I need to clarify the contents of saving as a saving structure and then the dual relationship between saving and consumption in NDI, which should be consistent with the saving structure. Jan Tinbergen [1956] reviewed Ramsey and stated that the utility function should be

measurable more easily (without an assumption of “bliss” as maximum utility). Tinbergen uses the marginal capital-output ratio instead of Ramsey’s direct relationship between consumption and wages each as flow. I find that both methods reach the same result. Note that Robert Solow [1992] (whose original exogenous growth model [1956] is based on a fixed rate of saving) only suggested a short version of using a discount rate and a taste parameter.

It is a main purpose of this paper to establish my method for estimating wages (modified wages; hereafter wages) and confirm its usefulness by showing empirical results using IMF data by country. In this case, I find, if I introduce technology that uses β^* into a golden rule and accordingly, the utility function of consumption, the empirical results are well-fitted. I need to review the golden rule in the literature, using variables under convergence. By extending equations in Kamiryō [2004a], I will prove that the golden rule in the literature assumes no technology. I will also prove that the rate of technological progress is deeply involved in a new golden rule and accordingly, the utility function of consumption. Kamiryō [2005c] will separately extend equations under convergence in more detail.

In Appendix, I attach tables and figures for the characteristics between consumption and wages using three dimensional graphs. Also I attach results of (ρ/ρ), comparing consumption with wages in OECD data by country.

2. The saving structure and the utility function

2.1 Relationship among saving ratios

In this section, I clarify that a compatible relationship between consumption in income and “estimated” wages in production, based on the saving structure (that is dual to the consumption structure) and the relative share of rental (that is dual to the ratio of wages to output). There is no literature to clarify the relationship between wages in GDP and “estimated” wages that satisfies optimum consump-

tion, as long as I have investigated.

I will start with the saving structure. I assume that dividends paid by corporations belong to household consumption and saving (since corporations do not consume), and saving is divided into corporate saving (before paying dividends) and household savings. An equal condition lying between the rate of saving and the relative share of rental, $s=\alpha$, presents the clue for setting up a saving structure. The condition of $s=\alpha$ is well known as the golden rule, where the rate of rental will be equal to the growth rate of output under convergence but without technology. Later I will discuss under what condition the golden rule holds in an endogenous growth.

In the saving structure, I distinguish three kinds of savings and each corresponding ratio in saving: (1) corporate saving and the retention ratio of corporations, s_{IT} , (or the payout ratio), (2) saved dividends and the ratio of saved dividends to dividends, $s_{SD/D}$, and (3) saved wages and the ratio of saved wages to wages, s_W , and/or, household saving as the sum of saved wages and saved dividends by households and the ratio of household saving to the sum of wages and dividends, s_H , where I simply assume, as I mentioned above, that dividends are saved by households, neglecting the dividends received and saved by corporations since it is difficult to estimate the ratio of corporate-saved dividends to dividends decided by corporations. I will show, first of all, each saving as a ratio to national disposable income, NDI (hereafter, output), so that each saving ratio is connected with the relative share of rental estimated in NDI. Thus, I need to clarify the relationships between each of the above three savings and each ratio to output. Using symbols, the saving structure is shown in comparison with consumption as follows:

Output $Y=NDI$, estimated returns/rental Π , estimated wages/compensation W : $Y= \Pi+W$ (hereafter, I abbreviate “estimated”).

Corporate saving S_{IT} , saved dividends S_D , saved wages S_W , total saving S : $S=$

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$S_{\Pi} + S_D + S_W$. Consumed dividends C_D , consumed wages C_W , total consumption C : $C = C_D + C_W$. And, $Y = S + C$, where $\Pi = S_{\Pi} + D$, $D = S_D + C_D$, $W = S_W + C_W$, and $Y = S + C = \Pi + W$.

I will show each saving or consumption divided by output, using small character:

$1 = s + c$, where $s = s_{SP/Y} + s_{SD/Y} + s_{SW/Y}$, $c = c_{CD/Y} + c_{CW/Y}$, $\alpha = \alpha_{S\Pi} + \alpha_D$, $d_{D/Y} = s_{SD/Y} + c_{CD/Y}$, $w_{W/Y} = s_{SW/Y} + c_{CW/Y}$, and $y = s + c = \alpha + w_{W/Y}$.²⁾

As the results, the retention ratio of corporations $S_{\Pi}/\Pi = s_{S\Pi/\Pi} = s_{\Pi}$, the ratio of saved dividends to dividends $S_D/D = s_{SD/Y}/d_{D/Y}$, and the ratio of household saving to the sum of wages and dividends $S_H/(W+D) = (s_{SW/Y} + s_{SD/Y})/(w_{W/Y} + d_{D/Y}) \equiv s_H$. The literature often uses s_{Π} and s_H , but without clarifying saved and consumed dividends or the relationship between the ratio of consumed dividends to output and the ratio of saved wages to output (for a version towards these relationships, see Robinson [1962]). I will show the equations before introducing a utility function and the equations after introducing a utility function.

Before introducing a utility function:

$$\alpha_{S\Pi} = \frac{S_{\Pi}}{Y} = \frac{S_{\Pi}}{\Pi} \cdot \frac{\Pi}{Y} = \alpha \cdot s_{\Pi} = s_{S\Pi/Y}. \quad (1)$$

$$\alpha_{SD} = \frac{S_D}{Y} = \frac{S_D}{D} \cdot \frac{D}{Y} = \alpha(1 - s_{\Pi})s_{SD/D} = s_{SD/Y}. \quad (2)$$

$$\alpha_{CD} = \frac{C_D}{Y} = \frac{\Pi - S_{\Pi} - S_D}{Y} = \alpha - s_{S\Pi/Y} - s_{SD/Y} = \alpha(1 - s_{\Pi})(1 - s_{SD/D}), \quad (3)$$

where $\alpha_D = \alpha_{SD} + \alpha_{CD} = \alpha - s_{S\Pi/Y}$.

$$s_{SW/Y} = \frac{S_W}{Y} = \frac{S_W}{W} \cdot \frac{W}{Y} = s_W(1 - \alpha), \text{ where } s_W = S_W/W. \quad (4)$$

$$s_{SH/Y} \equiv \frac{S_H}{Y} = \frac{S_W}{Y} + \frac{S_D}{Y} = s_W(1 - \alpha) + s_{SD/Y} \text{ or,} \quad (5)$$

2) In symbols, I distinguish the wage rate, w , with the ratio of wages to output, $w_{W/Y}$, where w is used for estimating the capital-labor ratio, using r/w .

$$s_{SH/Y} \equiv \frac{S_H}{Y} = \frac{S_W + S_D}{W + D} \cdot \frac{W + D}{Y} = s_H(1 - s_{\Pi/Y}), \text{ where } s_H = S_H/(W + D).$$

Therefore, paying attention to the dual relationship between saving and consumption,

$$\alpha = s_{\Pi/Y} + \alpha_{SD} + \alpha_{CD} = s_{\Pi/Y} + (\alpha - s_{\Pi/Y}), \text{ where } \alpha_D = \alpha_{SD} + \alpha_{CD} = \alpha(1 - s_{\Pi}). \quad (6)$$

$$s = s_{\Pi/Y} + s_{SD/Y} + s_{SW/Y} = s_{\Pi/Y} + s_{SH/Y}. \quad (7)$$

$$c = c_{CD/Y} + c_{CW/Y} = 1 - (s_{\Pi/Y} + s_{SD/Y} + s_{SW/Y}). \quad (8)$$

Thus, $s - \alpha = s_{SW/Y} - c_{CD/Y}$, or $s - \alpha = s_W(1 - \alpha) - ((1 - s_{\Pi})(1 - s_{SD/D})\alpha)$,³⁾

$$\text{where } s_{SW/Y} = s_W(1 - \alpha) \text{ and } c_{CD/Y} = \alpha(1 - s_{\Pi})(1 - s_{SD/D}). \quad (9)$$

$$\frac{s}{\alpha} = \frac{s_W}{\alpha} + (s_{\Pi} + s_{SD/D}(1 - s_{\Pi}) - s_W). \quad (10)$$

Since $c=1-s$ and $w_{W/Y}=1-\alpha$, Eq.9 is replaced by $c-w_{W/Y}$ and Eq.10 is replaced by $c/w_{W/Y}$.

3) Give the values of three saving ratios, $s-\alpha$ is a negative function of α : $s-\alpha=A-B\alpha$, where A is the ratio of saved wages to wages and B is the value of the ratio of saved wages to wages minus the product of the dividend payout ratio and the ratio of consumed dividends to dividends. This implies that economic policy can address the changes in these three ratios.

4) My model uses a concept of net investment that corresponds with saving (for detail, see Kamiryo, Black box [hopefully, 2005]). The investment structure should be shown using net investment, appropriately to connect with the saving structure. The relationship between the depreciation rate and the growth rate of capital is essential to the understanding of the relationship between the investment and saving structures, even if it comes from accounting identity. The depreciation rate in my endogenous model, even if net investment after depreciation is used, corresponds with the depreciation rate explicitly required for an exogenous model.

Net investment is finally connected with the relative share of rental using the ratio of investment to the relative share of rental: $\frac{i}{\alpha} = \frac{s}{\alpha} \cdot \frac{i}{s}$.

After introducing a utility function:

I distinguish the character of $s-\alpha$ with that of s/α (see Eqs. 9 and 10 above). I indicate that the character of s/α is much more fitted for the introduction of a utility function of consumption. Here I will conclusively use a key ratio derived from a utility function (for detail, see the next section). The key ratio is the ratio of the discount rate for consumption to the rate of rental under convergence: (rho/r) , where a discount rate, rho , is used for the present values of saving and consumption each as flow, and r is the optimum rate of rental and used for the present values of rental and wages each as flow. As a result, $w_{W/Y}$ and c are each expressed as,

$$(1-\alpha) = c / (rho / r) \text{ and } c = (rho / r)(1-\alpha), \tag{11}$$

$$\text{Or, } rho(1-\alpha) = r(1-s).$$

Note that in Eq.11, the ratio of wages to output, $1-\alpha$, cannot be replaced by the rate of rental, α , and the rate of consumption, c , cannot be replaced by the rate of saving, s ,

$$\alpha \neq s / (rho / r) \text{ and } s \neq (rho / r)\alpha. \tag{12}$$

Using Eq.11, the relationship between (rho/r) and s/α are derived as follows:

1. If $rho=r$, $1-s=1-\alpha$ or $s=\alpha$ or, if $(rho/r)=1$, $s/\alpha=1$.
2. If $rho<r$, $1-s<1-\alpha$ or $s>\alpha$ or, if $(rho/r)<1$, $s/\alpha>1$.
3. If $rho>r$, $1-s>1-\alpha$ or $s<\alpha$ or, if $(rho/r)>1$, $s/\alpha<1$.

Recall Eqs.7 to 10 and express these equations in Eq.12, then,

1. If $rho=r$ or $s=\alpha$, $s_{SW/Y} = c_{CD/Y}$ holds and thus, $s_W(1-\alpha) = \alpha(1-s_{\Pi})(1-s_{D/D})$,

$$\text{Or, } \alpha = \frac{s_W}{1-(s_{\Pi} + s_{SD/D})(1-s_{\Pi})-s_W} \text{ holds.} \tag{13}$$

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- 5) When we need to express the area of saving in three dimensional graphs, we calculate and express the area of consumption and then, by replacing this area with $1-c$, the area of saving is expressed.
 - 6) If $\alpha=s_W$, $s_W = s_{\Pi} + s_{SD/D}(1-s_{\Pi})$ holds but the retention ratio must be extremely low even if dividends are all consumed.

2. If $\rho < r$ or $s > \alpha$, $s_{SW/Y} > c_{CD/Y}$ holds. (14)

3. If $\rho > r$ or $s < \alpha$, $s_{SW/Y} < c_{CD/Y}$ holds. (15)

When the three saving ratios on the RHS of Eq.13 are each fixed, the golden rule will be maintained.

2.2 From the saving structure to the utility function

Now I will connect the above saving structure with a utility function in this section. I find that the rate of saving is closely connected with the relative share of rental in case study and empirical work, as I will show below. I find that the above ratio of the discount rate ρ to the rate of rental r , (ρ/r), will control the above two ratios, s and a . This ratio, (ρ/r), is supported by the idea of the utility function originated by Ramsey.

My framework for introducing a utility function into consumption will be justified by the following two assumptions: First, the marginal discount rate of consumption and the marginal rate of rental under convergence guarantee an optimum consumption as proved by Ramsey (apart from some restrictions below). Second, the marginal discount rate is equal to the discount rate on average. And also, the marginal rate of rental is equal to the rate of rental on average. The second assumption guarantees equilibrium under convergence. The above two assumptions present a base for a compatibility between optimum choice (lying between saving/investment and consumption) and equilibrium under convergence that uses real assets in national accounts. For the relationship between market equilibrium and equilibrium under convergence, I present a method for neutralizing of financial assets in Kamiryo [2004b].

Ramsey [1928, p.546] stresses the feeling of confrontation lying between the utility for consumption and the disutility for labor, where the production function is supported by using both labor and capital, and the rate of interest is applied to capital (see Eq.2). This is another expression of what I pursue—the relationship

between consumption and wages. Ramsey [ibid., p.547] shows an optimum path for a maximum consumption, using a calculus of variations based on $\int c dt$, where c is the ratio of consumption to output. Ramsey expresses the optimum consumption as a bliss, B , as a maximum value, but this B is difficult to measure as indicated by Tinbergen. However, Ramsey [ibid., p.556] states that the discount rate, ρ , is equal to $\partial f / \partial c$, where the maximum rate of consumption, labor, and capital are determined using the calculus of variations.⁷⁾

Tinbergen [1956, pp.604-605], instead of using Ramsey's B , intends to measure a corresponding rate of consumption by assuming that utility is measurable and using the marginal capital-output ratio as a capital coefficient and $m = \log(1 + \text{the discount rate})$ as a "psychological discount rate" comparable to an interest rate, since $\log(1+m) = m$ holds if m is small. As a result, he showed the area of the rate of consumption, c , and for the region of $c > 1$, he neglected each value using a dot: the higher the marginal capital-output ratio and m , the higher the rate of consumption: $c = m \times \text{the marginal capital coefficient}$.

I interpret his approach as follows: In equilibrium under convergence the marginal capital-output ratio is equal to the capital-output ratio on average. I denote this ratio as Ω as in my papers⁸⁾: $c = m \cdot \Omega$. The value of m , however, does not empirically correspond with the discount rate. Under $c = 0.9$, for example, if $\Omega = 2$, $m = 0.45$, and if $\Omega = 4$, $m = 0.225$, which is still too high. If I introduce the above (r/h) into Tinbergen's idea of the capital-output ratio, capital will be converted to rental as the product of capital and the rate of rental, r , and thus, $c = m \cdot \Omega$ is

7) In a state of equilibrium, $\frac{dx}{dt} = \frac{dc}{dt} = 0$, $x = f(a, c)$, $v(a) = \frac{\partial f}{\partial a} u(x)$, and $\frac{\partial f}{\partial c} = \rho$, where a is labor, c is capital, x is the rate of consumption. Also, $u(x)$ is the marginal rate of utility of a rate of consumption, and $v(a)$ is the marginal rate of disutility of a rate of labor. I pay attention to the treatment of the utility for consumption versus the disutility for labor.

8) I also stress the importance of the capital-output ratio. The range of this ratio should be carefully reviewed under convergence as shown in Kamiryō [2004c].

reduced to the above Eq.11 or 12. This is because for capital and labor, r is used and for saving and consumption ρ is used when (ρ/r) is introduced into the utility function.

In short, both Ramsey and Tinbergen show how to measure the utility function of consumption and I stress here that both different approaches for utility measurement will be reduced to Eq. 11 or 12 by using the ratio of ρ to r , (ρ/r) . Note that this ratio was explained by Ramsey [ibid., p.558] using an individual or his heirs: if $r > \rho$, he will save when he is young and if $r < \rho$ it may be negative, he or the class of men of this sort may borrow when young and pay back when old.

2.3 Relationship between the utility function and technological progress

Assume that the utility function is expressed simply by using $c(\rho/r)$ as I discussed above. Now, I raise two questions: (1) Is the utility function independent of technological progress? (2) If it is not, what is the relationship between the utility function and technological progress? I find that the growth rate of output is equal to the rate of rental under $s = \alpha$ if and only if the rate of technological progress is zero. The discount rate, ρ , is independent of technological progress, but the optimum rate of rental under convergence, $r = r^*$, is influenced by technological progress. Therefore, I indicate that the utility function is inevitably related to technological progress. This is now discussed briefly in this section (for detail, see Kamiryo [2005c]).

For the above questions, I will prepare a few equations, starting with the following Eq.23 that holds under convergence in Kamiryo [2004a, p.60]:

$$\Omega^* = \frac{\beta_{\delta=0}^* \cdot i (1 - \alpha)}{i (1 - \beta_{\delta=0}^*) (1 + n) + n (1 - \alpha)}, \text{ where if } \Omega^* = \Omega(0), \text{ the capital-output ratio is}$$

minimized or set at optimum.

The rate of rental under convergence, r^* , is obtained by solving $r^* = \alpha / \Omega^*$ at

optimum.

$$r^* \equiv \frac{\alpha}{\Omega^*} = \alpha \left(\frac{i(1 - \beta_{\delta=\alpha}^*)(1+n) + n(1-\alpha)}{\beta_{\delta=\alpha}^* \cdot i(1-\alpha)} \right).$$

Or,
$$\frac{r^*}{\alpha} = \frac{i(1 - \beta_{\delta=\alpha}^*)(1+n)}{\beta_{\delta=\alpha}^* \cdot i(1-\alpha)} + \frac{n}{\beta_{\delta=\alpha}^* \cdot i} \quad (16)$$

The growth rate of output under convergence, g_Y^* , is, in the literature, shown as $g_Y^* = \frac{g_A^*(1+n)}{1-\alpha} + n$ (consistently with Solow [1956]). Since the rate of technological progress, g_A^* , is shown in my endogenous growth model as $g_A^* = i(1 - \beta_{\delta=\alpha}^*)$, the growth rate of output is formulated by inserting $g_A^* = i(1 - \beta_{\delta=\alpha}^*)$ into g_Y^* :

$$g_Y^* = \frac{g_A^*(1+n)}{1-\alpha} + n = \frac{i(1 - \beta_{\delta=\alpha}^*)(1+n)}{1-\alpha} + n. \quad (17)$$

The relationship between the rate of rental and the growth rate of output under convergence is now derived by using $A = \frac{i(1 - \beta_{\delta=\alpha}^*)(1+n)}{1-\alpha}$ for the above r^* in Eq.16 and $B = \frac{i(1 - \beta_{\delta=\alpha}^*)}{1-\alpha}$ for the above in Eq.17.

$$r^* = \left(\frac{\alpha}{\beta_{\delta=\alpha}^* \cdot i} \right) g_Y^*. \quad (18)$$

$$\text{or } g_Y^* = \left(\frac{\beta_{\delta=\alpha}^* \cdot i}{\alpha} \right) r^*. \quad (19)$$

In Eqs.18 or Eq.19, the relationship between r^* and g_Y^* is reduced to:

$$\text{When } r^* = g_Y^*, \alpha = \beta_{\delta=\alpha}^* \cdot i \quad \alpha / (\beta_{\delta=\alpha}^* \cdot i) = 1 \text{ holds since.} \quad (20)$$

Eq.20 is useful to understand the character of the golden rule that includes the rate of technological progress. Eq.20 suggests that the relative share of rental under convergence may differ from the current relative share of rental: $\alpha < \alpha^*$, where $\alpha^* \equiv \alpha / \beta^*$. This idea will present a useful device when we estimate *alpha* from consumption, by using (*rho/r*) (see empirical results in 3.3 below).

1. If the rate of technological progress is zero or the structural reform parameter, $\beta_{\delta=\alpha}^*$, is 1.0, the relative share of rental is equal to the rate of saving assuming that the rate of saving equals the rate of investment or exports equals imports.
2. The higher the rate of technological progress the lower the rate of saving and vice versa assuming that the relative share of rental is fixed.

Under the optimum convergence, $\Omega(0)=\Omega^*$ and $r(0)=r^*$ (see in Kamiryō [PhD 2003]). Therefore, the utility function of consumption is shown as $c(rho/r^*)$. In short, when my model is used, an endogenous rate of technological progress influences the relationship between the rate of consumption and the relative share of rental. Or, three fixed ratios in saving have already reflected technological progress.

3. Case study and empirical results using the utility function

3.1 Case study between saving and wages

In this section, I will explain the above saving structure and the relationship between the rate of saving and the relative share of rental, first using case study shown as Tables 1 and 2. Second, I will apply the above relationship to the Japanese national accounts in 1993 to 2002, by dividing total accounts into the government sector and the private sector. Third, I will apply the above relationship to fifteen countries whose data come from OECD national accounts statistics.

Before stating, I will explain the relationship between saving and returns/rental or between consumption and wages using the utility function of consumption, (rho/r) . Saving is dual to consumption. When saving is determined by the retention ratio, the ratio of saved dividends to dividends, and the ratio of household saving to wages, the rate of saving is, at the same time, determined from the utility function of consumption, $c(rho/r)$. In other words, when $c(rho/r)$ is deter-

mined, at the same time, the above three saving ratios are determined and balanced so that the sum of saving and consumption equals national disposable income. In these cases, when consumption is determined, at the same time, returns/rental and wages are newly determined, each value being different from that in GDP.

Table 1 Given $C/W=\rho/r$ in national disposable accounts, saving ratios are determined with the relative share of rental

Case	$s=S/Y$	$c=C/Y$	$C/W=\rho/r$	$W/Y=(1-\alpha)$	$\Pi/Y=\alpha$	S_H/Y	S_H/Y	$S_H/(D+W)$
1	0	1	1.0500	0.9524	0.0476	0.0190	-0.0190	-0.0187
2	0.05	0.95	1.0250	0.9268	0.0732	0.0293	0.0207	0.0201
3	0.1	0.9	1.0	0.9000	0.1000	0.0400	0.0600	0.0576
4	0.15	0.85	0.9757	0.8712	0.1288	0.0515	0.0985	0.0934
5	0.2	0.8	0.9500	0.8421	0.1579	0.0632	0.1368	0.1282
6	0.25	0.75	0.9250	0.8108	0.1892	0.0757	0.1743	0.1611
7	0.3	0.7	0.9000	0.7778	0.2222	0.0889	0.2111	0.1923
8	0.35	0.65	0.8750	0.7429	0.2571	0.1029	0.2471	0.2217
9	0.4	0.6	0.8500	0.7059	0.2941	0.1176	0.2824	0.2491

Note: 1. $C/W=\rho/r$: This presents how to estimate wages, when s or c is given. Case 3, $s=\alpha=0.1$, is a base.

From $c(\rho/r)$, consumption, C , is theoretically derived, but in statistics, given C , W should be estimated.

2. The table replaces $Y-S+C$ by $Y-\Pi+W$ using $\rho/r=W/C$, for $s < \alpha$ and $s > \alpha$.

3. The table assumes that the retention ratio $S_H/\Pi (=1-\text{payout ratio}, D/\Pi)$ is 0.4.

4. As a result, S_H/Y and $S_H/(D+W)$ are calculated. The literature may show S_H/W .

First, Table 1 shows a case which starts with the rate of saving and Table 2 shows a case which starts with the rate of saving and the relative share of rental. Both in Tables 1 and 2, the utility function of consumption, $c(\rho/r)$, integrates consistently each value of saving, consumption, rental, and wages or interrelated ratios. Figure 1 shows each ratio in Table 1, where the above three

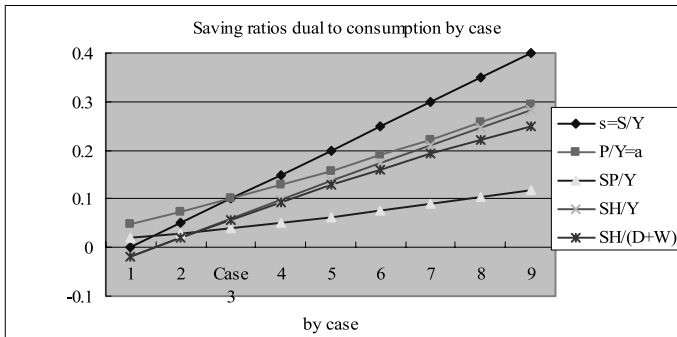


Figure 1 Given $C/W=\rho/r$ in national disposable accounts, saving ratios are determined with the relative share of rental

saving ratios are fixed and suggest that if each ratio changes together with the change in (rho/r) each line will not be a linear.

In Tables 1 and 2, the ratio of consumption to wages is 1.0 when the rate of saving, s , is equal to the relative share of rental, α , where the golden rule/age is realized. And, if $s < \alpha$ this ratio gradually decreases below 1.0, and if $s > \alpha$ it gradually increases above 1.0.⁹⁾

Table 2 Given three kinds of saving ratios with the relative share of rental, $\rho/\rho=C/W$ is determined

given	s	0	0.05	0.1	0.15	0.2	0.25	0.3
given	$alpha$	0.06	0.08	0.1	0.12	0.14	0.16	0.18
	$s/alpha$	0	0.625	1.0000	1.25	1.4286	1.5625	1.6667
s_{SHH}	$s-alpha$	-0.06	-0.03	0	0.03	0.06	0.09	0.12
0.35	$s_{SII} \gamma$	0.021	0.028	0.035	0.042	0.049	0.056	0.063
s_{SDD}	$\alpha_{DII} \gamma$	0.039	0.052	0.065	0.078	0.091	0.104	0.117
0.6	$s_{SD} \gamma$	0.0234	0.0312	0.0390	0.0468	0.0546	0.0624	0.0702
	$c_{CD} \gamma$	0.0156	0.0208	0.0260	0.0312	0.0364	0.0416	0.0468
	$s_{SI} \gamma$	-0.0444	-0.0092	0.0260	0.0612	0.0964	0.1316	0.1668
s_{SHY}	$s_{SHY} - s_{SDY}$	-0.0210	0.0220	0.0650	0.1080	0.1510	0.1940	0.2370
	s_{SIW}	-0.0472	-0.0100	0.0289	0.0695	0.1121	0.1567	0.2034
	$s_{SH(D-W)}$	-0.0215	0.0226	0.0674	0.1127	0.1588	0.2055	0.2529
	$c_{CW} \gamma$	0.9844	0.9292	0.8740	0.8188	0.7636	0.7084	0.6532
$c-C_{CDY} + c_{CW} \gamma$		1.0000	0.9500	0.9000	0.8500	0.8000	0.7500	0.7000
	$s_{SIW} - c_{CDY}$	-0.06	-0.03	0.00	0.03	0.06	0.09	0.12
	$c-w_{II} \gamma$	0.06	0.03	0	-0.03	-0.06	-0.09	-0.12
	$\rho/r=c/w_{II} \gamma$	1.0638	1.0326	1.0000	0.9659	0.9302	0.8929	0.8537

Note: 1. $s-alpha = (c-w_{II} \gamma)$; dual each other. And, $s-alpha = s_{SHY} - c_{CDY}$ if related parameters are given.

2. The literature often assumes that the above saved wages equal consumed dividends.

3. The ratio of household saving, in this table, is totally adjusted by fixing s_{SHH} and s_{SDD} .

It is suggested in Table 2 that the higher the rate of saving the higher the difference between s and α . Is it possible for policy-makers to realize the golden rule/age when the rate of saving is extremely high? How can consumption increase when the rate of saving is high? An only way is to spend all dividends for consumption, where the ratio of saved dividends to dividends is zero. In any case, it is shown that the rate of saving cannot separate from $alpha$ beyond a

9) Recall that Ramsey [1928, p.556] uses two kinds of the rate of rental: the demand curve for capital as $r = \partial f / \partial K$ and the discount rate, ρ , as the supply line. I find that this intersect corresponds with the point of the above $C/W=1.0$ and his demand rate of capital corresponds with the rate of rental under convergence in my model.

certain range.

3.2 Case study with three dimensional graphs

Let me now explain some cases using three dimensional graphs. This section first shows a basic case using Tables 3 and 4 together with Figures 2 and 3. Second, I will clarify the characteristics of (rho/r) using Table 5-1, where the golden rule/age is deeply involved. The process to calculate Table 5-1 will be shown by case in tables of Appendixes (Tables A1 to A3), starting with Table 4.

As shown in Figures 2 and 3 corresponding with Tables 3 and 4, the shape of the area of the ratio of consumption to output, c , is rather simple. The higher (rho/r) and “1-alpha,” the higher the ratio of consumption to output is. Here, first of all, I pay attention to the upper limit of (rho/r) , where if the value of (rho/r) is a little bit higher than 1.0 (e.g., 1.05 or 1.07), the ratio of consumption to output, c , becomes beyond 1.0: $c > 1$. When (rho/r) is 1.0, by definition, it is under the golden age/rule, where $s = alpha$. The estimation of (rho/r) is closely related to the level of $s = alpha$. In other words, it is inevitable for the estimation of wages (or rental) using consumption (or saving) to know the relationship between (rho/r) and the level of the golden age. This is shown as Table 5-1, “Table of (rho/r) by the level of golden age,” together with Figure 4.

Table 5-1 was calculated changing each or two of the three saving ratios, by the level of the golden age. Consumption is the current consumption and saving is future consumption as an alternative, yet, $1 = c + s$ and c is dual to s and s is dual to c . Note that when I finalize the values of (rho/r) in Table 5-1, I use c (or $1-s$) and $1-alpha$. I raise here two questions for Table 5-1 (or Figure 4): (1) Why is (rho/r) higher when $1-alpha$ is lower (the rate of rental is higher)? How can I interpret this tendency? (2) Why is (rho/r) higher when the golden age is lower? How can I interpret this tendency? I conclude as follows:

1. The line of $(rho/r) = 1.0$ constitutes a diagonal line downwards to the right in

Table 3 The utility function of $c(\rho/r)$ by $1-\alpha$: as a base

ρ is the discount rate of the utility function & r is the rate of rental under convergence.

Case 1	r	ρ							
		ρ/r	0.2	0.7	0.75	0.8	0.85	0.9	0.95
	0.16	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
	0.17	1.1765	0.7647	0.8235	0.8824	0.9412	1.0000	1.0588	1.1176
	0.18	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
	0.19	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9474	1.0000
	0.2	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
	0.21	0.9524	0.6190	0.6667	0.7143	0.7619	0.8095	0.8571	0.9048
	0.22	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
	0.23	0.8696	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261
	0.24	0.8333	0.5417	0.5833	0.6250	0.6667	0.7083	0.7500	0.7917

The z axis: for ρ/r . The above idea comes from both F.P. Ramsey [1928] and J Tinbergen [1956].

Case 2	r	ρ							
		ρ/r	0.1	0.7	0.75	0.8	0.85	0.9	0.95
	0.08	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
	0.085	1.1765	0.7647	0.8235	0.8824	0.9412	1.0000	1.0588	1.1176
	0.09	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
	0.095	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9474	1.0000
	0.1	1	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
	0.105	0.9524	0.6190	0.6667	0.7143	0.7619	0.8095	0.8571	0.9048
	0.11	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
	0.115	0.8696	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261
	0.12	0.8333	0.5417	0.5833	0.6250	0.6667	0.7083	0.7500	0.7917

The z axis: for ρ/r .

Case 3	r	ρ							
		ρ/r	0.1	0.7	0.75	0.8	0.85	0.9	0.95
	0.09	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
	0.095	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9474	1.0000
	0.1	1	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
	0.105	0.9524	0.6190	0.6667	0.7143	0.7619	0.8095	0.8571	0.9048
	0.11	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
	0.115	0.8696	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261
	0.125	0.8000	0.5200	0.5600	0.6000	0.6400	0.6800	0.7200	0.7600
	0.13	0.7692	0.5000	0.5385	0.5769	0.6154	0.6538	0.6923	0.7308
	0.135	0.7407	0.4815	0.5185	0.5556	0.5926	0.6296	0.6667	0.7037

The z axis: for ρ/r .

Case 4	r	ρ							
		ρ/r	0.1	0.7	0.75	0.8	0.85	0.9	0.95
	0.07	1.4286	0.9286	1.0000	1.0714	1.1429	1.2143	1.2857	1.3571
	0.075	1.3333	0.8667	0.9333	1.0000	1.0667	1.1333	1.2000	1.2667
	0.08	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
	0.085	1.1765	0.7647	0.8235	0.8824	0.9412	1.0000	1.0588	1.1176
	0.09	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
	0.095	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9474	1.0000
	0.1	1	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500
	0.105	0.9524	0.6190	0.6667	0.7143	0.7619	0.8095	0.8571	0.9048
	0.11	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636

The z axis: for ρ/r .

Table 5-1. In the area above this line, (rho/r) is below 1.0, where the situation is more saving-oriented under $r > rho$. The lower the (rho/r) when (rho/r) is farther above from this line. In the area below this line, (rho/r) is above 1.0, where the situation is more consumption-oriented under $rho > r$. The higher the (rho/r) when (rho/r) is farther below from this

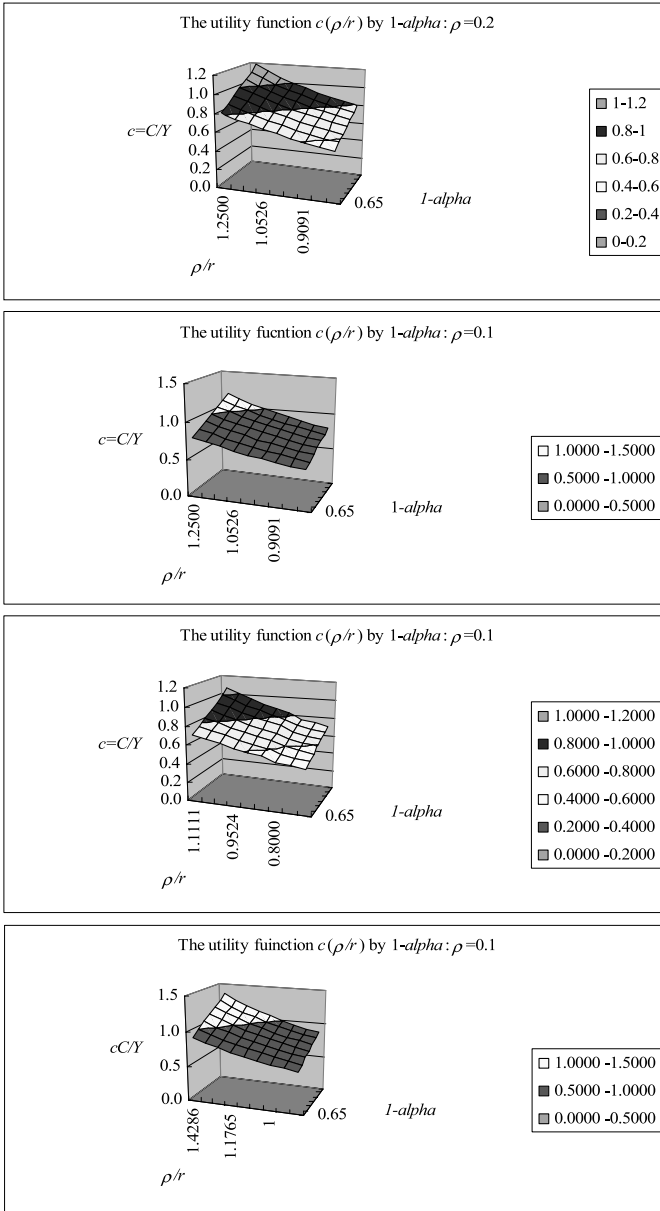


Figure 2 The utility function of $c(p/r)$ by $1-\alpha$: for Table 3

Table 4 The rate of saving that is dual to the utility function of $c(\rho/r)$ by $1-\alpha$

Case 1		ρ							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.							
r	ρ/r	0.2	0.65	0.7	0.75	0.8	0.85	0.9	0.95
		0.24	0.8333	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250
0.23	0.8696	0.2353	0.1765	0.1176	0.0588	0.0000	-0.0588	-0.1176	
0.22	0.9091	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0556	
0.21	0.9524	0.3158	0.2632	0.2105	0.1579	0.1053	0.0526	0.0000	
0.2	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.19	1.0526	0.3810	0.3333	0.2857	0.2381	0.1905	0.1429	0.0952	
0.18	1.1111	0.4091	0.3636	0.3182	0.2727	0.2273	0.1818	0.1364	
0.17	1.1765	0.4348	0.3913	0.3478	0.3043	0.2609	0.2174	0.1739	
0.16	1.2500	0.4583	0.4167	0.3750	0.3333	0.2917	0.2500	0.2083	

The z axis is for ρ/r . The X axis is for n , and the Y axis is for β .

Case 2		ρ							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.							
r	ρ/r	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
		0.12	0.8333	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250
0.115	0.8696	0.2353	0.1765	0.1176	0.0588	0.0000	-0.0588	-0.1176	
0.11	0.9091	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0556	
0.105	0.9524	0.3158	0.2632	0.2105	0.1579	0.1053	0.0526	0.0000	
0.1	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.095	1.0526	0.3810	0.3333	0.2857	0.2381	0.1905	0.1429	0.0952	
0.09	1.1111	0.4091	0.3636	0.3182	0.2727	0.2273	0.1818	0.1364	
0.085	1.1765	0.4348	0.3913	0.3478	0.3043	0.2609	0.2174	0.1739	
0.08	1.2500	0.4583	0.4167	0.3750	0.3333	0.2917	0.2500	0.2083	

The z axis is for ρ/r .

Case 3		ρ							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.							
r	ρ/r	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
		0.099	1.0101	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000
0.105	0.9524	0.6842	0.7368	0.7895	0.8421	0.8947	0.9474	1.0000	
0.1	1	0.6500	0.7000	0.7500	0.8000	0.8500	0.9000	0.9500	
0.095	1.0526	0.6190	0.6667	0.7143	0.7619	0.8095	0.8571	0.9048	
0.09	1.1111	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636	
0.085	1.1765	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261	
0.08	1.2500	0.5200	0.5600	0.6000	0.6400	0.6800	0.7200	0.7600	
0.075	1.3333	0.5000	0.5385	0.5769	0.6154	0.6538	0.6923	0.7308	
0.07	1.4286	0.4815	0.5185	0.5556	0.5926	0.6296	0.6667	0.7037	

The z axis is for ρ/r .

Case 4		ρ							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.							
r	ρ/r	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
		0.13	0.7692	0.0714	0.0000	-0.0714	-0.1429	-0.2143	-0.2857
0.125	0.8000	0.1333	0.0667	0.0000	-0.0667	-0.1333	-0.2000	-0.2667	
0.12	0.8333	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250	-0.1875	
0.115	0.8696	0.2353	0.1765	0.1176	0.0588	0.0000	-0.0588	-0.1176	
0.11	0.9091	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0556	
0.105	0.9524	0.3158	0.2632	0.2105	0.1579	0.1053	0.0526	0.0000	
0.1	1	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	
0.095	1.0526	0.3810	0.3333	0.2857	0.2381	0.1905	0.1429	0.0952	
0.09	1.1111	0.4091	0.3636	0.3182	0.2727	0.2273	0.1818	0.1364	

The z axis is for ρ/r .

- line. I understand these characteristics are reasonable.
- The lower the golden age, where $s=alpha$, the more consumption-oriented the area is. For example, in the case of $s=alpha=0.05$, the situation is completely consumption-oriented. Then, does this situation show the lowest

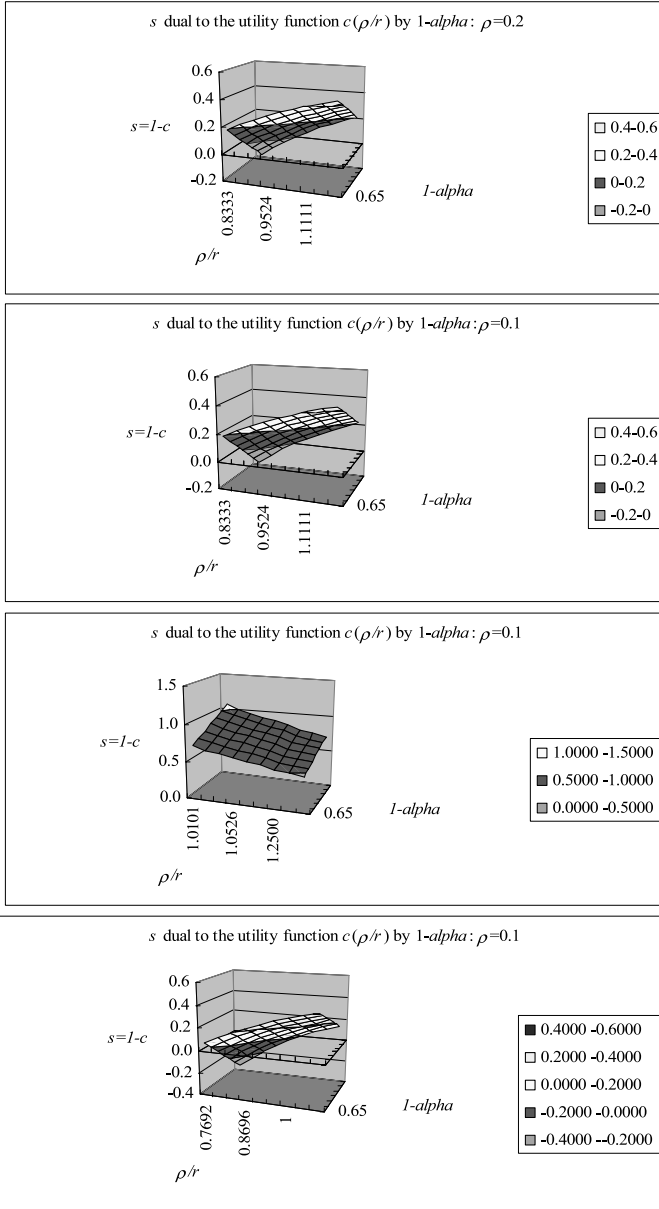


Figure 3 The rate of saving, s , dual to the utility function of $c(\rho/r)$ by $1 - \alpha$: for Table 4

Table 5-1 Table of (rho/r) by the level of golden age, $s=\alpha$, where $\beta^*=1.0$ and $i/s=1.0$

$1-\alpha$		0.65	0.7	0.75	0.8	0.85	0.9	0.95
α		0.35	0.3	0.25	0.2	0.15	0.1	0.05
$s = \alpha = 0.35$	s	0.3500	0.3093	0.2686	0.2279	0.1871	0.1464	0.1057
	c=1-s	0.6500	0.6907	0.7314	0.7721	0.8129	0.8536	0.8943
	ρ/r	1.0000	0.9867	0.9752	0.9652	0.9563	0.9484	0.9414
$s = \alpha = 0.30$	s	0.3400	0.3000	0.2600	0.2200	0.1800	0.1400	0.1000
	c	0.6600	0.7000	0.7400	0.7800	0.8200	0.8600	0.9000
	ρ/r	1.0154	1.0000	0.9867	0.9750	0.9647	0.9556	0.9474
$s = \alpha = 0.25$	s	0.3260	0.2880	0.2500	0.2120	0.1740	0.1360	0.0980
	c	0.6740	0.7120	0.7500	0.7880	0.8260	0.8640	0.9020
	ρ/r	1.0370	1.0172	1.0000	0.9850	0.9718	0.9600	0.9495
$s = \alpha = 0.20$	s	0.3126	0.2750	0.2375	0.2000	0.1625	0.1250	0.0875
	c	0.6874	0.7250	0.7625	0.8000	0.8375	0.8750	0.9125
	ρ/r	1.0576	1.0357	1.0166	1.0000	0.9853	0.9722	0.9605
$s = \alpha = 0.15$	s	0.2968	0.2601	0.2234	0.1867	0.1500	0.1134	0.0767
	c	0.7032	0.7399	0.7766	0.8133	0.8500	0.8866	0.9233
	ρ/r	1.0819	1.0570	1.0355	1.0166	1.0000	0.9852	0.9719
$s = \alpha = 0.10$	s	0.2500	0.2200	0.1900	0.1600	0.1300	0.1000	0.0700
	c	0.7500	0.7800	0.8100	0.8400	0.8700	0.9000	0.9300
	ρ/r	1.1538	1.1143	1.0800	1.0500	1.0235	1.0000	0.9789
$s = \alpha = 0.05$	s	0.2001	0.1751	0.1501	0.1251	0.1001	0.0750	0.0500
	c	0.7999	0.8249	0.8499	0.8749	0.8999	0.9250	0.9500
	ρ/r	1.2306	1.1784	1.1332	1.0937	1.0588	1.0277	1.0000

Note: saving machines rental by changing each of s_{SDF} , s_{SD} , and s_{SRE} by the level of the golden age.

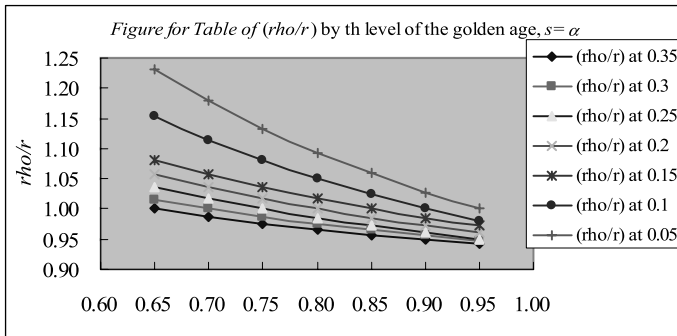


Figure 4 Figure for Table of (rho/r) by the level of the golden age, $s=\alpha$: for Table 5-1

alpha? This differs from the real

- The lower the golden age, where $s=\alpha$, the more consumption-oriented the area is. For example, in the case of $s=\alpha=0.05$, the situation is completely consumption-oriented. Then, does this situation show the lowest

alpha? This differs from the real

world: does the US show a low *alpha*? No, it does not and we need a suitable interpretation.

If *alpha* is low, the rate of rental is also low and cannot absorb funds from the world assuming a fixed moderate capital-output ratio, $\Omega(0)$: if *alpha*=0.05 and $\Omega(0)$ =2.5, then the rate of rental, $r(0)$, will be 0.02, which is not attractive at all in the world money market. Recall the above Eq.20, $\alpha = \beta_{\delta=\alpha}^* \cdot i$ under the golden age, when technological progress is not zero. A modified *alpha* is set $\alpha^* \equiv \alpha / \beta_{\delta=\alpha}^*$. If $\beta_{\delta=\alpha}^* = 0.7$ and *alpha*=0.1, then the modified *alpha* is $0.1/0.7=0.1428$. This implies that *alpha* is estimated at a higher value than that in Table 5-1. I will briefly discuss this problem in 3.3 soon below and also in Kamiryō [2005c] in more detail.¹⁰⁾

Finally, I raise two propositions derived from Table 5-1 and Figure 4.

Proposition 1: The value of (*rho/r*) and the ratio of consumption to output will be hyperbolically higher if the relative share of rental increases.

Proposition 2: The higher the (*rho/r*), the more consumption-oriented the situation is, if the level of $s=alpha$ at the golden age becomes lower, assuming no technological progress. On the other hand, the level of (*rho/r*) is more mitigated if the rate of technological progress is higher.

In short, in my endogenous growth model, the golden rule is modified and influenced by the level of β^* and thus, the level of $s=alpha$ will be higher than that with no technological progress. This corresponds with such that technology will induce investment as stressed by Schumpeter.

10) When *alpha* is estimated at a higher value, usually the capital-labor will be estimated at a higher value. When capital is unknown, if I divide $alpha/(1-alpha)$ by the ratio of the rate of rental to the wage rate, r/w , the capital-labor ratio, $k(0)$, will be estimated: if the rate of rental is lower r/w will be lower and thus, the capital-labor ratio and capital will be higher, assuming that *alpha* is fixed. This idea comes from Kamiryō [2004c]. In this respect, *alpha* is indirectly related to capital.

3.3 Empirical results in the Japanese national accounts

This section, using (rho/r) , shows estimated wages and rental based on national disposable income as the sum of saving and consumption, using the Japanese national accounts, 1993 to 2002. The results are more vividly shown when the total economy is divided into the government sector and the private sector (compare Tables 6-1 with Tables 6-2 and 6-3, together with Figure 5).

The Japanese economy has changed significantly last one decade in terms of the relationship between saving and $alpha$. This change is expressed by the transition of the ratio of s to $alpha$: $s/alpha$. When $s/alpha=1.0$, the situation is under the golden rule/age. The higher the value of $s/alpha$ beyond 1.0, the more saving-oriented it is and vice versa. The value of $s/alpha$ has declined from 1.25 in 1993 to 0.75 in 2002, slightly influenced by the value of (rho/r) as seen in Table 6-1 by case. Japan rapidly turned to consumption-oriented economy during this decade.

Table 6-1 Measurement of wages using the utility function: Cases of Japan (Open S-I Approach)

For total economy: Japanese cases

		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Final consumpt	C	334189.1	344000.5	352096.3	362349.8	364961.7	367224.5	368276.1	370896	372136.2	371944.7
	Y=S+C	402296.5	407054.3	411920.9	423474.5	428522.4	396183.8	419292.3	419970.9	411570.5	406983.1
	c=C/Y	0.8307	0.8451	0.8548	0.8557	0.8517	0.9269	0.8783	0.8831	0.9042	0.9139
	s=S/Y	0.1693	0.1549	0.1452	0.1443	0.1483	0.0731	0.1217	0.1169	0.0958	0.0861
	$L_{CV}=(S-Sc)/Sc$	3.1710	2.1915	1.7713	1.1233	1.0937	0.8611	0.7296	0.2463	(0.0884)	(0.3712)
	(S-D/Y)	0.0354	0.0305	0.0230	0.0172	0.0309	(0.0277)	0.0316	0.0295	0.0289	0.0329
Case1	(rho/r)	0.960	0.960	0.960	0.960	0.960	1.020	0.980	0.990	1.015	1.030
as a base	W=C/(p/r)	348113.6	358333.9	366767.0	377447.7	380168.4	360024.0	375791.9	374642.4	366636.7	361111.4
	1-c=W/Y	0.8653	0.8803	0.8904	0.8913	0.8872	0.9087	0.8963	0.8921	0.8908	0.8873
	alpha	0.1347	0.1197	0.1096	0.1087	0.1128	0.0913	0.1037	0.1079	0.1092	0.1127
	$s/alpha$	1.2570	1.2942	1.3249	1.3280	1.3145	0.8009	1.1728	1.0827	0.8776	0.7638
	Rental	54182.9	48720.4	45153.9	46026.8	48354.0	36159.8	43500.4	45328.5	44933.8	44571.7
Case 2	(rho/r)	0.960	0.960	0.964	0.964	0.964	0.984	0.975	0.975	0.979	0.984
	W=C/(p/r)	348113.6	358333.9	365416.5	376068.2	378745.5	373142.1	377827.1	380498.5	37964.5	37994.4
	1-c=W/Y	0.8653	0.8803	0.8871	0.8881	0.8838	0.9418	0.9011	0.9066	0.9232	0.9288
	alpha	0.1347	0.1197	0.1129	0.1119	0.1162	0.0582	0.0989	0.0940	0.0768	0.0712
	$s/alpha$	1.2570	1.2942	1.2864	1.2894	1.2769	1.2568	1.2303	1.2433	1.2477	1.2087
	Rental	54183	48720	46504	47406	49777	23042	41465	39472	31606	28989
Case 3	(rho/r)	0.960	0.960	0.960	0.960	0.960	1.012	1.025	1.014	1.014	1.020
	W=C/(p/r)	348113.6	358333.9	366767.0	377447.7	380168.4	362960.8	39352.1	368889.1	367160.0	364691.2
	1-c=W/Y	0.8653	0.8803	0.8904	0.8913	0.8872	0.9161	0.8570	0.8712	0.8921	0.8961
	alpha	0.1347	0.1197	0.1096	0.1087	0.1128	0.0839	0.1430	0.1288	0.1079	0.1039
	$s/alpha$	1.2570	1.2942	1.3249	1.3280	1.3145	0.8717	0.8511	0.9074	0.8880	0.8285
	Rental	54183	48720	45154	46027	48354	33223	59940	54082	44410	42292
Case 4	(rho/r)	0.943	0.943	0.943	0.943	0.943	1.014	1.018	1.027	1.036	1.045
	W=C/(p/r)	354531.1	364937.3	373504.1	384389.2	387133.5	362317.7	361768.5	361170.1	359279.9	356666.9
	1-c=W/Y	0.8813	0.8965	0.9067	0.9077	0.9034	0.9145	0.8628	0.8600	0.8729	0.8749
	alpha	0.1187	0.1035	0.0953	0.0923	0.0966	0.0855	0.1372	0.1400	0.1271	0.1251
	$s/alpha$	1.4259	1.4971	1.5573	1.5639	1.5357	0.8551	0.8869	0.8346	0.8241	0.6882
	Rental	42765	42117	38417	39085	41389	33866	57524	58306	52291	50916
Case 5	(rho/r)	0.962	0.963	0.964	0.964	0.964	1.021	1.007	1.012	1.017	1.022
	W=C/(p/r)	347474.5	357354.6	365255.2	375740.4	378407.4	359689.0	365565.0	366512.9	365915.6	363925.6
	1-c=W/Y	0.8637	0.8779	0.8867	0.8873	0.8831	0.9079	0.8719	0.8727	0.8891	0.8942
	alpha	0.1363	0.1221	0.1133	0.1127	0.1169	0.0921	0.1281	0.1273	0.1109	0.1058
	$s/alpha$	1.2423	1.2687	1.2820	1.2805	1.2683	0.7935	0.9495	0.9180	0.8637	0.8138
	Rental	54822	49700	46666	47734	50115	36495	53727	53458	45655	43058

Table 6-2 Measurement of wages using the utility function: Cases of Japan (Open S-I Approach)

The government sector		1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
Excl.pension	C_{it}	30966.2	31023.3	32411.6	33108.1	34150.7	35197.9	36389.4	36974.3	37872.2	38119.1
	$Y_{it}-C_{it}/p$	39261.1	32428.9	31462.9	31803.2	33457.8	24592.4	19009.1	21221.1	17956	10095.5
	$S_{it}-C_{it}/Y_{it}$	0.7658	0.9567	1.0302	1.0410	1.0207	1.4313	1.9143	1.7423	2.1092	3.8436
	$s_{it}-S_{it}/Y_{it}$	0.2342	0.0433	(0.0302)	(0.0410)	(0.0207)	(0.4313)	(0.9143)	(0.7423)	(1.1092)	(2.8436)
	$L_{t,excl}(U_{it}-S_{it}/S_{it})$ ($S_{it}-C_{it}/Y_{it}$)	0.4479 (0.3399)	2.1296 (0.6167)	3.4040 (0.7517)	4.0836 (0.6986)	3.8715 (0.5698)	(4.2401)	(2.3008)	(2.1746)	(1.7172)	(1.3804)
Case1	(rho,r)	0.960	0.960	0.960	0.960	0.960	1.020	0.980	0.990	1.015	1.030
	$W_{it}-C_{it}/p$	31319.0	32315.9	33762.1	34487.6	35573.6	34507.7	37132.0	37347.8	37312.5	37389.3
	W_{it}/Y_{it}	0.7977	0.9965	1.0731	1.0844	1.0632	1.4032	1.9534	1.7599	2.0780	3.7317
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0731)	(0.0844)	(0.0632)	(0.4032)	(0.9534)	(0.7599)	(1.0780)	(2.7317)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	0.4126	0.4861	0.3275	1.0666	0.9590	0.9768	1.0289	1.0410
Case 2	(rho,r)	0.960	0.960	1.000	1.000	1.000	1.025	1.020	1.020	1.020	1.020
	$W_{it}-C_{it}/p$	31319.0	32315.9	32411.6	33108.1	34150.7	34339.4	35675.9	36249.3	37129.6	37755.9
	W_{it}/Y_{it}	0.7977	0.9965	1.0302	1.0410	1.0207	1.3963	1.8768	1.7082	2.0678	3.7682
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0302)	(0.0410)	(0.0207)	(0.3963)	(0.8768)	(0.7082)	(1.0678)	(2.7682)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	1.0000	1.0000	1.0000	1.0881	1.0428	1.0482	1.0387	1.0272
Case 3	(rho,r)	0.960	0.960	0.960	0.960	0.960	0.940	0.980	0.960	0.960	0.940
	$W_{it}-C_{it}/p$	31319.0	32315.9	33762.1	34487.6	35573.6	37444.6	37132.0	38514.9	39450.2	40969.1
	W_{it}/Y_{it}	0.7977	0.9965	1.0731	1.0844	1.0632	1.5228	1.9534	1.8149	2.1970	4.0889
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0731)	(0.0844)	(0.0632)	(0.5228)	(0.9534)	(0.8149)	(1.1970)	(3.0889)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	0.4126	0.4861	0.3275	0.8252	0.9590	0.9109	0.9266	0.9206
Case 4	(rho,r)	0.970	0.970	0.970	0.970	0.970	1.000	1.000	1.000	1.000	1.000
	$W_{it}-C_{it}/p$	30996.1	31982.8	33414.0	34132.1	35206.9	35197.9	36389.4	36974.3	37872.2	38511.0
	W_{it}/Y_{it}	0.7895	0.9862	1.0620	1.0732	1.0523	1.4313	1.9143	1.7423	2.1092	3.8436
	$\alpha_{it}p_{it}$	0.2105	0.0138	(0.0620)	(0.0732)	(0.0523)	(0.4313)	(0.9143)	(0.7423)	(1.1092)	(2.8436)
	$S_{it}/\beta_{it}p_{it}$	1.1125	3.1507	0.4862	0.5603	0.3961	1.0000	1.0000	1.0000	1.0000	1.0000
Case 5	(rho,r)	0.980	0.990	1.005	1.010	1.010	1.030	1.030	1.030	1.035	1.040
	$W_{it}-C_{it}/p$	30679.8	31336.7	32250.3	32780.3	33812.6	34172.7	35329.5	35897.4	36591.5	37029.8
	W_{it}/Y_{it}	0.7814	0.9663	1.0250	1.0307	1.0106	1.3896	1.8586	1.6916	2.0378	3.6958
	$\alpha_{it}p_{it}$	0.2186	0.0337	(0.0250)	(0.0307)	(0.0106)	(0.3896)	(0.8586)	(0.6916)	(1.0378)	(2.6958)
	$S_{it}/\beta_{it}p_{it}$	1.0715	1.2869	1.2048	1.3355	1.9531	1.0770	1.0649	1.0734	1.0687	1.0548
The private sector	(rho,r)	0.960	0.960	0.960	0.960	0.960	1.020	0.980	0.990	1.015	1.030
	$W_{it}-C_{it}/p$	316794.7	326017.9	333004.9	342960.1	344594.8	325516.3	338659.9	337294.6	329324.1	323722.0
	W_{it}/Y_{it}	0.8726	0.8703	0.8753	0.8756	0.8722	0.8760	0.8461	0.8459	0.8367	0.8155
	$\alpha_{it}p_{it}$	0.1274	0.1297	0.1247	0.1244	0.1278	0.1240	0.1539	0.1541	0.1633	0.1845
	$S_{it}/\beta_{it}p_{it}$	1.2740	1.2683	1.2807	1.2816	1.2731	0.8587	1.1099	1.0549	0.9232	0.8674
Case 2	(rho,r)	0.960	0.960	0.960	0.960	0.960	0.980	0.970	0.970	0.975	0.980
	$W_{it}-C_{it}/p$	316794.7	326017.9	333004.9	342960.1	344594.8	338802.7	342151.2	344249.2	342834.9	340238.5
	W_{it}/Y_{it}	0.8726	0.8703	0.8753	0.8756	0.8722	0.9118	0.8548	0.8633	0.8710	0.8571
	$\alpha_{it}p_{it}$	0.1274	0.1297	0.1247	0.1244	0.1278	0.0882	0.1452	0.1367	0.1290	0.1429
	$S_{it}/\beta_{it}p_{it}$	1.2740	1.2683	1.2807	1.2816	1.2731	2.067	1.1766	1.1895	1.1688	1.1200
Case 3	(rho,r)	0.960	0.960	0.960	0.960	0.960	1.020	1.030	1.020	1.020	1.030
	$W_{it}-C_{it}/p$	316794.7	326017.9	333004.9	342960.1	344594.8	325516.3	322220.1	327374.2	327709.8	323722.0
	W_{it}/Y_{it}	0.8726	0.8703	0.8753	0.8756	0.8722	0.8760	0.8500	0.8210	0.8036	0.8155
	$\alpha_{it}p_{it}$	0.1274	0.1297	0.1247	0.1244	0.1278	0.1240	0.1950	0.1790	0.1674	0.1845
	$S_{it}/\beta_{it}p_{it}$	1.2740	1.2683	1.2807	1.2816	1.2731	0.8587	0.8762	0.9083	0.9006	0.8674
Case 4	(rho,r)	0.940	0.940	0.940	0.940	0.940	1.015	1.020	1.030	1.040	1.050
	$W_{it}-C_{it}/p$	323535.0	332954.5	340090.1	350257.1	351926.6	327119.8	325379.1	324195.8	321407.7	317555.9
	W_{it}/Y_{it}	0.8912	0.8888	0.8939	0.8943	0.8908	0.8803	0.8129	0.8130	0.8166	0.8000
	$\alpha_{it}p_{it}$	0.1088	0.1112	0.1061	0.1057	0.1092	0.1197	0.1871	0.1870	0.1834	0.2000
	$S_{it}/\beta_{it}p_{it}$	1.4914	1.4794	1.5055	1.5074	1.4895	0.8897	0.9131	0.8695	0.8220	0.8000
Case 5	(rho,r)	0.960	0.960	0.960	0.960	0.960	1.020	1.005	1.010	1.015	1.020
	$W_{it}-C_{it}/p$	316794.7	326017.9	333004.9	342960.1	344594.8	325516.3	330235.5	330615.5	329324.1	326985.8
	W_{it}/Y_{it}	0.8726	0.8703	0.8753	0.8756	0.8722	0.8760	0.8250	0.8291	0.8367	0.8235
	$\alpha_{it}p_{it}$	0.1274	0.1297	0.1247	0.1244	0.1278	0.1240	0.1750	0.1709	0.1633	0.1765
	$S_{it}/\beta_{it}p_{it}$	1.2740	1.2683	1.2807	1.2816	1.2731	0.8587	0.9764	0.9515	0.9232	0.9067
The government sector	(rho,r)	0.960	0.960	0.960	0.960	0.960	1.020	0.980	0.990	1.015	1.030
	$W_{it}-C_{it}/p$	31319.0	32315.9	33762.1	34487.6	35573.6	34507.7	37132.0	37347.8	37312.5	37389.3
	W_{it}/Y_{it}	0.7977	0.9965	1.0731	1.0844	1.0632	1.4032	1.9534	1.7599	2.0780	3.7317
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0731)	(0.0844)	(0.0632)	(0.4032)	(0.9534)	(0.7599)	(1.0780)	(2.7317)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	0.4126	0.4861	0.3275	1.0666	0.9590	0.9768	1.0289	1.0410
Case 2	(rho,r)	0.960	0.960	1.000	1.000	1.000	1.025	1.020	1.020	1.020	1.020
	$W_{it}-C_{it}/p$	31319.0	32315.9	32411.6	33108.1	34150.7	34339.4	35675.9	36249.3	37129.6	37755.9
	W_{it}/Y_{it}	0.7977	0.9965	1.0302	1.0410	1.0207	1.3963	1.8768	1.7082	2.0678	3.7682
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0302)	(0.0410)	(0.0207)	(0.3963)	(0.8768)	(0.7082)	(1.0678)	(2.7682)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	1.0000	1.0000	1.0000	1.0881	1.0428	1.0482	1.0387	1.0272
Case 3	(rho,r)	0.960	0.960	0.960	0.960	0.960	0.940	0.980	0.960	0.960	0.940
	$W_{it}-C_{it}/p$	31319.0	32315.9	33762.1	34487.6	35573.6	37444.6	37132.0	38514.9	39450.2	40969.1
	W_{it}/Y_{it}	0.7977	0.9965	1.0731	1.0844	1.0632	1.5228	1.9534	1.8149	2.1970	4.0889
	$\alpha_{it}p_{it}$	0.2023	0.0035	(0.0731)	(0.0844)	(0.0632)	(0.5228)	(0.9534)	(0.8149)	(1.1970)	(3.0889)
	$S_{it}/\beta_{it}p_{it}$	1.1577	12.4431	0.4126	0.4861	0.3275	0.8252	0.9590	0.9109	0.9266	0.9206
Case 4	(rho,r)	0.970	0.970	0.970	0.970	0.970	1.000	1.000	1.000	1.000	1.000
	$W_{it}-C_{it}/p$	30996.1	31982.8	33414.0	34132.1	35206.9	35197.9	36389.4	36974.3	37872.2	38511.0
	W_{it}/Y_{it}	0.7895	0.9862	1.0620	1.0732	1.0523	1.4313	1.9143	1.7423	2.1092	3.8436
	$\alpha_{it}p_{it}$	0.2105	0.0138	(0.0620)	(0.0732)	(0.0523)	(0.4313)	(0.9143)	(0.7423)	(1.1092)	(2.8436)
	$S_{it}/\beta_{it}p_{it}$	1.1125	3.1507	0.4862	0.5603	0.3961	1.0000	1.0000	1.0000	1.0000	1.0000
Case 5	(rho,r)	0.980	0.990	1.005	1.010	1.010	1.030	1.030	1.030	1.035	1.040
	$W_{it}-C_{it}/p$	30679.8	31336.7	32250.3	32780.3	33812.6	34172.7	35329.5	35897.4	36591.5	37029.8
	W_{it}/Y_{it}	0.7814	0.9663	1.0250	1.0307	1.0106	1.3896				

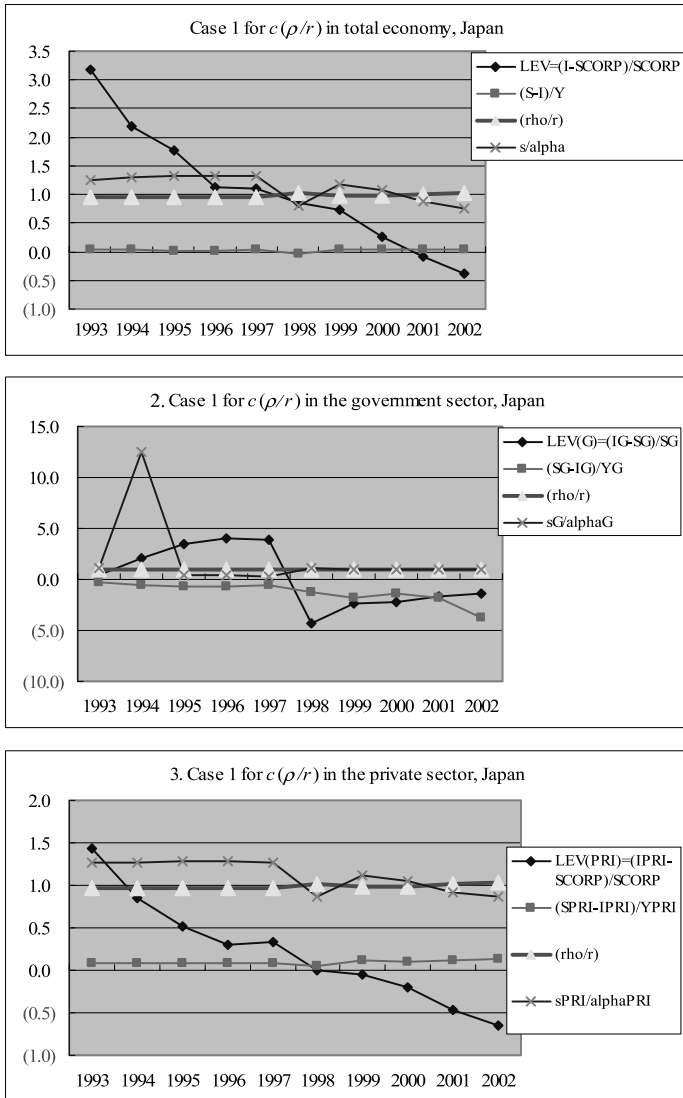


Figure 5 The relationship between saving and rental using the utility function: cases of Japan: for Table 6

The above tendency is significantly stronger in the government sector, but α becoming negative as seen in most EU countries. And, the above tendency should be weaker in the private sector, but note that the private sector has decreased borrowings without facing at “crowding out,” as seen in Figure 5. In other words, the leverage defined as the ratio of net investment minus corporate saving to corporate saving has decreased continuously in the decade. This corresponds with the decrease in bad debts and also in the capital-output ratio. Under these circumstances, how can the value of (ρ/r) be discriminated by sector? This will be discussed more in Kamiryo [2005c].

3.4 Empirical results in the OECD national accounts

Finally, in most countries in EU, the results derived using OECD National Accounts Statistics data in 1967 to 2003, show that in the long-term the value of s/α significantly has varied due to budget deficits but the value of (ρ/r) has compulsively remained within some narrow ranges. This is because if (ρ/r) increased continuously the value of α also increases, resulting in the increase in the capital-output ratio without limit,¹¹⁾ which destroys an economic system.

For my empirical analysis, I need four values¹²⁾ (which I call raw data in statistics) together with the determination of (ρ/r) : GDP, national disposable income, consumption, and actual compensation/wages in GDP. I investigated twenty five countries available in OECD Statistics, but finally I found that ten countries publish all of the above four values. Thus, I added five countries to my investigation, although national disposable income and/or actual wages are

11) I discussed this problem in Kamiryo [2004c], where the capital-output ratio has its upper limit by club, classified using many developing and advanced countries.

12) In the neat future I intend to use together “employed persons” (available in OECD raw data) instead of population (available in IFS and GFS, IMF). In this case, I hope I can distinguish increasing returns to capital (IRC) with decreasing returns to capital (DRC) more accurately.

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often not available after 1996. The results are shown in Appendix (Figures A4
with OECD data). Thus,

Proposition 3: If the value of (rho/r) is determined within a certain limit or
within a range of Table 5-1, the ratio of consumption to output, c , and/or $alpha$
may fluctuate significantly in a short term, but in the long-term, c and/or $alpha$
will return back to a normal level by economic policies.

4. Conclusion

This paper discussed the relationship between consumption and com-
pensation. I synthesize consumption and compensation with both optimum con-
sumption in income and maximum rental in production. For consumption, I use
the discount rate for consumers and the rate of rental for production and integrate
the relationship between consumption and compensation by using (rho/r) , main-
taining each character of production and income.

I reviewed Ramsey [1928] and Tinbergen [1956] together with Sen Amarta
Kumar [1957, 1961]. This is because Solow [1992] indicated the necessity of
introducing the utility function of consumption and Tinbergen assumed and
advocated that the utility function should be measurable. And, I found in this
paper that both approaches finally are integrated to a simple utility function of
consumption, $c(rho/r)$. My approach differs from Tinbergen, yet I got a hint
from him and expressed the area of $c(rho/r)$, by using three dimensional
graphs and presenting Table 5-1 (abbreviating 5-2 and 5-3 for calculation in
this paper).

I showed the above results using cases together with three dimensional graphs,
and also testing the Japanese national accounts in 1993 to 2002 and OECD
National Statistics in 1967 to 2003. As the results, I found three propositions
that were related to a golden rule/age consistent with technology. These proposi-
tions are useful when we estimate wages or rental by country and year, using the

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values of (rho/r) as in Table 5-1.¹³⁾

Finally, I add that I am soon applying my method (that is uniquely related to a golden rule consistent with technology) to many countries “by sector,” using the data of IFS and GFS of IMF, which covers many countries in the world. And also, I must justify how to apply a technology-golden rule to the private sector. If the private sector constitutes an economy, income must be equal to production/output as “net product” stated by Bailey Martin, J. [1971, 33, 257–259].

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13) A question may still remain: how can we determine the level of $s=alpha$ by country? The higher the rate of saving the higher the $alpha$, but at what rate of saving $alpha$ should be equal to the rate of saving? If I follow Solow [1958], the golden age by country may converge by club of convergence and the range of the golden age will be narrower than the range of rate of saving among countries. Two factors will reply to the above question: (1) the influence of technology by country as discussed in this paper and (2) the introduction of the differences between saving and net investment by sector (the balance of payment=budget deficit + the difference of saving and net investment in the private sector).

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Table A1-1 Structure of saving and consumption as a base to be consistent with the utility function

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Figure A4-1 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

Figure A4-2 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

Figure A4-3 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

Figure A4-4 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

Tables A4-1 to A4-5: OECD data, applying my method to these data by country

Table A1-1 Structure of saving and consumption as a base to be consistent with the utility function

Case 1	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
<i>alpha</i>	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$S_{SP} =$
α_{SPY}	0.2275	0.195	0.1625	0.13	0.0975	0.065	0.0325	0.65
α_{DY}	0.1225	0.105	0.0875	0.07	0.0525	0.035	0.0175	$S_{SD} =$
s_{SDY}	0.0526	0.0450	0.0375	0.0300	0.0225	0.0150	0.0075	0.429
c_{CDY}	0.0699	0.0600	0.0500	0.0400	0.0300	0.0200	0.0100	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$S_{SW} =$
s_{SWY}	0.0325	0.0350	0.0375	0.0400	0.0425	0.0450	0.0475	0.05
c_{CWY}	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025	
<i>s</i>	0.3126	0.2750	0.24	0.2	0.1625	0.1250	0.0875	
<i>c</i>	0.6874	0.7250	0.7625	0.8000	0.8375	0.8750	0.9125	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0576	1.0357	1.0166	1.0000	0.9853	0.9722	0.9605	

Case 2	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
<i>alpha</i>	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$S_{SP} =$
α_{SPY}	0.1575	0.135	0.1125	0.09	0.0675	0.045	0.0225	0.45
α_{DY}	0.1925	0.165	0.1375	0.11	0.0825	0.055	0.0275	$S_{SD} =$
s_{SDY}	0.1155	0.0990	0.0825	0.0660	0.0495	0.0330	0.0165	0.6
c_{CDY}	0.0770	0.0660	0.0550	0.0440	0.0330	0.0220	0.0110	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$S_{SW} =$
s_{SWY}	0.0358	0.0385	0.0413	0.0440	0.0468	0.0495	0.0523	0.055
c_{CWY}	0.6143	0.6615	0.7088	0.7560	0.8033	0.8505	0.8978	
<i>s</i>	0.3088	0.2725	0.24	0.2	0.1638	0.1275	0.0913	
<i>c</i>	0.6913	0.7275	0.7638	0.8000	0.8363	0.8725	0.9088	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0635	1.0393	1.0183	1.0000	0.9838	0.9694	0.9566	

Case 3	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
<i>alpha</i>	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$S_{SP} =$
α_{SPY}	0.175	0.15	0.125	0.1	0.075	0.05	0.025	0.5
α_{DY}	0.175	0.15	0.125	0.1	0.075	0.05	0.025	$S_{SD} =$
s_{SDY}	0.1260	0.1080	0.0900	0.0720	0.0540	0.0360	0.0180	0.72
c_{CDY}	0.0490	0.0420	0.0350	0.0280	0.0210	0.0140	0.0070	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$S_{SW} =$
s_{SWY}	0.0390	0.0420	0.0450	0.0480	0.0510	0.0540	0.0570	0.06
c_{CWY}	0.6110	0.6580	0.7050	0.7520	0.7990	0.8460	0.8930	
<i>s</i>	0.3400	0.3	0.26	0.2200	0.1800	0.1400	0.1000	
<i>c</i>	0.6600	0.7000	0.7400	0.7800	0.8200	0.8600	0.9000	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0154	1.0000	0.9867	0.9750	0.9647	0.9556	0.9474	

Case 4	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
<i>alpha</i>	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$S_{SP} =$
α_{SPY}	0.14	0.12	0.1	0.08	0.06	0.04	0.02	0.4
α_{DY}	0.21	0.18	0.15	0.12	0.09	0.06	0.03	$S_{SD} =$
s_{SDY}	0.0840	0.0720	0.0600	0.0480	0.0360	0.0240	0.0120	0.4
c_{CDY}	0.1260	0.1080	0.0900	0.0720	0.0540	0.0360	0.0180	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$S_{SW} =$
s_{SWY}	0.0260	0.0280	0.0300	0.0320	0.0340	0.0360	0.0380	0.04
c_{CWY}	0.6240	0.6720	0.7200	0.7680	0.8160	0.8640	0.9120	
<i>s</i>	0.2500	0.2	0.19	0.1600	0.1300	0.1	0.0700	
<i>c</i>	0.7500	0.7800	0.8100	0.8400	0.8700	0.9000	0.9300	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.1538	1.1143	1.0800	1.0500	1.0235	1.0000	0.9789	

Table A1-2 The utility function of $c(\rho/r)$ by $1-\alpha$: consistent with saving and consumption as a base

Count backward

Case 1

ρ is the discount rate of the utility function and r is the rate of profit under convergence.

		ρ						
		0.05						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
		$1-\alpha$						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0466	1.0730	0.6975	0.7511	0.8048	0.8584	0.9121	0.9657	1.0194
0.0473	1.0576	0.6874	0.7403	0.7932	0.8461	0.8990	0.9519	1.0047
0.0483	1.0357	0.6732	0.7250	0.7767	0.8285	0.8803	0.9321	0.9839
0.0492	1.0166	0.6608	0.7116	0.7625	0.8133	0.8641	0.9150	0.9658
0.0500	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0507	0.9853	0.6404	0.6897	0.7390	0.7882	0.8375	0.8867	0.9360
0.0514	0.9722	0.6319	0.6805	0.7292	0.7778	0.8264	0.8750	0.9236
0.0521	0.9605	0.6243	0.6724	0.7204	0.7684	0.8164	0.8645	0.9125
0.0526	0.9500	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025

The z axis: for ρ/r . The above idea comes from both F.P. Ramsey [1928] and J Tinbergen [1956].

Count backward

Case 2

		ρ						
		0.1						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
		$1-\alpha$						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0917	1.0900	0.7085	0.7630	0.8175	0.8720	0.9265	0.9810	1.0355
0.0940	1.0635	0.6913	0.7444	0.7976	0.8508	0.9039	0.9571	1.0103
0.0962	1.0393	0.6755	0.7275	0.7795	0.8314	0.8834	0.9354	0.9873
0.0982	1.0183	0.6619	0.7128	0.7638	0.8147	0.8656	0.9165	0.9674
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.1016	0.9838	0.6395	0.6887	0.7379	0.7871	0.8363	0.8854	0.9346
0.1032	0.9694	0.6301	0.6786	0.7271	0.7756	0.8240	0.8725	0.9210
0.1045	0.9566	0.6218	0.6696	0.7174	0.7653	0.8131	0.8609	0.9088
0.1064	0.9400	0.6110	0.6580	0.7050	0.7520	0.7990	0.8460	0.8930

The z axis: for ρ/r .

Count backward

Case 3

		ρ						
		0.1						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
		$1-\alpha$						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0971	1.0300	0.6695	0.7210	0.7725	0.8240	0.8755	0.9270	0.9785
0.0985	1.0154	0.6600	0.7108	0.7615	0.8123	0.8631	0.9138	0.9646
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.1014	0.9867	0.6413	0.6907	0.7400	0.7893	0.8387	0.8880	0.9373
0.1026	0.9750	0.6338	0.6825	0.7313	0.7800	0.8288	0.8775	0.9263
0.1037	0.9647	0.6271	0.6753	0.7235	0.7718	0.8200	0.8682	0.9165
0.1047	0.9556	0.6211	0.6689	0.7167	0.7644	0.8122	0.8600	0.9078
0.1056	0.9474	0.6158	0.6632	0.7105	0.7579	0.8053	0.8526	0.9000
0.1067	0.9370	0.6091	0.6559	0.7028	0.7496	0.7965	0.8433	0.8902

The z axis: for ρ/r .

Count backward

Case 4

		ρ						
		0.1						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
		$1-\alpha$						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0833	1.2000	0.7800	0.8400	0.9000	0.9600	1.0200	1.0800	1.1400
0.0867	1.1538	0.7500	0.8077	0.8654	0.9231	0.9808	1.0385	1.0962
0.0897	1.1143	0.7243	0.7800	0.8357	0.8914	0.9471	1.0029	1.0586
0.0926	1.0800	0.7020	0.7560	0.8100	0.8640	0.9180	0.9720	1.0260
0.0952	1.0500	0.6825	0.7350	0.7875	0.8400	0.8925	0.9450	0.9975
0.0977	1.0235	0.6653	0.7165	0.7676	0.8188	0.8700	0.9212	0.9724
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.1022	0.9789	0.6363	0.6853	0.7342	0.7832	0.8321	0.8811	0.9300
0.1053	0.9500	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025

The z axis: for ρ/r .

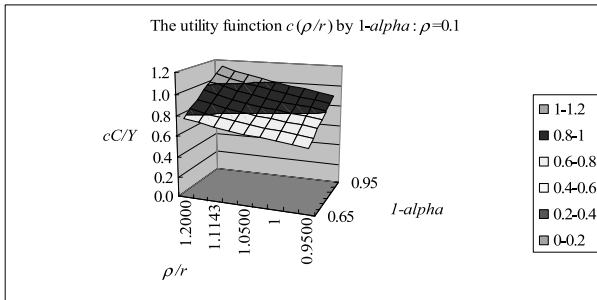
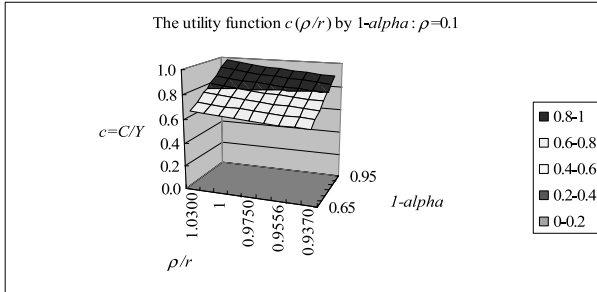
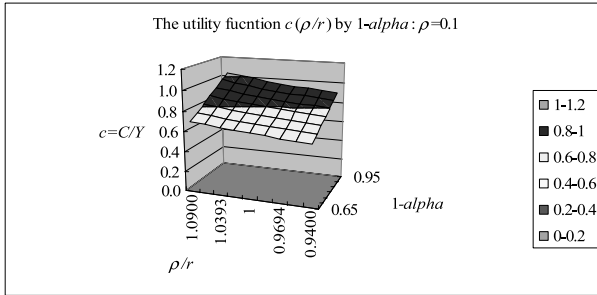
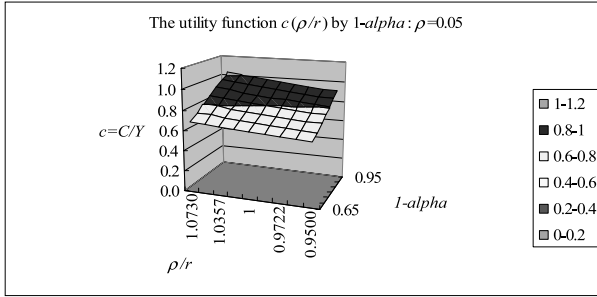


Figure A1-2 The utility function of $c(\rho/r)$ by $1-\alpha$

Table A1-3 The rate of saving dual to the utility function: consistent with saving and consumption as a base

Count backward

Case 1

ρ is the discount rate of the utility function and r is the rate of profit under convergence.

$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.

$r=\rho/(C/W)$	ρ							
	0.05	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0466	1.0730	0.3026	0.2489	0.1953	0.1416	0.0880	0.0343	-0.0194
0.0473	1.0576	0.3126	0.2597	0.2068	0.1539	0.1010	0.0481	-0.0047
0.0483	1.0357	0.3268	0.2750	0.2233	0.1715	0.1197	0.0679	0.0161
0.0492	1.0166	0.3392	0.2884	0.2375	0.1867	0.1359	0.0850	0.0342
0.0500	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.0507	0.9853	0.3596	0.3103	0.2610	0.2118	0.1625	0.1133	0.0640
0.0514	0.9722	0.3681	0.3195	0.2708	0.2222	0.1736	0.1250	0.0764
0.0521	0.9605	0.3757	0.3276	0.2796	0.2316	0.1836	0.1355	0.0875
0.0526	0.9500	0.3825	0.3350	0.2875	0.2400	0.1925	0.1450	0.0975

The z axis: for ρ/r .

Count backward

Case 2

$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.

$r=\rho/(C/W)$	ρ							
	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0917	1.0900	0.2915	0.2370	0.1825	0.1280	0.0735	0.0190	-0.0355
0.0940	1.0635	0.3088	0.2556	0.2024	0.1492	0.0961	0.0429	-0.0103
0.0962	1.0393	0.3245	0.2725	0.2205	0.1686	0.1166	0.0646	0.0127
0.0982	1.0183	0.3381	0.2872	0.2363	0.1853	0.1344	0.0835	0.0326
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.1016	0.9838	0.3605	0.3113	0.2621	0.2129	0.1638	0.1146	0.0654
0.1032	0.9694	0.3699	0.3214	0.2729	0.2244	0.1760	0.1275	0.0790
0.1045	0.9566	0.3782	0.3304	0.2826	0.2347	0.1869	0.1391	0.0913
0.1064	0.9400	0.3890	0.3420	0.2950	0.2480	0.2010	0.1540	0.1070

The z axis: for ρ/r .

Count backward

Case 3

$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.

$r=\rho/(C/W)$	ρ							
	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0971	1.0300	0.3305	0.2790	0.2275	0.1760	0.1245	0.0730	0.0215
0.0985	1.0154	0.3400	0.2892	0.2385	0.1877	0.1369	0.0862	0.0354
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.1014	0.9867	0.3587	0.3093	0.2600	0.2107	0.1613	0.1120	0.0627
0.1026	0.9750	0.3663	0.3175	0.2688	0.2200	0.1713	0.1225	0.0738
0.1037	0.9647	0.3729	0.3247	0.2765	0.2282	0.1800	0.1318	0.0835
0.1047	0.9556	0.3789	0.3311	0.2833	0.2356	0.1878	0.1400	0.0922
0.1056	0.9474	0.3842	0.3368	0.2895	0.2421	0.1947	0.1474	0.1000
0.1067	0.9370	0.3910	0.3441	0.2973	0.2504	0.2036	0.1567	0.1099

The z axis: for ρ/r .

Count backward

Case 4

$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.

$r=\rho/(C/W)$	ρ							
	0.1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0833	1.2000	0.2200	0.1600	0.1000	0.0400	-0.0200	-0.0800	-0.1400
0.0897	1.1143	0.2500	0.1923	0.1346	0.0769	0.0192	-0.0385	-0.0962
0.0926	1.0800	0.2757	0.2200	0.1643	0.1086	0.0529	-0.0029	-0.0586
0.0952	1.0500	0.2980	0.2440	0.1900	0.1360	0.0820	0.0280	-0.0260
0.0977	1.0235	0.3175	0.2650	0.2125	0.1600	0.1075	0.0550	0.0025
0.1000	1.0000	0.3347	0.2835	0.2324	0.1812	0.1300	0.0788	0.0276
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.1022	0.9789	0.3637	0.3147	0.2658	0.2168	0.1679	0.1189	0.0700
0.1053	0.9500	0.3825	0.3350	0.2875	0.2400	0.1925	0.1450	0.0975

The z axis: for ρ/r .

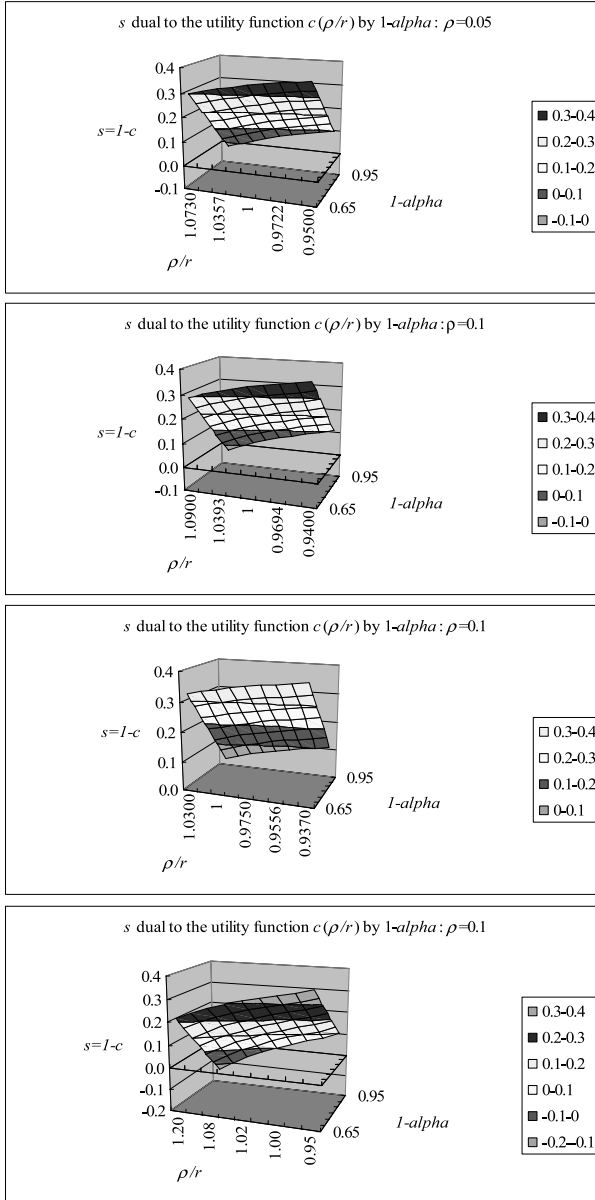


Figure A1-3 The rate of saving dual to the utility function of $c(\rho/r)$ by $1 - \alpha$

Table A2-1 Structure of saving and consumption as a base to be consistent with the utility function

Case 1	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	S_{SP} at $\alpha = s$
$\alpha_{SP/Y}$	0.1323	0.1350	0.1325	0.1300	0.0900	0.0600	0.0350	0.65
$\alpha_{D/Y}$	0.2177	0.1650	0.1175	0.0700	0.0600	0.0400	0.0150	S_{SD} at $\alpha = s$
$S_{SD/Y}$	0.0894	0.0660	0.0470	0.0300	0.0270	0.0180	0.0078	0.429
$C_{CD/Y}$	0.1284	0.0990	0.0705	0.0400	0.0330	0.0220	0.0072	
$W_{W/Y} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	S_{SW} at $\alpha = s$
$S_{SW/Y}$	0.0137	0.0212	0.0311	0.0400	0.0735	0.1038	0.1311	0.05
$C_{CW/Y}$	0.6364	0.6788	0.7190	0.7600	0.7765	0.7962	0.8189	
s	0.2353	0.2222	0.21	0.2	0.1905	0.1818	0.1739	
c	0.7647	0.7778	0.7895	0.8000	0.8095	0.8182	0.8261	$C_{CW/Y} + C_{CD/Y}$
$p/r = c/w_{W/Y}$	1.1765	1.1111	1.0526	1.0000	0.9524	0.9091	0.8696	
S_{SP}^m	0.3779	0.45	0.53	0.65	0.6	0.6	0.7	
S_{SD}^m	0.4104	0.4	0.4	0.429	0.45	0.45	0.52	
S_{SW}^m	0.021	0.0303	0.0414	0.05	0.0864	0.1153	0.138	
Case 2	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	S_{SP} at $\alpha = s$
$\alpha_{SP/Y}$	0.1323	0.1350	0.1325	0.1300	0.0900	0.0600	0.0350	0.45
$\alpha_{D/Y}$	0.217735	0.165	0.1175	0.07	0.06	0.04	0.015	S_{SD} at $\alpha = s$
$S_{SD/Y}$	0.0894	0.0660	0.0470	0.0300	0.0270	0.0180	0.0078	0.6
$C_{CD/Y}$	0.1284	0.0990	0.0705	0.0400	0.0330	0.0220	0.0072	
$W_{W/Y} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	S_{SW} at $\alpha = s$
$S_{SW/Y}$	0.0137	0.0212	0.0311	0.0400	0.0735	0.1038	0.1311	0.055
$C_{CW/Y}$	0.6364	0.6788	0.7190	0.7600	0.7765	0.7962	0.8189	
s	0.2353	0.2222	0.21	0.2	0.1905	0.1818	0.1739	
c	0.7647	0.7778	0.7895	0.8000	0.8095	0.8182	0.8261	$C_{CW/Y} + C_{CD/Y}$
$p/r = c/w_{W/Y}$	1.1765	1.1111	1.0526	1.0000	0.9524	0.9091	0.8696	
S_{SP}^m	0.3779	0.45	0.53	0.65	0.6	0.6	0.7	
S_{SD}^m	0.4104	0.4	0.4	0.429	0.45	0.45	0.52	
S_{SW}^m	0.021	0.0303	0.0414	0.05	0.0864	0.1153	0.138	
Case 3	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	S_{SP} at $\alpha = s$
$\alpha_{SP/Y}$	0.2100	0.1800	0.1500	0.1200	0.0900	0.0600	0.0300	0.5
$\alpha_{D/Y}$	0.14	0.12	0.1	0.08	0.06	0.04	0.02	S_{SD} at $\alpha = s$
$S_{SD/Y}$	0.0630	0.0540	0.0450	0.0360	0.0270	0.0180	0.0090	0.72
$C_{CD/Y}$	0.0770	0.0660	0.0550	0.0440	0.0330	0.0220	0.0110	
$W_{W/Y} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	S_{SW} at $\alpha = s$
$S_{SW/Y}$	0.0428	0.0660	0.0907	0.1167	0.1438	0.2020	0.2303	0.06
$C_{CW/Y}$	0.6072	0.6340	0.6593	0.6833	0.7062	0.6980	0.7197	
s	0.3158	0.3	0.29	0.2727	0.2608	0.2800	0.2693	
c	0.6842	0.7000	0.7143	0.7273	0.7392	0.7200	0.7307	$C_{CW/Y} + C_{CD/Y}$
$p/r = c/w_{W/Y}$	1.0526	1.0000	0.9524	0.9091	0.8696	0.8000	0.7692	
S_{SP}^m	0.6	0.6	0.6	0.6	0.6	0.6	0.6	
S_{SD}^m	0.45	0.45	0.45	0.45	0.45	0.45	0.45	
S_{SW}^m	0.0659	0.0943	0.1209	0.1459	0.1692	0.2244	0.2424	
Case 4	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	S_{SP} at $\alpha = s$
$\alpha_{SP/Y}$	0.0175	0.0300	0.0500	0.0400	0.0450	0.0400	0.0200	0.4
$\alpha_{D/Y}$	0.3325	0.27	0.2	0.16	0.105	0.06	0.03	S_{SD} at $\alpha = s$
$S_{SD/Y}$	0.0000	0.0270	0.0200	0.0240	0.0210	0.0210	0.0120	0.4
$C_{CD/Y}$	0.3325	0.2430	0.1800	0.1360	0.0840	0.0390	0.0180	
$W_{W/Y} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	S_{SW} at $\alpha = s$
$S_{SW/Y}$	0.1159	0.0680	0.0476	0.0471	0.0393	0.0390	0.0624	0.04
$C_{CW/Y}$	0.5341	0.6320	0.7024	0.7529	0.8107	0.8610	0.8876	
s	0.1334	0.1	0.12	0.111	0.1053	0.1	0.0944	
c	0.8666	0.8750	0.8824	0.8889	0.8947	0.9000	0.9056	$C_{CW/Y} + C_{CD/Y}$
$p/r = c/w_{W/Y}$	1.3333	1.2500	1.1765	1.1111	1.0526	1.0000	0.9533	
S_{SP}^m	0.05	0.1	0.2	0.2	0.3	0.4	0.4	
S_{SD}^m	0	0.1	0.1	0.15	0.2	0.35	0.4	
S_{SW}^m	0.1782	0.0971	0.0635	0.0589	0.0462	0.0433	0.0657	

Table A2-2 The utility function of $c(\rho/r)$ by $1-\alpha$: consistent with saving and consumption as a base

Case 1		ρ is the discount rate of the utility function & r is the rate of rental under convergence.						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0400	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
0.0425	1.1765	0.7647	0.8236	0.8824	0.9412	1.0000	1.0589	1.1177
0.0450	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0555
0.0475	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9473	1.0000
0.0500	1	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.0525	0.9524	0.6191	0.6667	0.7143	0.7619	0.8095	0.8572	0.9048
0.0550	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
0.0575	0.8696	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261
0.0602	0.8310	0.5402	0.5817	0.6233	0.6648	0.7064	0.7479	0.7895

The z axis: for ρ/r . The above idea comes from both F.P. Ramsey [1928] and J Tinbergen [1956].

Case 2		ρ						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0800	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
0.0850	1.1765	0.7647	0.8236	0.8824	0.9412	1.0000	1.0589	1.1177
0.0900	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0555
0.0950	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9473	1.0000
0.1000	1	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.1050	0.9524	0.6191	0.6667	0.7143	0.7619	0.8095	0.8572	0.9048
0.1100	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
0.1150	0.8696	0.5652	0.6087	0.6522	0.6957	0.7391	0.7826	0.8261
0.1203	0.8310	0.5402	0.5817	0.6233	0.6648	0.7064	0.7479	0.7895

The z axis: for ρ/r .

Case 3		ρ						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0900	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
0.0950	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9473	1.0000
0.1000	1	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.1050	0.9524	0.6191	0.6667	0.7143	0.7619	0.8095	0.8572	0.9048
0.1100	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
0.1150	0.8696	0.5652	0.6087	0.6522	0.6957	0.7392	0.7826	0.8261
0.1250	0.8000	0.5200	0.5600	0.6000	0.6400	0.6800	0.7200	0.7600
0.1300	0.7692	0.5000	0.5384	0.5769	0.6154	0.6538	0.6923	0.7307
0.1351	0.7400	0.4810	0.5180	0.5550	0.5920	0.6290	0.6660	0.7030

The z axis: for ρ/r .

Case 4		ρ						
		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0700	1.4286	0.9286	1.0000	1.0714	1.1429	1.2143	1.2857	1.3571
0.0750	1.3333	0.8666	0.9333	1.0000	1.0666	1.1333	1.2000	1.2666
0.0800	1.2500	0.8125	0.8750	0.9375	1.0000	1.0625	1.1250	1.1875
0.0850	1.1765	0.7647	0.8236	0.8824	0.9412	1.0000	1.0589	1.1177
0.0900	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0555
0.0950	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9473	1.0000
0.1000	1	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.1049	0.9533	0.6196	0.6673	0.7149	0.7626	0.8103	0.8579	0.9056
0.1053	0.9500	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025

The z axis: for ρ/r .

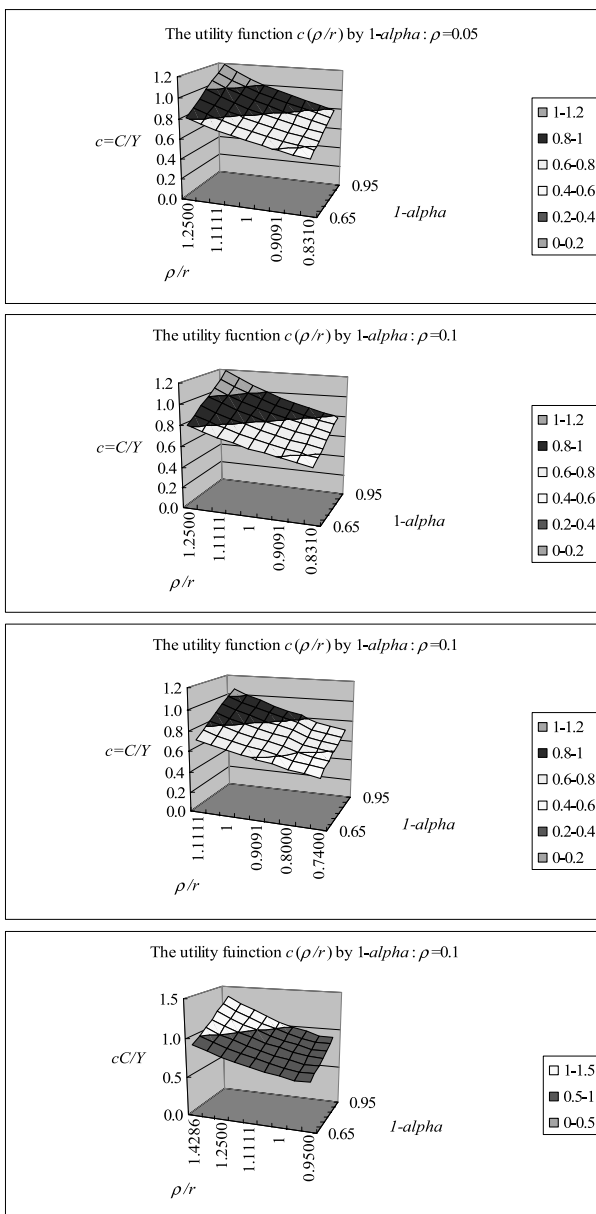


Figure A2-2 The utility function of $c(\rho/r)$ by $1-\alpha$

Table A2-3 The rate of saving dual to the utility function: consistent with saving and consumption as a base

Count backward

		ρ						
		0.05						
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
Case 1								$1-\alpha$
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0400	1.2500	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250	-0.1875
0.0425	1.1765	0.2353	0.1764	0.1176	0.0588	0.0000	-0.0589	-0.1177
0.0450	1.1111	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0555
0.0475	1.0526	0.3158	0.2632	0.2106	0.1579	0.1053	0.0527	0.0000
0.0500	1.0	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.0525	0.9524	0.3809	0.3333	0.2857	0.2381	0.1905	0.1428	0.0952
0.0550	0.9091	0.4091	0.3636	0.3182	0.2727	0.2273	0.1818	0.1364
0.0575	0.8696	0.4348	0.3913	0.3478	0.3043	0.2609	0.2174	0.1739
0.0602	0.8310	0.4599	0.4183	0.3768	0.3352	0.2937	0.2521	0.2106

The z axis: for ρ/r .

Count backward

		ρ						
		0.1						
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
Case 2								$1-\alpha$
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0800	1.2500	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250	-0.1875
0.0850	1.1765	0.2353	0.1764	0.1176	0.0588	0.0000	-0.0589	-0.1177
0.0900	1.1111	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0555
0.0950	1.0526	0.3158	0.2632	0.2106	0.1579	0.1053	0.0527	0.0000
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.1050	0.9524	0.3809	0.3333	0.2857	0.2381	0.1905	0.1428	0.0952
0.1100	0.9091	0.4091	0.3636	0.3182	0.2727	0.2273	0.1818	0.1364
0.1150	0.8696	0.4348	0.3913	0.3478	0.3043	0.2609	0.2174	0.1739
0.1203	0.8310	0.4599	0.4183	0.3768	0.3352	0.2937	0.2521	0.2106

The z axis: for ρ/r .

Count backward

		ρ						
		0.1						
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
Case 3								$1-\alpha$
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0900	1.1111	0.7222	0.7778	0.8333	0.8889	0.9444	1.0000	1.0556
0.0950	1.0526	0.6842	0.7368	0.7895	0.8421	0.8947	0.9473	1.0000
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.1050	0.9524	0.6191	0.6667	0.7143	0.7619	0.8095	0.8572	0.9048
0.1100	0.9091	0.5909	0.6364	0.6818	0.7273	0.7727	0.8182	0.8636
0.1150	0.8696	0.5652	0.6087	0.6522	0.6957	0.7392	0.7826	0.8261
0.1200	0.8000	0.5200	0.5600	0.6000	0.6400	0.6800	0.7200	0.7600
0.1300	0.7692	0.5000	0.5384	0.5769	0.6154	0.6538	0.6923	0.7307
0.1351	0.7400	0.4810	0.5180	0.5550	0.5920	0.6290	0.6660	0.7030

The z axis: for ρ/r .

Count backward

		ρ						
		0.1						
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						
Case 4								$1-\alpha$
$r=\rho/(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95
0.0700	1.4286	0.0714	0.0000	-0.0714	-0.1429	-0.2143	-0.2857	-0.3571
0.0750	1.3333	0.1334	0.0667	0.0000	-0.0666	-0.1333	-0.2000	-0.2666
0.0800	1.2500	0.1875	0.1250	0.0625	0.0000	-0.0625	-0.1250	-0.1875
0.0850	1.1765	0.2353	0.1765	0.1176	0.0588	0.0000	-0.0589	-0.1177
0.0900	1.1111	0.2778	0.2222	0.1667	0.1111	0.0556	0.0000	-0.0555
0.0950	1.0526	0.3158	0.2632	0.2106	0.1579	0.1053	0.0527	0.0000
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05
0.1049	0.9533	0.3804	0.3327	0.2851	0.2374	0.1897	0.1421	0.0944
0.1053	0.9500	0.3825	0.3350	0.2875	0.2400	0.1925	0.1450	0.0975

The z axis: for ρ/r .

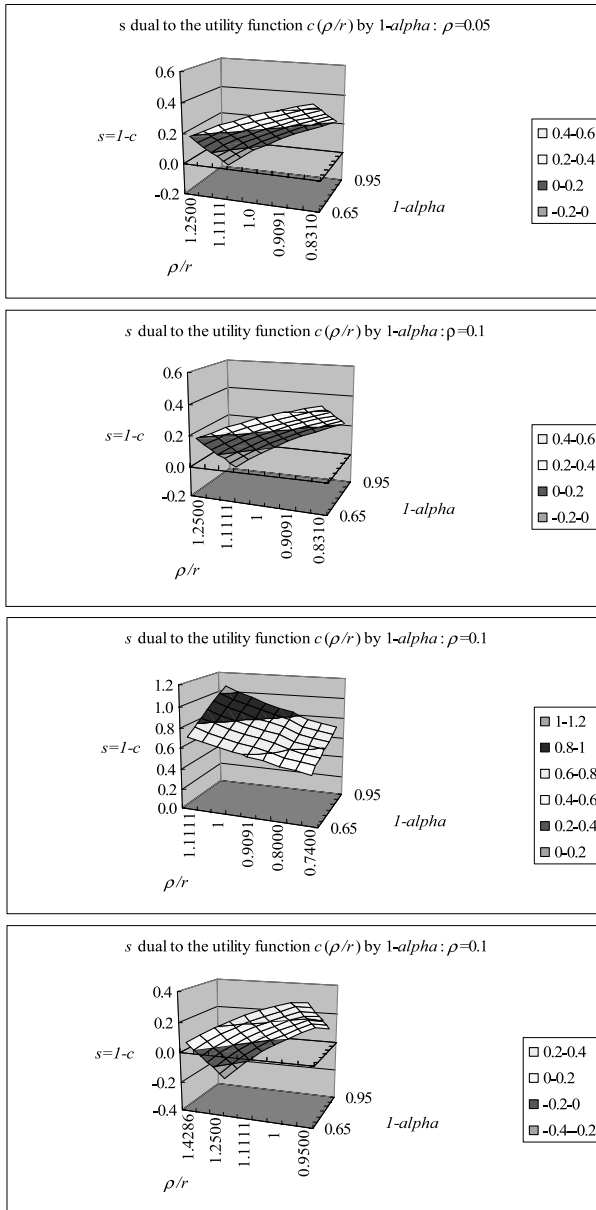


Figure A2-3 The rate of saving dual to the utility function of $c(\rho/r)$ by $1-\alpha$

Table A3-1 Structure of saving and consumption as a base to be consistent with the utility function

Case 1	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$s_{SP} =$
α_{SPY}	0.2275	0.195	0.1625	0.13	0.0975	0.065	0.0325	0.65
α_{DY}	0.1225	0.105	0.0875	0.07	0.0525	0.035	0.0175	$s_{SD} =$
s_{SDY}	0.0526	0.0450	0.0375	0.0300	0.0225	0.0150	0.0075	0.429
c_{CDY}	0.0699	0.0600	0.0500	0.0400	0.0300	0.0200	0.0100	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$s_{SW} =$
s_{SWY}	0.0325	0.0350	0.0375	0.0400	0.0425	0.0450	0.0475	0.05
c_{CWY}	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025	
s	0.3126	0.2750	0.24	0.2	0.1625	0.1250	0.0875	
c	0.6874	0.7250	0.7625	0.8000	0.8375	0.8750	0.9125	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0576	1.0357	1.0166	1.0000	0.9853	0.9722	0.9605	

Case 2	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$s_{SP} =$
α_{SPY}	0.1575	0.135	0.1125	0.09	0.0675	0.045	0.0225	0.45
α_{DY}	0.1925	0.165	0.1375	0.11	0.0825	0.055	0.0275	$s_{SD} =$
s_{SDY}	0.1155	0.0990	0.0825	0.0660	0.0495	0.0330	0.0165	0.6
c_{CDY}	0.0770	0.0660	0.0550	0.0440	0.0330	0.0220	0.0110	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$s_{SW} =$
s_{SWY}	0.0358	0.0385	0.0413	0.0440	0.0468	0.0495	0.0523	0.055
c_{CWY}	0.6143	0.6615	0.7088	0.7560	0.8033	0.8505	0.8978	
s	0.3088	0.2725	0.24	0.2	0.1638	0.1275	0.0913	
c	0.6913	0.7275	0.7638	0.8000	0.8363	0.8725	0.9088	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0635	1.0393	1.0183	1.0000	0.9838	0.9694	0.9566	

Case 3	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$s_{SP} =$
α_{SPY}	0.175	0.15	0.125	0.1	0.075	0.05	0.025	0.5
α_{DY}	0.175	0.15	0.125	0.1	0.075	0.05	0.025	$s_{SD} =$
s_{SDY}	0.1260	0.1080	0.0900	0.0720	0.0540	0.0360	0.0180	0.72
c_{CDY}	0.0490	0.0420	0.0350	0.0280	0.0210	0.0140	0.0070	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$s_{SW} =$
s_{SWY}	0.0390	0.0420	0.0450	0.0480	0.0510	0.0540	0.0570	0.06
c_{CWY}	0.6110	0.6580	0.7050	0.7520	0.7990	0.8460	0.8930	
s	0.3400	0.3	0.26	0.2200	0.1800	0.1400	0.1000	
c	0.6600	0.7000	0.7400	0.7800	0.8200	0.8600	0.9000	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.0154	1.0000	0.9867	0.9750	0.9647	0.9556	0.9474	

Case 4	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1-alpha
alpha	0.35	0.3	0.25	0.2	0.15	0.1	0.05	$s_{SP} =$
α_{SPY}	0.14	0.12	0.1	0.08	0.06	0.04	0.02	0.4
α_{DY}	0.21	0.18	0.15	0.12	0.09	0.06	0.03	$s_{SD} =$
s_{SDY}	0.0840	0.0720	0.0600	0.0480	0.0360	0.0240	0.0120	0.4
c_{CDY}	0.1260	0.1080	0.0900	0.0720	0.0540	0.0360	0.0180	
$w_{WY} = 1 - \alpha$	0.65	0.7	0.75	0.8	0.85	0.9	0.95	$s_{SW} =$
s_{SWY}	0.0260	0.0280	0.0300	0.0320	0.0340	0.0360	0.0380	0.04
c_{CWY}	0.6240	0.6720	0.7200	0.7680	0.8160	0.8640	0.9120	
s	0.2500	0.2	0.19	0.1600	0.1300	0.1	0.0700	
c	0.7500	0.7800	0.8100	0.8400	0.8700	0.9000	0.9300	$c_{CWY} + c_{CDY}$
$\rho/r = c/w_{WY}$	1.1538	1.1143	1.0800	1.0500	1.0235	1.0000	0.9789	

Table A3-2 The utility function of $c(\rho/r)$ by $1-\alpha$: consistent with saving and consumption as a base

Count backward

		r		ρ is the discount rate of the utility function & r is the rate of rental under convergence.						
		0.05		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						$1-\alpha$
Case 1	$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.0537	1.0730	0.6975	0.7511	0.8048	0.8584	0.9121	0.9657	1.0194		
0.0529	1.0576	0.6874	0.7403	0.7932	0.8461	0.8990	0.9519	1.0047		
0.0518	1.0357	0.6732	0.7250	0.7767	0.8285	0.8803	0.9321	0.9839		
0.0508	1.0166	0.6608	0.7116	0.7625	0.8133	0.8641	0.9150	0.9658		
0.0500	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95		
0.0493	0.9853	0.6404	0.6897	0.7390	0.7882	0.8375	0.8867	0.9360		
0.0486	0.9722	0.6319	0.6805	0.7292	0.7778	0.8264	0.8750	0.9236		
0.0480	0.9605	0.6243	0.6724	0.7204	0.7684	0.8164	0.8645	0.9125		
0.0475	0.9500	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025		

The z axis: for ρ/r . The above idea comes from both F.P. Ramsey [1928] and J Tinbergen [1956].

Count backward

		r		ρ is the discount rate of the utility function & r is the rate of rental under convergence.						
		0.1		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						$1-\alpha$
Case 2	$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1090	1.0900	0.7085	0.7630	0.8175	0.8720	0.9265	0.9810	1.0355		
0.1063	1.0635	0.6913	0.7444	0.7976	0.8508	0.9039	0.9571	1.0103		
0.1039	1.0393	0.6755	0.7275	0.7795	0.8314	0.8834	0.9354	0.9873		
0.1018	1.0183	0.6619	0.7128	0.7638	0.8147	0.8656	0.9165	0.9674		
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95		
0.0984	0.9838	0.6395	0.6887	0.7379	0.7871	0.8363	0.8854	0.9346		
0.0969	0.9694	0.6301	0.6786	0.7271	0.7756	0.8240	0.8725	0.9210		
0.0957	0.9566	0.6218	0.6696	0.7174	0.7653	0.8131	0.8609	0.9088		
0.0940	0.9400	0.6110	0.6580	0.7050	0.7520	0.7990	0.8460	0.8930		

The z axis: for ρ/r .

Count backward

		r		ρ is the discount rate of the utility function & r is the rate of rental under convergence.						
		0.1		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						$1-\alpha$
Case 3	$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1030	1.0300	0.6695	0.7210	0.7725	0.8240	0.8755	0.9270	0.9785		
0.1015	1.0154	0.6600	0.7108	0.7615	0.8123	0.8631	0.9138	0.9646		
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95		
0.0987	0.9867	0.6413	0.6907	0.7400	0.7893	0.8387	0.8880	0.9373		
0.0975	0.9750	0.6338	0.6825	0.7313	0.7800	0.8288	0.8775	0.9263		
0.0965	0.9647	0.6271	0.6753	0.7235	0.7718	0.8200	0.8682	0.9165		
0.0956	0.9556	0.6211	0.6689	0.7167	0.7644	0.8122	0.8600	0.9078		
0.0947	0.9474	0.6158	0.6632	0.7105	0.7579	0.8053	0.8526	0.9000		
0.0937	0.9370	0.6091	0.6559	0.7028	0.7496	0.7965	0.8433	0.8902		

The z axis: for ρ/r .

Count backward

		r		ρ is the discount rate of the utility function & r is the rate of rental under convergence.						
		0.1		$c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$.						$1-\alpha$
Case 4	$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1200	1.2000	0.7800	0.8400	0.9000	0.9600	1.0200	1.0800	1.1400		
0.1154	1.1538	0.7500	0.8077	0.8654	0.9231	0.9808	1.0385	1.0962		
0.1114	1.1143	0.7243	0.7800	0.8357	0.8914	0.9471	1.0029	1.0586		
0.1080	1.0800	0.7020	0.7560	0.8100	0.8640	0.9180	0.9720	1.0260		
0.1050	1.0500	0.6825	0.7350	0.7875	0.8400	0.8925	0.9450	0.9975		
0.1024	1.0235	0.6653	0.7165	0.7676	0.8188	0.8700	0.9212	0.9724		
0.1000	1	0.65	0.7	0.75	0.8	0.85	0.9	0.95		
0.0979	0.9789	0.6363	0.6853	0.7342	0.7832	0.8321	0.8811	0.9300		
0.0950	0.9500	0.6175	0.6650	0.7125	0.7600	0.8075	0.8550	0.9025		

The z axis: for ρ/r .

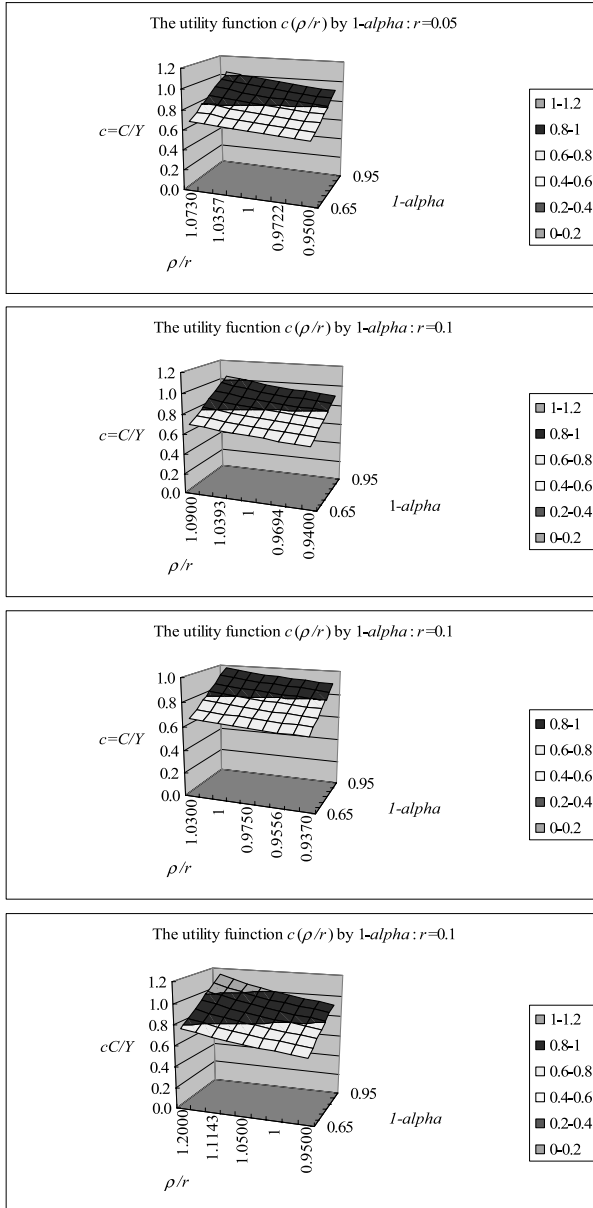


Figure A3-2 The utility function of $c(\rho/r)$ by $1-\alpha$

Table A3-3 The rate of saving dual to the utility function: consistent with saving and consumption as a base

Count backward

		<i>r</i>							
		0.05							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$							
		$1-\alpha$							
$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.0537	1.0730	0.3026	0.2489	0.1953	0.1416	0.0880	0.0343	-0.0194	
0.0529	1.0576	0.3126	0.2597	0.2068	0.1539	0.1010	0.0481	-0.0047	
0.0518	1.0357	0.3268	0.2750	0.2233	0.1715	0.1197	0.0679	0.0161	
0.0508	1.0166	0.3392	0.2884	0.2375	0.1867	0.1359	0.0850	0.0342	
0.0500	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.0493	0.9853	0.3596	0.3103	0.2610	0.2118	0.1625	0.1133	0.0640	
0.0486	0.9722	0.3681	0.3195	0.2708	0.2222	0.1736	0.1250	0.0764	
0.0480	0.9605	0.3757	0.3276	0.2796	0.2316	0.1836	0.1355	0.0875	
0.0475	0.9500	0.3825	0.3350	0.2875	0.2400	0.1925	0.1450	0.0975	

The z axis: for ρ/r .

Count backward

		<i>r</i>							
		0.1							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$							
		$1-\alpha$							
$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1090	1.0900	0.2915	0.2370	0.1825	0.1280	0.0735	0.0190	-0.0355	
0.1063	1.0635	0.3088	0.2556	0.2024	0.1492	0.0961	0.0429	-0.0103	
0.1039	1.0393	0.3245	0.2725	0.2205	0.1686	0.1166	0.0646	0.0127	
0.1018	1.0183	0.3381	0.2872	0.2363	0.1853	0.1344	0.0835	0.0326	
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.0984	0.9838	0.3605	0.3113	0.2621	0.2129	0.1638	0.1146	0.0654	
0.0969	0.9694	0.3699	0.3214	0.2729	0.2244	0.1760	0.1275	0.0790	
0.0957	0.9566	0.3782	0.3304	0.2826	0.2347	0.1869	0.1391	0.0913	
0.0940	0.9400	0.3890	0.3420	0.2950	0.2480	0.2010	0.1540	0.1070	

The z axis: for ρ/r .

Count backward

		<i>r</i>							
		0.1							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$							
		$1-\alpha$							
$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1030	1.0300	0.3305	0.2790	0.2275	0.1760	0.1245	0.0730	0.0215	
0.1015	1.0154	0.3400	0.2892	0.2385	0.1877	0.1369	0.0862	0.0354	
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.0987	0.9867	0.3587	0.3093	0.2600	0.2107	0.1613	0.1120	0.0627	
0.0975	0.9750	0.3663	0.3175	0.2688	0.2200	0.1713	0.1225	0.0738	
0.0965	0.9647	0.3729	0.3247	0.2765	0.2282	0.1800	0.1318	0.0835	
0.0956	0.9556	0.3789	0.3311	0.2833	0.2356	0.1878	0.1400	0.0922	
0.0947	0.9474	0.3842	0.3368	0.2895	0.2421	0.1947	0.1474	0.1000	
0.0937	0.9370	0.3910	0.3441	0.2973	0.2504	0.2036	0.1567	0.1099	

The z axis: for ρ/r .

Count backward

		<i>r</i>							
		0.1							
		$s=1-c=(\rho/r)(1-\alpha)$, where $\rho/r=C/W$							
		$1-\alpha$							
$\rho=r(C/W)$	ρ/r	0.65	0.7	0.75	0.8	0.85	0.9	0.95	
0.1200	1.2000	0.2200	0.1600	0.1000	0.0400	-0.0200	-0.0800	-0.1400	
0.1154	1.1143	0.2500	0.1923	0.1346	0.0769	0.0192	-0.0385	-0.0962	
0.1114	1.0800	0.2757	0.2200	0.1643	0.1086	0.0529	-0.0029	-0.0586	
0.1080	1.0500	0.2980	0.2440	0.1900	0.1360	0.0820	0.0280	-0.0260	
0.1050	1.0235	0.3175	0.2650	0.2125	0.1600	0.1075	0.0550	0.0025	
0.1024	1.0000	0.3347	0.2835	0.2324	0.1812	0.1300	0.0788	0.0276	
0.1000	1	0.35	0.3	0.25	0.2	0.15	0.1	0.05	
0.0979	0.9789	0.3637	0.3147	0.2658	0.2168	0.1679	0.1189	0.0700	
0.0950	0.9500	0.3825	0.3350	0.2875	0.2400	0.1925	0.1450	0.0975	

The z axis: for ρ/r .

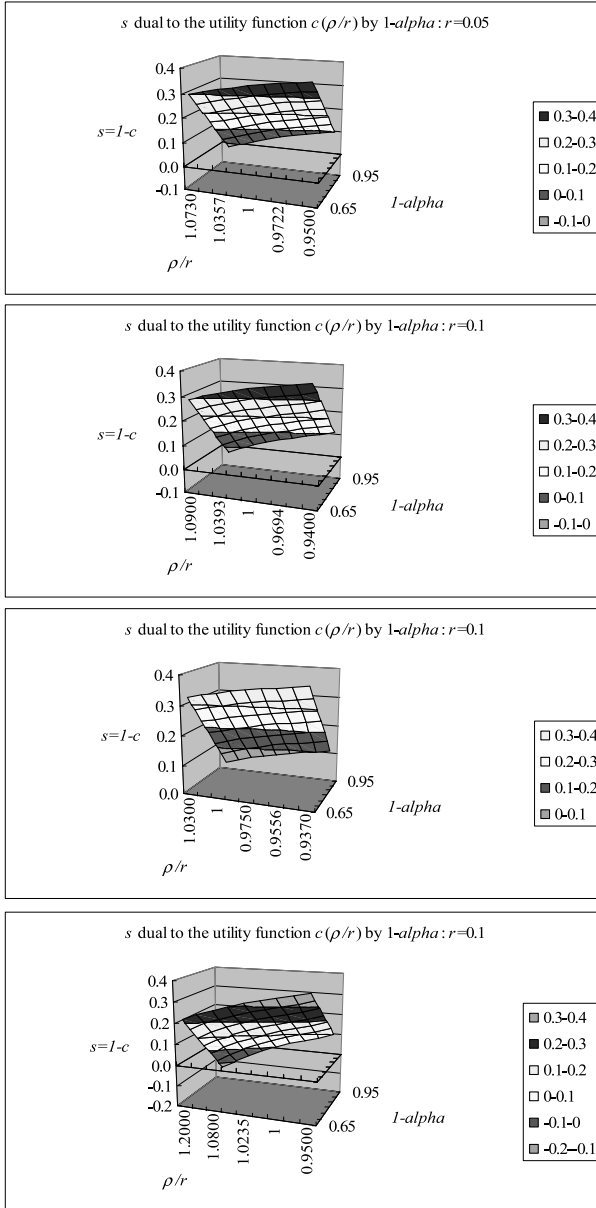


Figure A3-3 The rate of saving dual to the utility function of $c(\rho/r)$ by $1-\alpha$

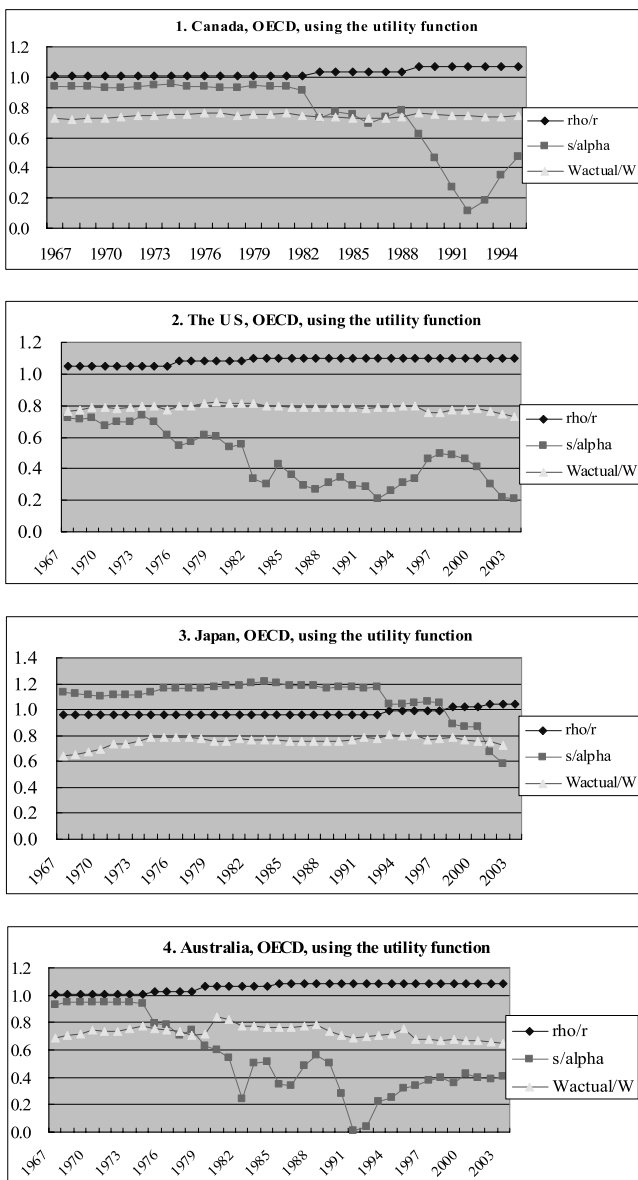


Figure A4-1 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

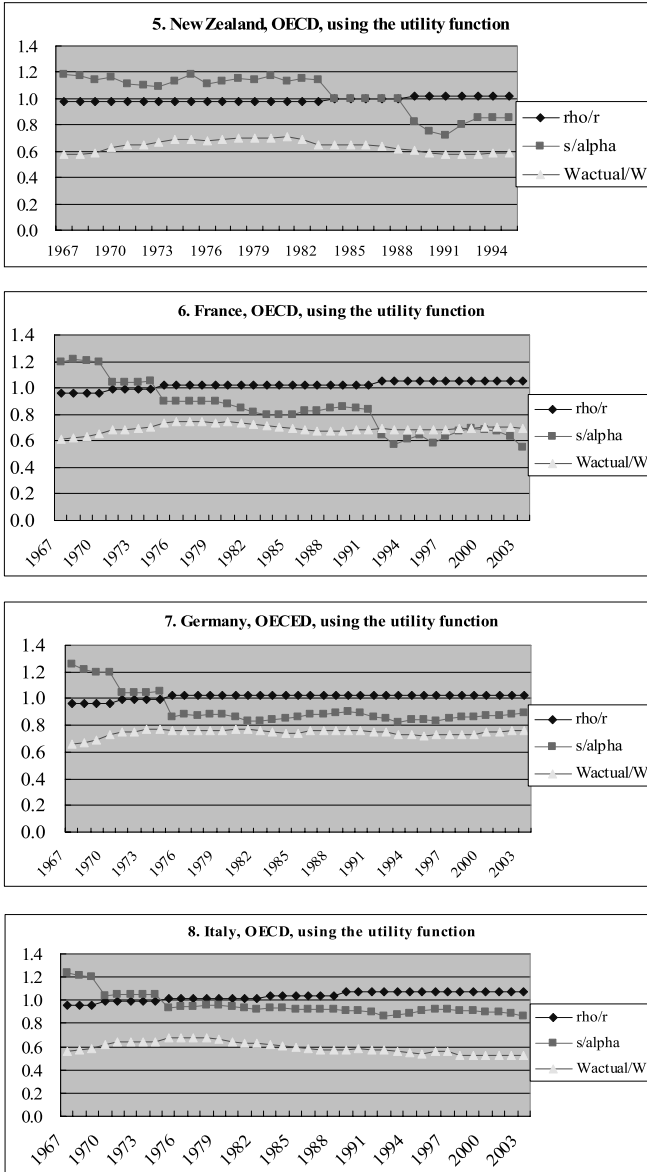


Figure A4-2 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

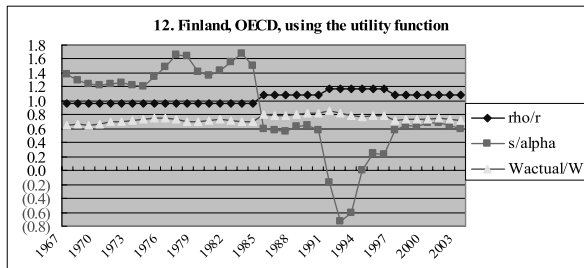
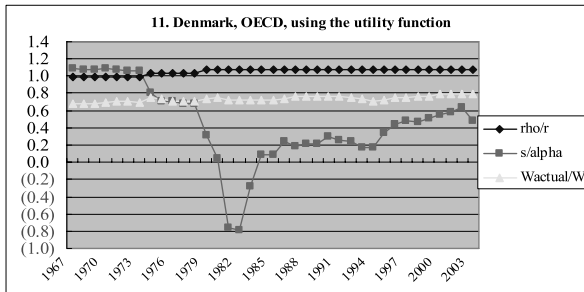
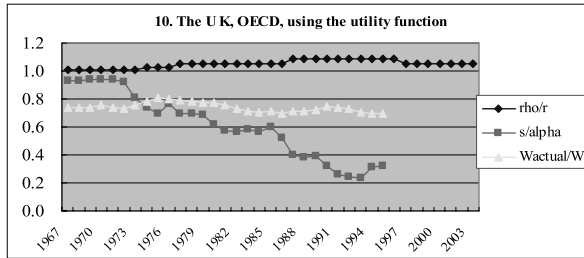
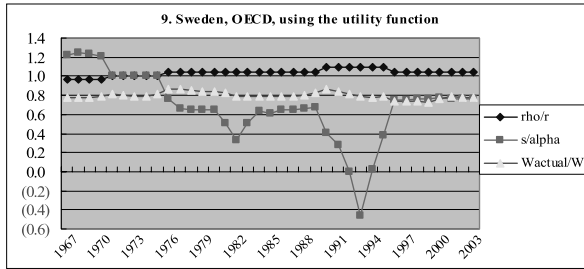
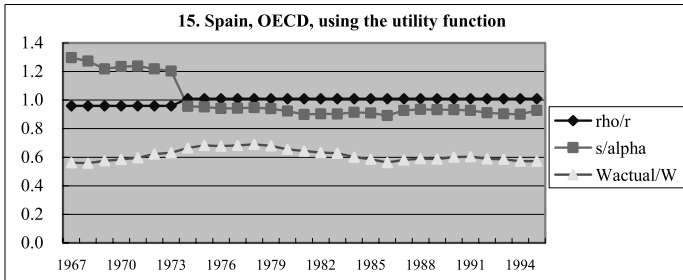
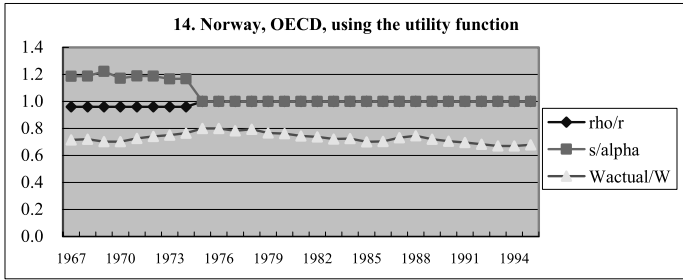
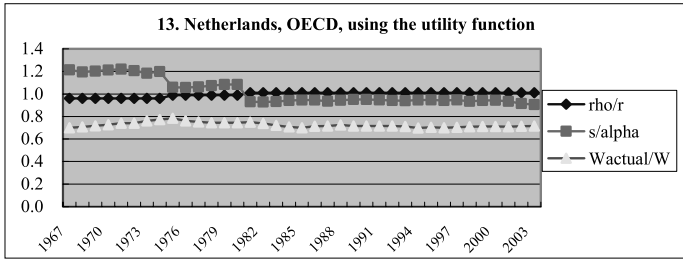


Figure A4-3 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data



Notes:

Data source: OECD National Statistics by year and my estimation of (ρ/r) by year and country.

OECD data are GDP, actual wages, national disposable income, and consumption.

In some countries, national disposable income and/or actual wages are not available after 1995.

As the result, in these countries the ratio of saving to $\bar{\alpha}$ and the ratio of actual wages in GDP to estimated compensation/wages in NDI cannot be shown in figures.

In several countries, the ratio of saving to $\bar{\alpha}$ fluctuates due to budget deficits, but this ratio will recover from critical situations in the long-term by taking appropriate policies.

Figure A4-4 Utility coefficient, ρ/r , and the ratio of saving to the relative share of rental using OECD data

