Changes in Elasticity of Substitution *sigma* in KEWT 3.09, with Seven Vital Aspects: 58 Countries and Three Areas by Sector, 1990–2007

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This paper, using the endogenous-data of KEWT 3.09 by country, focuses on how the endogenous wage rate, $w = W / L$, changes with the rate of return by year (or, how the marginal rate of substitution, $MRS = \frac{\Delta(r/w)}{r/w}$, changes), against the change in the capital-labor ratio, i.e., how the elasticity of substitution *sigma*, $\sigma = \frac{\Delta k / k}{\Delta(r/w)/\Delta(r/w)}$, changes. Without the change in *sigma*, the author's 'endogenous' does not exist. In the endogenous-equilibrium, price level is shown by the 'relative' price = 1.0 by year, but independently between two years. The price-equilibrium in the literature holds with actual prices *P* by year, but it is difficult to measure *P*. In both equilibriums, therefore, the differences of prices between two years must be commonly estimated by using external consumers' price index, *CPI*, or the similar.

The conventional Cobb-Douglas production function is based on the price-equilibrium and holds by year. However, it is unable to measure the elasticity of substitution consistently over years. The endogenous Cobb-Douglas production function, on the other hand, is based on the endogenous-equilibrium, where the endogenous-data are consistent over years so that the elasticity of substitution is measured consistently over years. In short, the elasticity of substitution changes by year in accordance with the endogenous-equilibrium. Assume that the price-equilibrium equals the endogenous-equilibrium in the long run. Even in this case, price changes between two years must depend on external indicators such as *CPI*.

The endogenous-data also shows the transitional path at one fiscal year by using endogenous recursive programming. In this recursive programming, the relative price is also 1.0, where $P \cdot Y = r \cdot K + w \cdot L$ and $p = 1.0 = P \cdot Y / Y$. Then, the elasticity of substitution remains 1.0, continuously by time/year (for the proof, see Figure 21 at the end of this paper): the rate of change in the capital-labor ratio $k = K / L$ is negatively equal to the rate of change in the *MRS*.

The author's endogenous model is based on the (conventional) Cobb-Douglas production function under constant returns to scale. This Cobb-Douglas production function, however, has hidden parameters such as *beta*, *delta*, and *lambda*. '1 − *beta**' determines a base for the rate of technological progress, as the ratio of qualitative investment to total investment: $g_A^* = i_A^* = i\,(1 - \beta^*)$. The coefficient of diminishing returns, *delta*, is measured using the capital-output ratio $\Omega^* = K / Y$ and $B = (1 - \beta^*) / \beta^*$: $\delta_0 = 1 - \frac{LN(1/\Omega^*)}{LN(B^*)}$. The endogenous speed of convergence, $1 / \lambda^*$, is: $\frac{1}{\lambda^*} = \frac{1}{(1-\alpha)n + (1-\delta_0)g_A^*}$ while externally, $(1 - \alpha)\,(n + g_{A(EXTERNA)})$ was earlier formulated for the denominator (see, e.g., Barro and Sala-i-Martin, 1989). Using these endogenous parameters, data and model are consistent with each other by year and over years.

This paper, before showing *sigma* and *MRS*, illustrates six most influential aspects of the endogenous-data. These aspects are: endogenous taxes and given deficit related to equilibrium (S1 in Figures S1 to S12, below), government deflation rate related to huge deficit (S2), the difference between the growth rate of the current technology level and the endogenous rate of technological progress (S3), *alpha* hyperbolic curve convex to downwards, which is the product of the capital-output ratio and the rate of return at convergence (S4), the government sector's *alpha*, similarly to S4, but rigorously reflecting huge deficit (S5), and the private sector's national taste as the ratio of the discount rate of consumption goods to that of saving/capital goods (S6).

The above six aspects overwhelmingly imply that budget surplus/deficit determines the economic power by country and under different national taste (see S6). And, a country maintains sustainability with environmental 'technology and investment', which strengthen the endogenous-data most consistently. Improvement in 'eco and green' is not costs but the everlasting promoter to maintaining better endogenous equilibrium. Typically, if the capital-output ratio goes up due to government deficit and its investment as in Japan (see S5), it damages the base for economic growth miserably. One more, throughout countries, the relationship between growth and inequality becomes robust by appropriate policies taken by country, as strongly suggested by S6 (Figure S11 and 12). The shift from capital investment to human investment in education and welfare helps an economy to stop inequality as a whole.

$MRS = \frac{\Delta(r/w)}{r/w}$ and $\sigma = \frac{\Delta k/k}{\Delta(r/w)/\Delta(r/w)}$ integrate the above six aspects although each aspect is interrelated to each other. The endogenous model is never 'partial' and as a result, guarantees full employment with a low inflation rate. Endogenous equilibrium is a base for full employment. This implies that deficit is a vital clue to determine the future of an economy. Figures in this paper remind past policies in 1990/1995 to 2007 by country. Each country has taken different policies and the results differ surprisingly. Nevertheless, numerical system/mechanism is common to each country. National taste still holds by country: National taste of the total economy uses a common taste function to its propensity to consume. Government neutral taste uses the same taste function to its propensity to consume. As a result, private taste function to its propensity to consume differs surprisingly. National taste coexists with culture and system.

Results reflect philosophy, culture, and will of policy-makers. When the lines and curves of the above seven aspects (including *sigma* and *MRS*) are simple, smooth, and robust, these imply that endogenous equilibrium holds without extra manipulation. When these lines and curves are rather complicated and irregular, endogenous equilibrium holds but with changing policies in the short run and/or instability of political situations. In other words, if economic, fiscal, and financial policies are consistent with each other in the long run, sustainability of an economy is strengthened, regardless of whether government is apparently democratic or not. The target of sustainability is the welfare of people including next generations. Even in a democratic society, if its level is selfish, group-oriented, and short-sighted, the level of democracy is inferior to an economy whose philosophical target is steady and robust. Figures of seven aspects give readers a real image of democratic level by country.

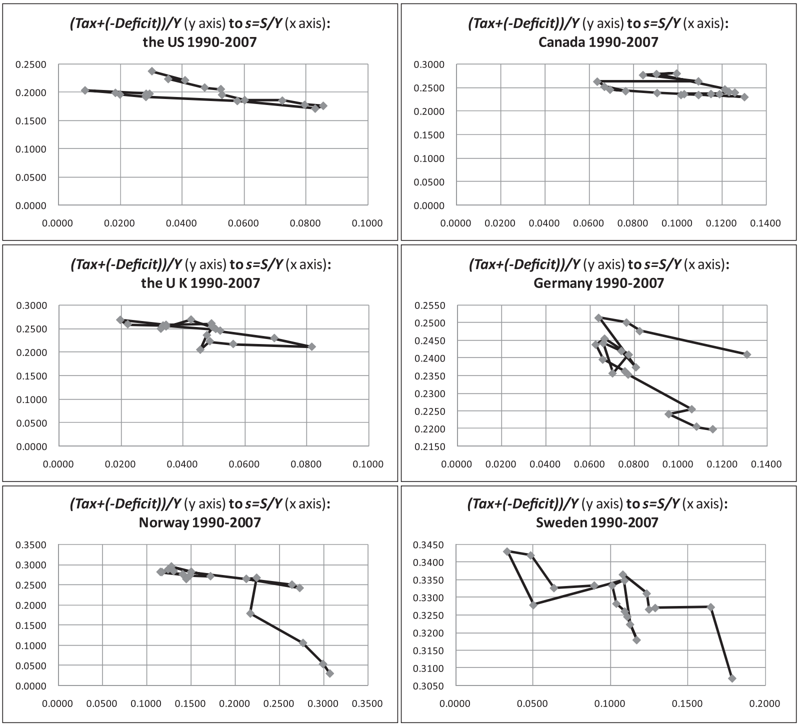
Finally, the author stresses, it is required for policy-makers to publish rough results obtained from such as the above seven aspects with clear-cut explanations by year: First endogenous results, second, disclosures and explanations, and third understanding of people’s will in accordance with each own culture. Global cooperation will be strengthened towards sustainable economy by country.

Questions and aspects	From usual C-D producticn function	To 'endogenous' C-D production fucntion
In an open economy		
1. Common base:	Constant returns to scale.	Constant returns to scale.
	National accounts in 'continuous.'	National accounts in 'discrete' and no differntial.
$Y = AK^{\alpha}L^{1-\alpha}$ $y = AK^{\alpha}$	Price-equilibrium by year, with P.	Endogenous-equilibirum by year, with $P = 1.0$.
$A = k^{1-\alpha} / \Omega$ (2003, Note 11).	(Between two years, there is no measure of price differences).	(Between two years, there is no measure of price differences).
	There is no excessive profit.	There is no excessive profit.
	Relative share of capital α is used.	Relative share of capital α is explicitly used.
2. Essential difference:	Capital K & the rate of return r is independent, using statistics-data.	Capital K & the rate of return are consistent in a whole mumerical system, endogenous-data.
Partial versus general.	($Y = C + S = W + \Pi$ is not strict).	($Y = C + S = W + \Pi$ is an essential prerequisite).
Short versus long over years.	No implicit parameters are used.	Three basic endogenous parameters are used*.
	Rate of tech.progress is given.	Rate of tech.progress is endogenously measured.
	In the long run, full-employment under the growth rate of population.	The level of under-, full-, and over-employment is enodgenously measured by compaing the actual growth rate of population with endogenous.
	The speed of convergence is used, externally under constant returns to capital.	The endogenous speed of convergence is used, under diminishing returns to capital, *delta* $> \alpha$, and under increasing returns to capital, *delta* $< \alpha$.

*: endogenous parameters: $\beta^* = \dfrac{\Omega^*\{n(1-\alpha)+i(1+n)\}}{i(1-\alpha)+\Omega^*\cdot(1+n)}$ $\delta_0 = 1 - \dfrac{LN(1/\Omega^*)}{LN(B^*)}$ $\lambda^* = (1-\alpha)\,n + (1+\delta_0)\,g_A^*$

Note: At the C-D production function, capital K and labor L are stocks, where each stock and flow are consistent.In the discrete case, the endogenous rate of tech. progress, $g^*_{A\,(flow)}$, is independent of the current technology, $g_{A(STOCK)}$; since the relative price level p is 1.0 by fiscal year, but the diference of price level P between two years is unknown.

Endogenous taxes (1)



Data source: KEWT 3.09 of fifty-eight countries and three areas on average by sector, 1990–2007, whose ten original data come from *International Financial Statistics Yearbook*, IMF. (hereafter, abbreviated)

Figure S1 Endogenous taxes, deficit, and the propensity to save: the US, Canada, the UK, Germany, Norway, and Sweden

Endogenous taxes (2)

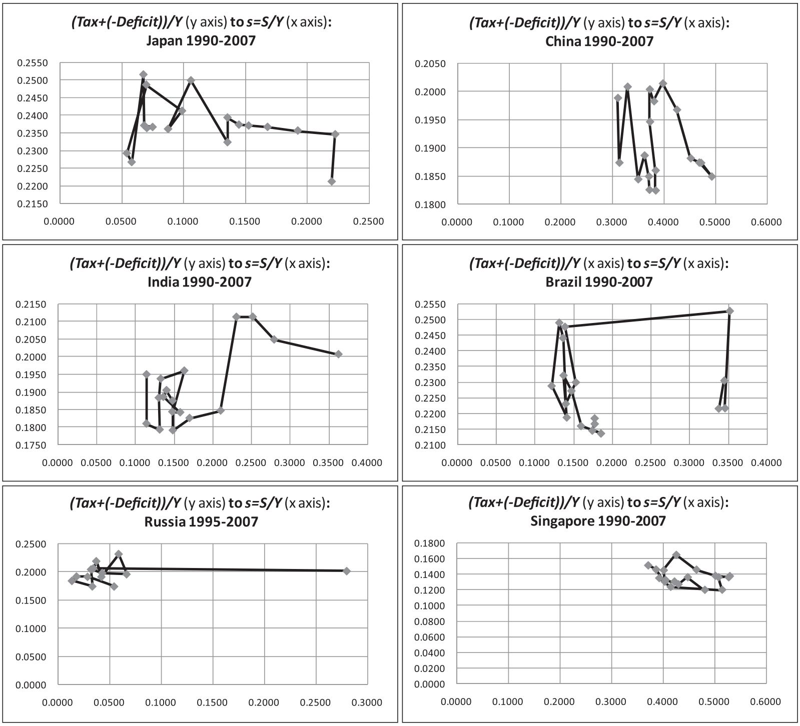


Figure S2 Endogenous taxes, deficit, and the propensity to save: Japan, China, India, Brazil, Russia, and Singapore

Government rate of inflation (1)

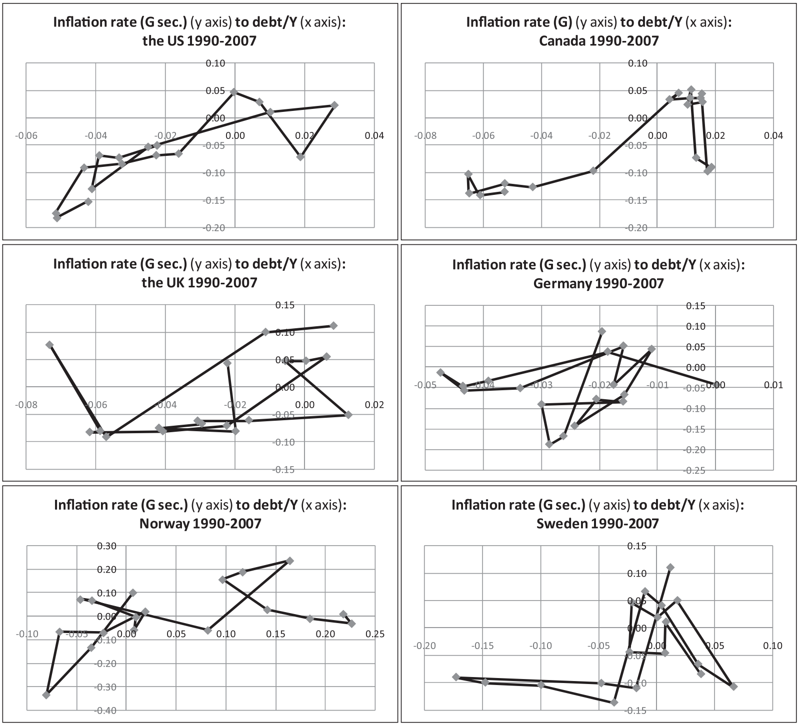


Figure S3 Government rate of inflation and the ratio of debt to output: the US, Canada, the UK, Germany, Norway, and Sweden

Government rate of inflation (2)

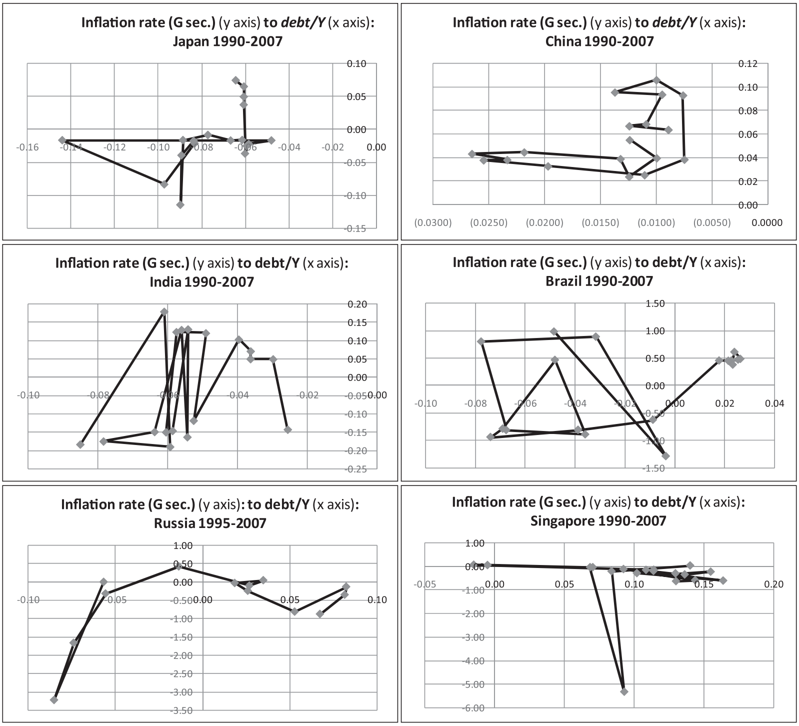


Figure S4 Government rate of inflation and the ratio of debt to output: Japan, China, India, Brazil, Russia, and Singapore

Technological progress: stock and flow (1)

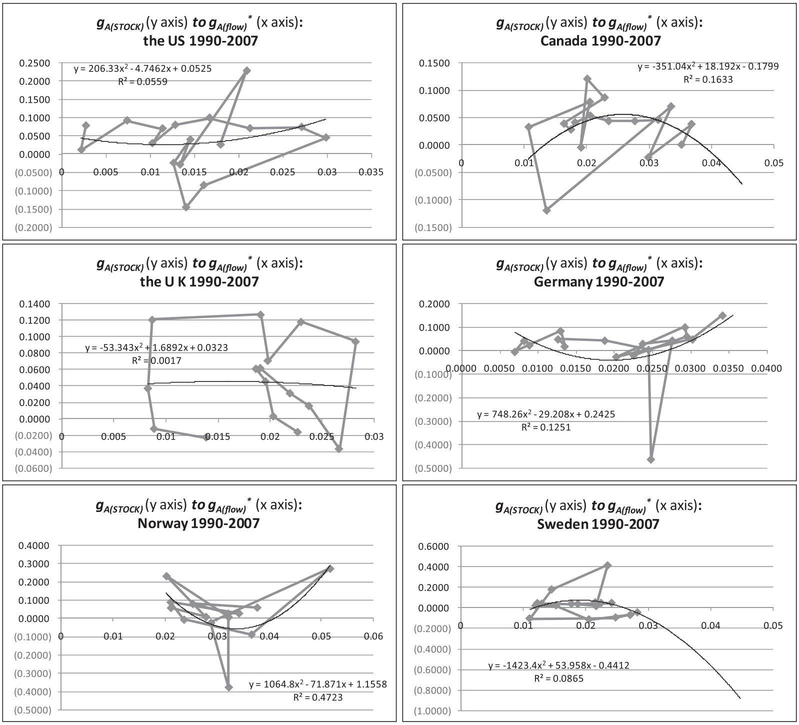


Figure S5 Growth rates of the current level of technology and the rate of tech. progress at convergence: the US, Canada, the UK, Germany, Norway, and Sweden

Technological progress: stock and flow (2)

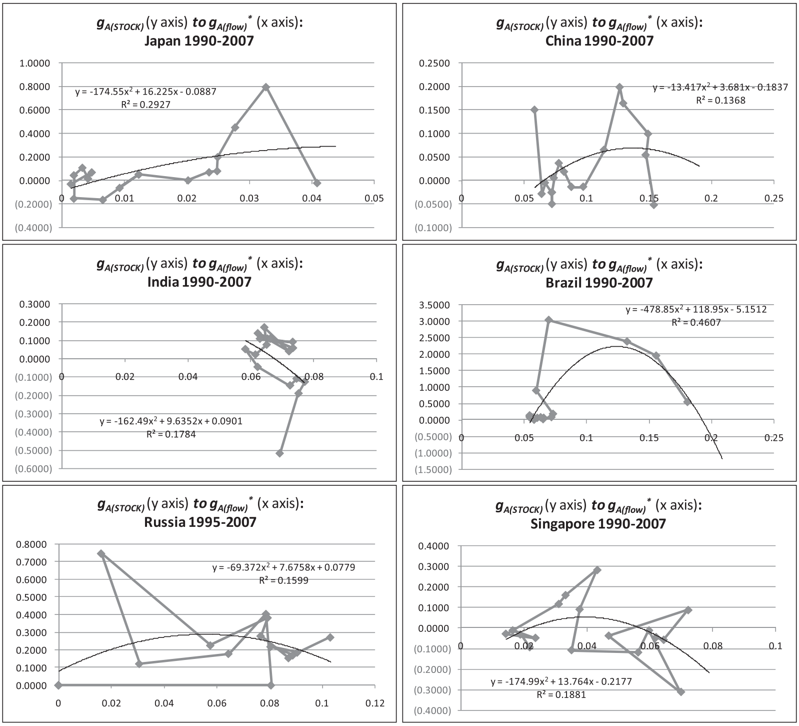


Figure S6 Growth rates of the current level of technology and the rate of tech. progress at convergence: Japan, China, India, Brazil, Russia, and Singapore

Growth and inequality: for the stable curve of *alpha* (1)

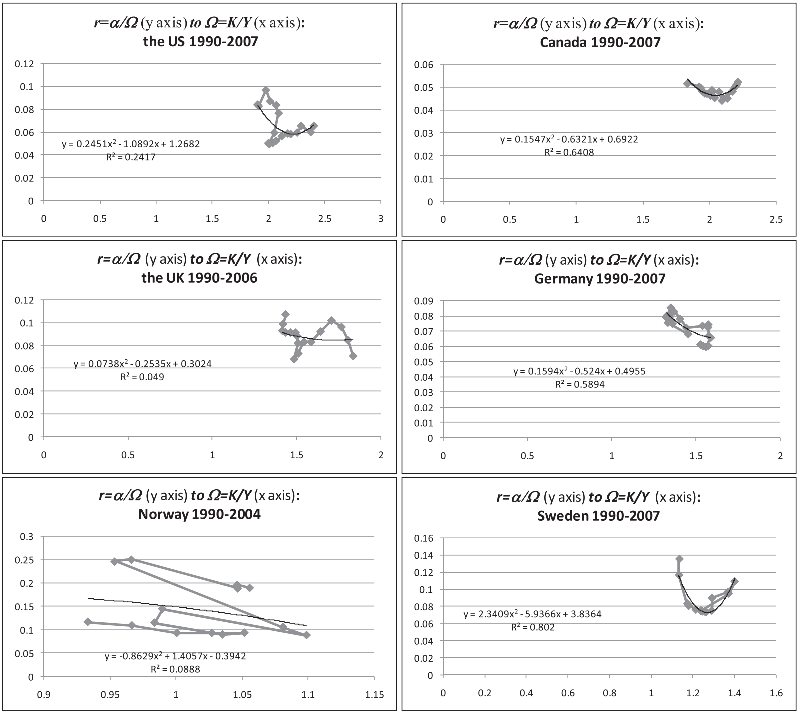


Figure S7 *Alpha*'s curve as the product of the rate of return and the capital-output ratio: the US, Canada, the UK, Germany, Norway, and Sweden

Growth and inequality: for the stable curve of *alpha* (2)

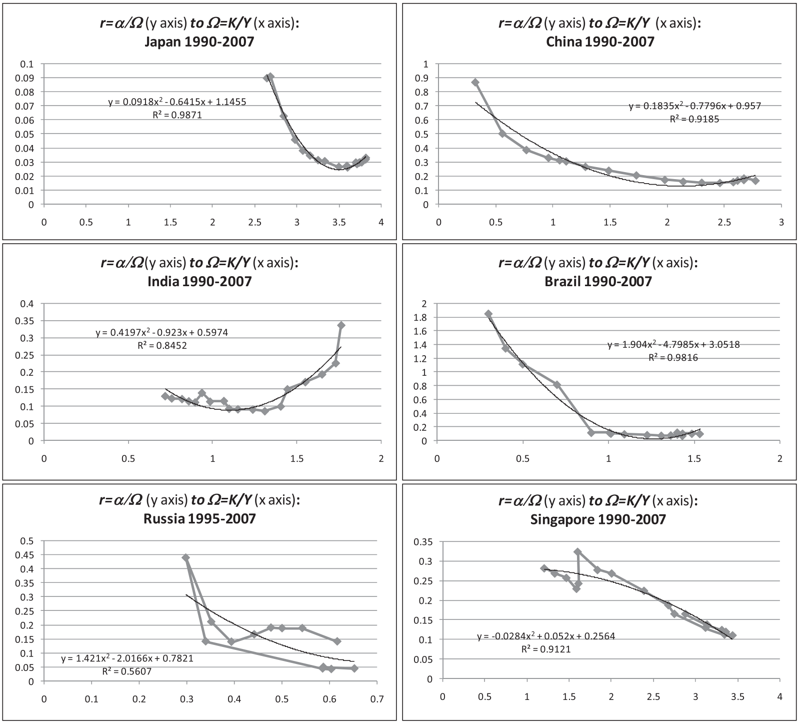


Figure S8 *Alpha*'s curve as the product of the rate of return and the capital-output ratio: Japan, China, India, Brazil, Russia, and Singapore

Government *alpha* through government deficit (1)

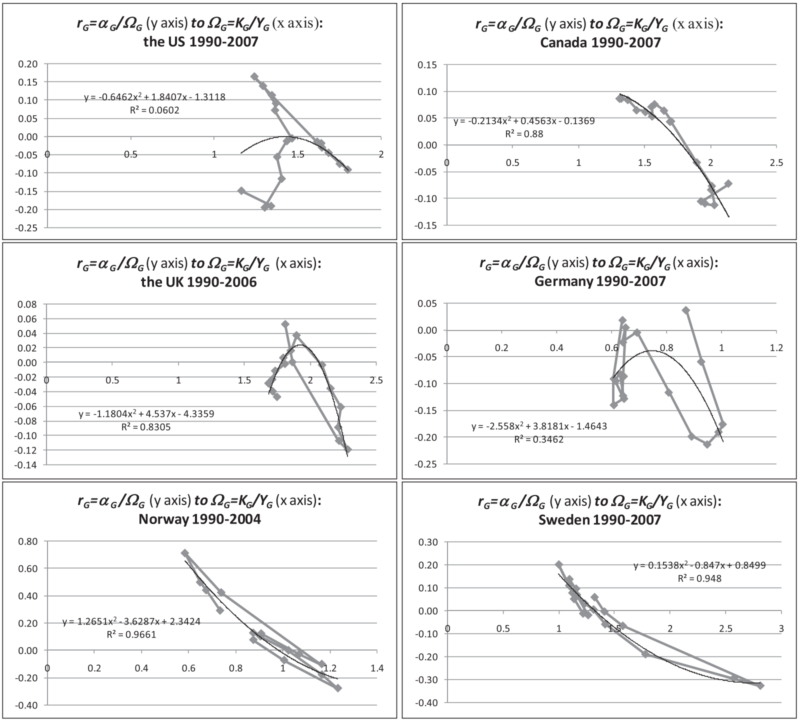


Figure S9 Government *alpha*'s curve as its product of the rate of return and the capital-output ratio: the US, Canada, the UK, Germany, Norway, and Sweden

Government *alpha* through government deficit (2)

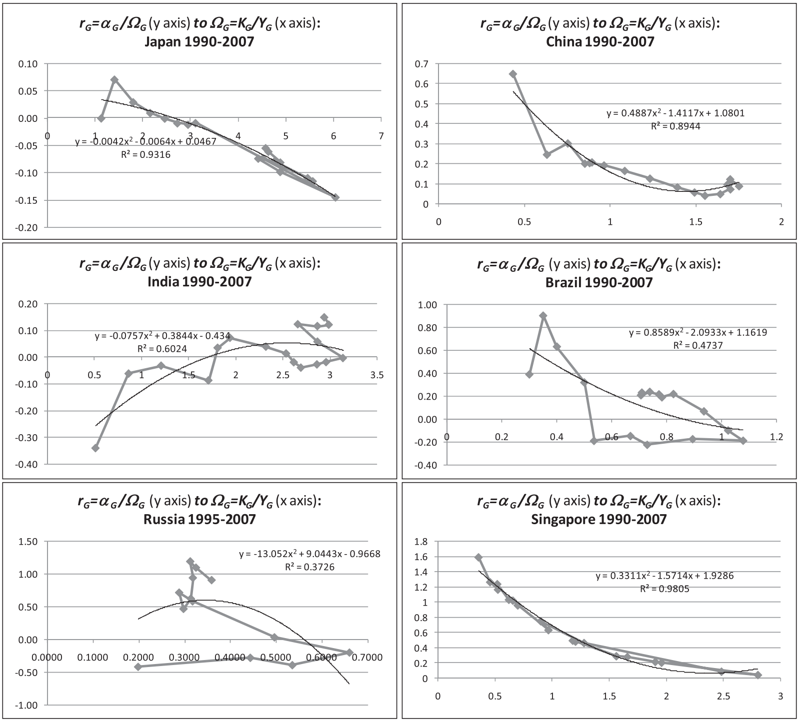


Figure S10 Government *alpha*'s curve as its product of the rate of return and the capital-output ratio: Japan, China, India, Brazil, Russia, and Singapore

National taste differs by country in equilibrium (1)

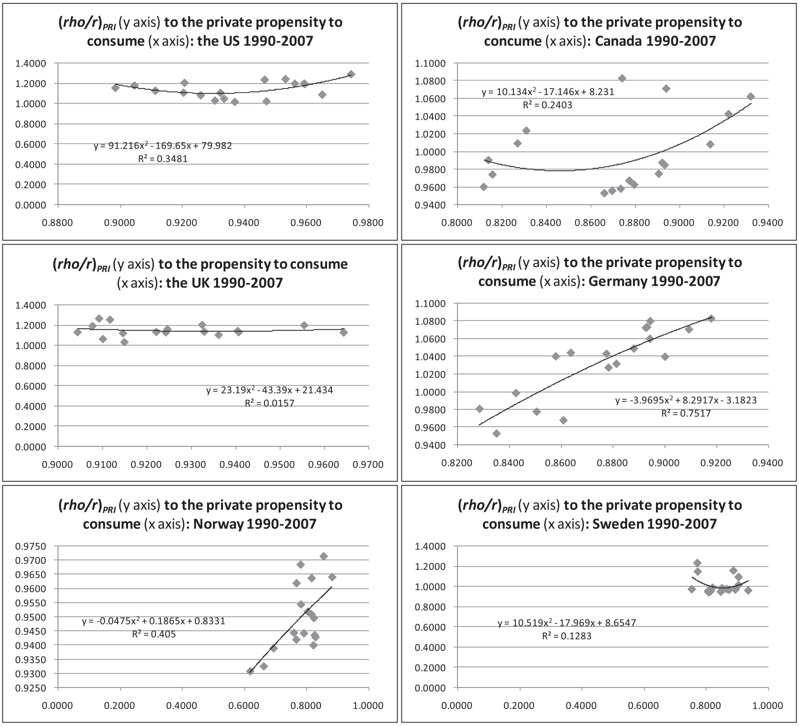


Figure S11　Private sector's national taste with its propensity to consume: the US, Canada, the UK, Germany, Norway, and Sweden

National taste differs by country in equilibrium (2)

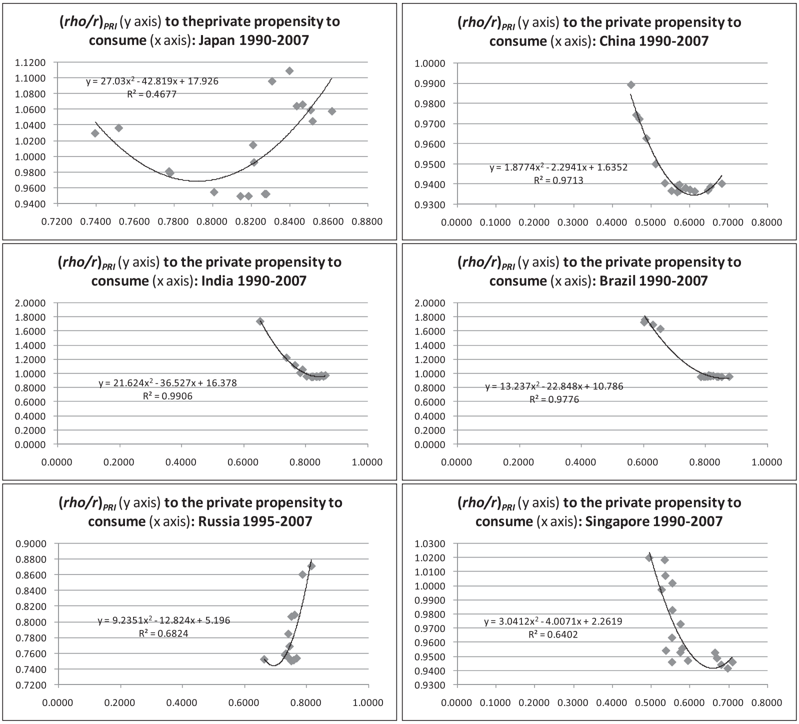
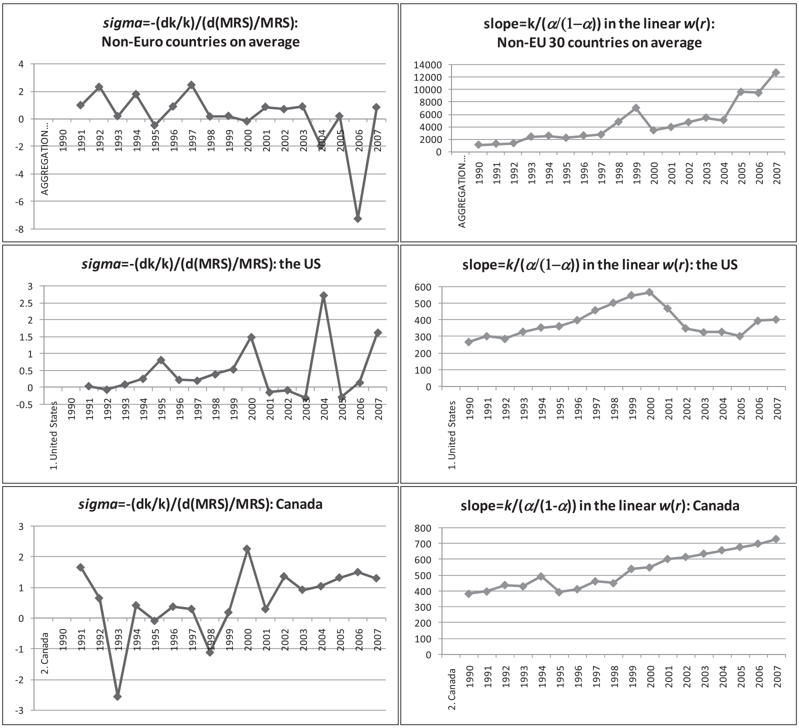


Figure S12 Private sector's national taste with its propensity to consume: Japan, China, India, Brazil, Russia, and Singapore

Hereafter, for *sigma* in KEWT3.09, 58 countries and 3 areas



Data source: KEWT 3.09 of fifty-eight countries and three areas on average by sector, 1990-2007, whose ten original data come from *International Financial Statistics Yearbook*, IMF. (hereafter, abbreviated)

Figure 1 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Non-EU 30 countries on average, the US, and Canada

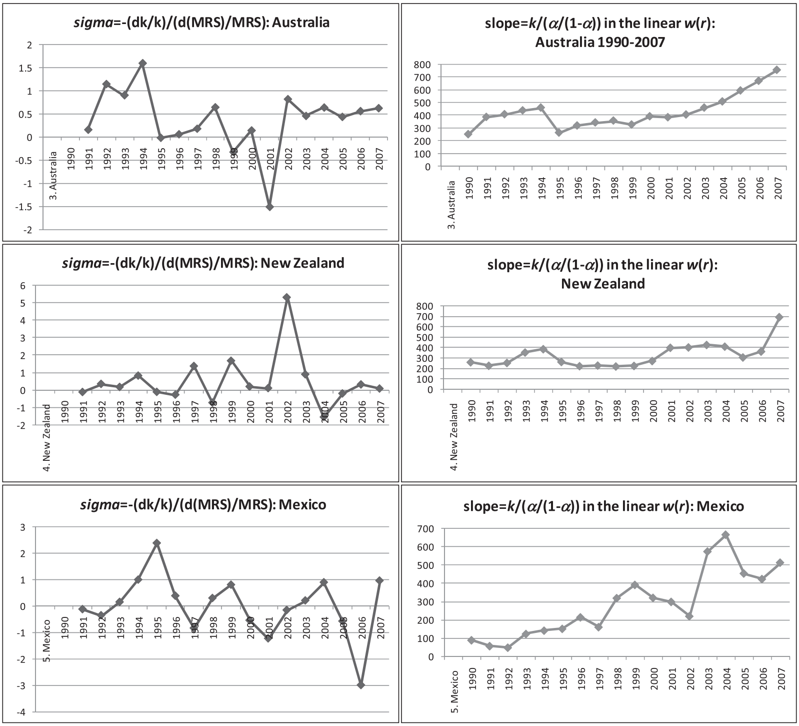


Figure 2　Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Australia, New Zealand, and Mexico

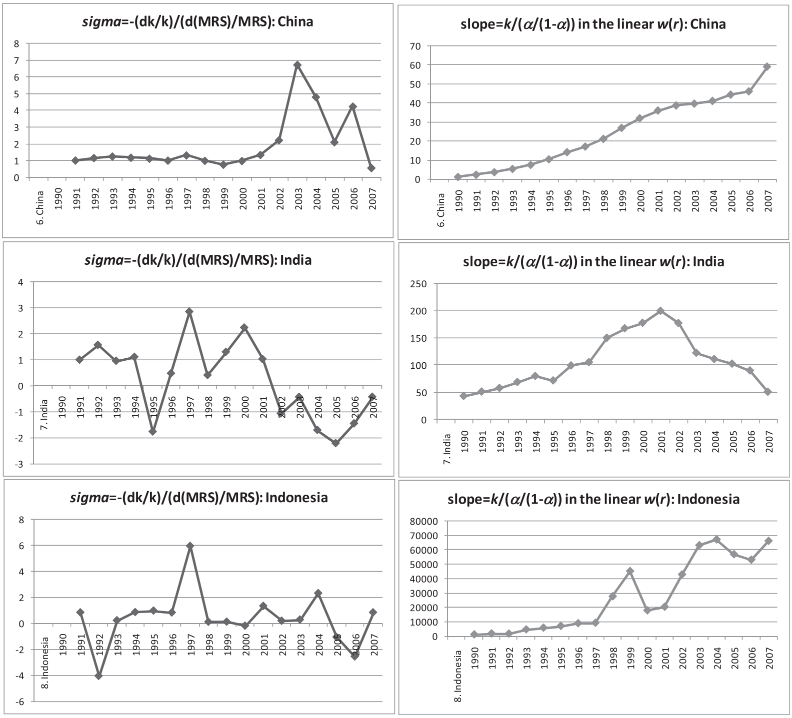


Figure 3 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: China, India, and Indonesia

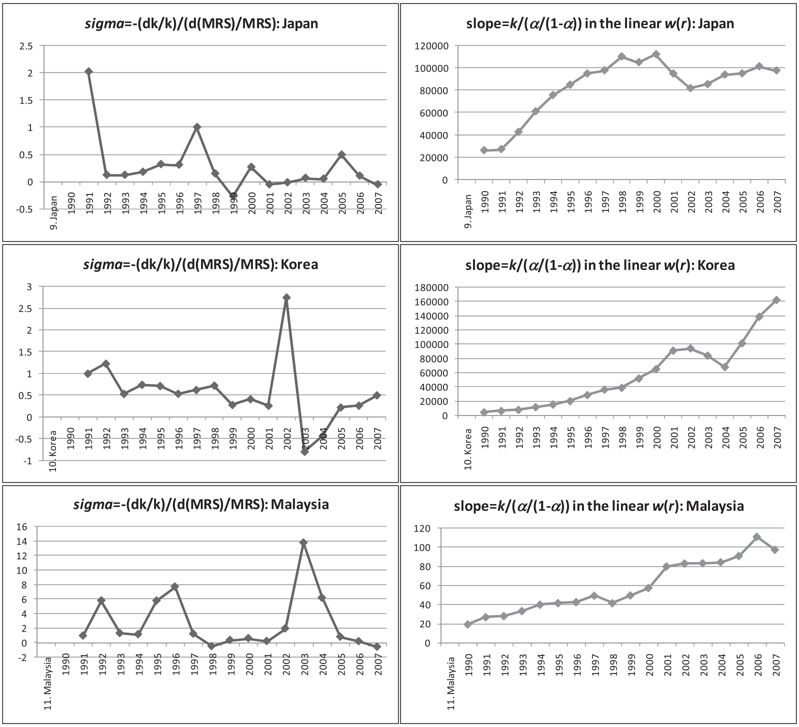


Figure 4　Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Japan, Korea, and Malaysia

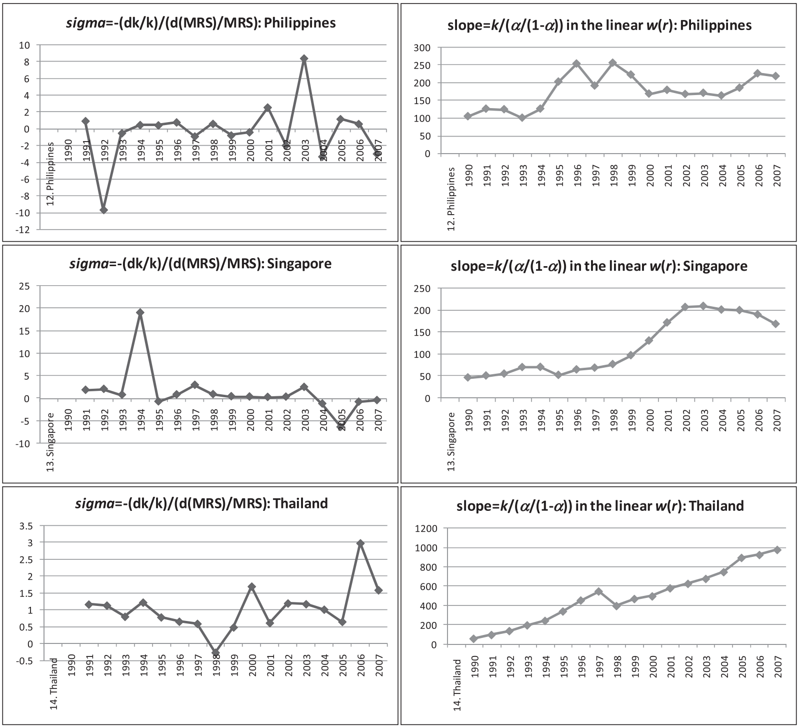


Figure 5 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Philippines, Singapore, and Thailand

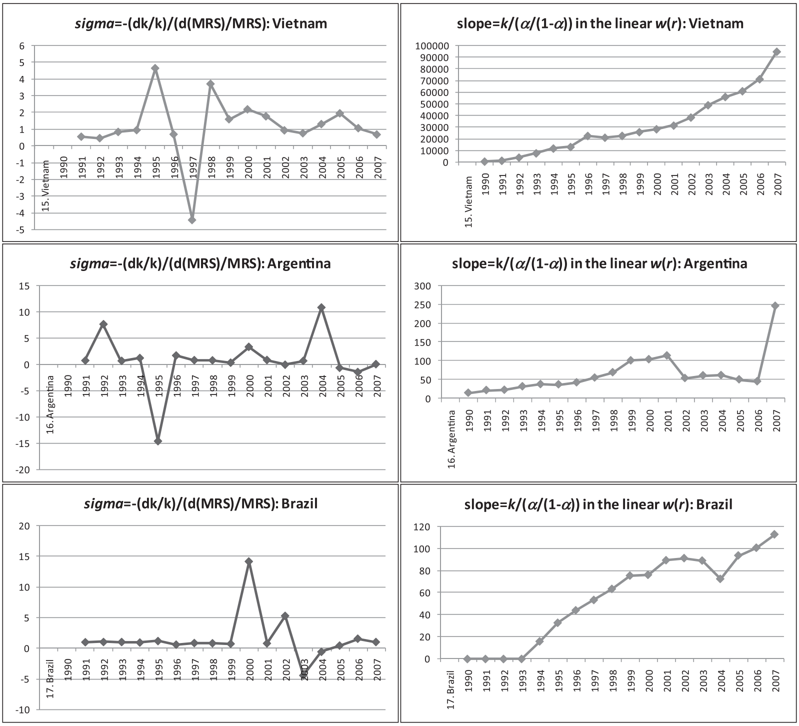


Figure 6　Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Vietnam, Argentina, and Brazil

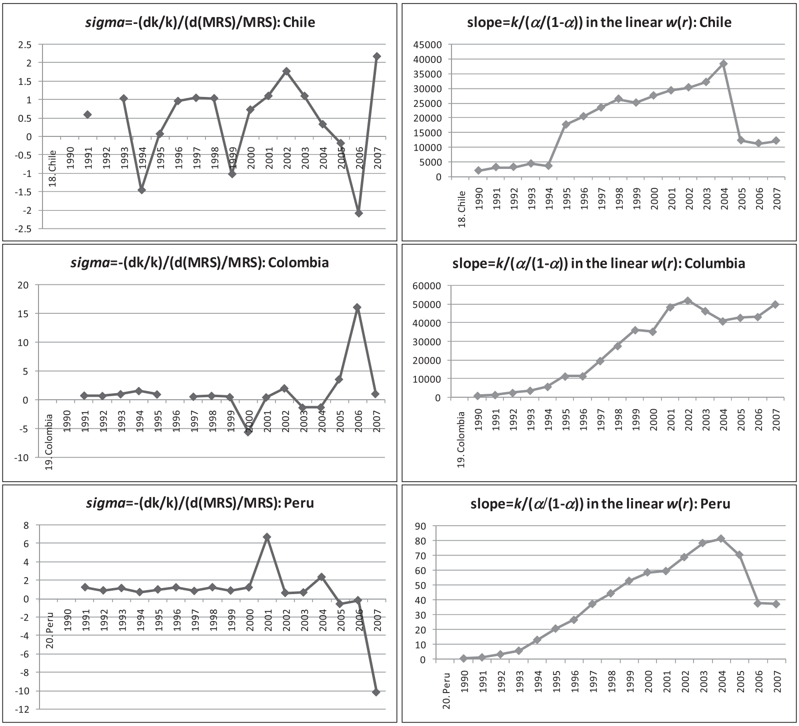


Figure 7 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Chile, Colombia, and Peru

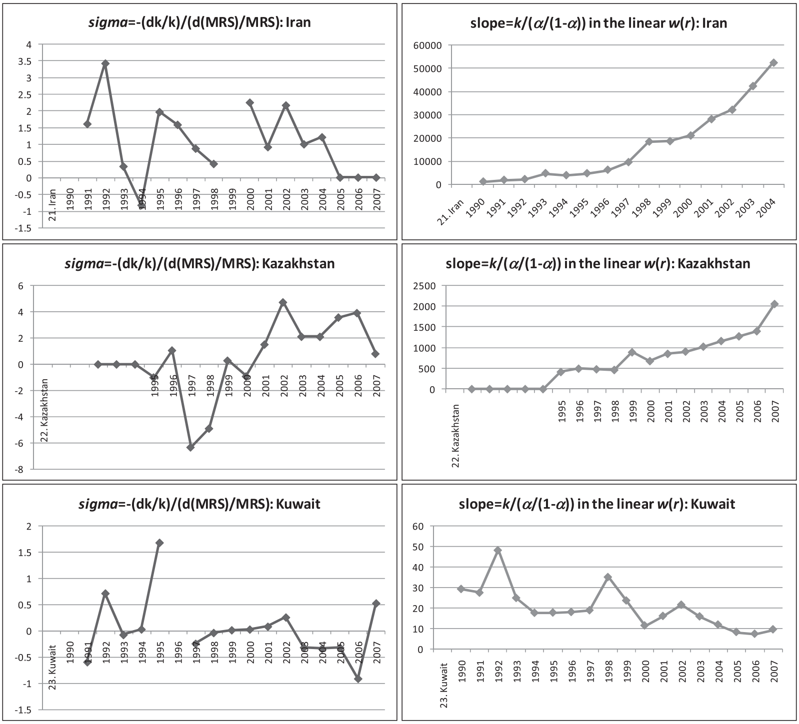


Figure 8 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Iran, Kazakhstan, and Kuwait

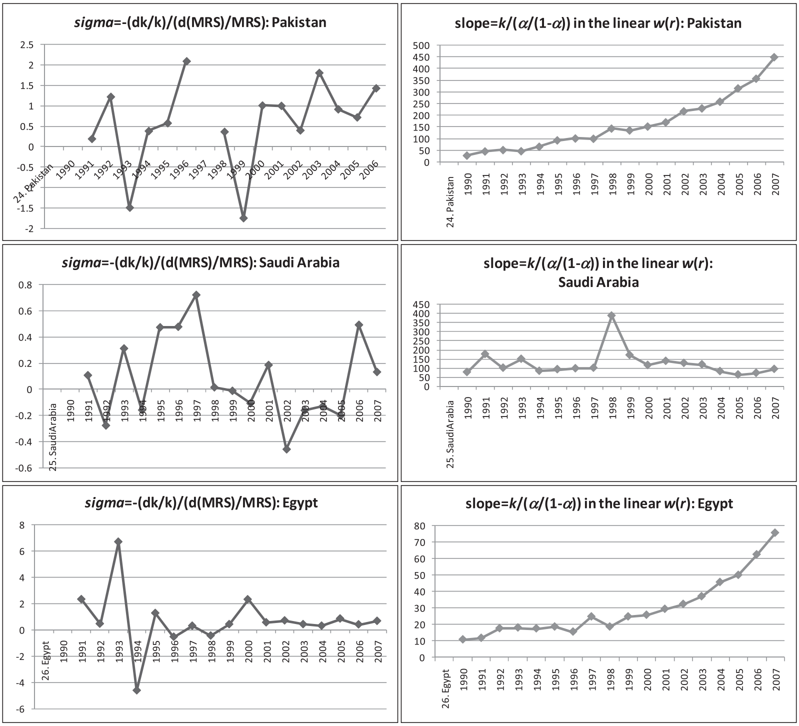


Figure 9 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Pakistan, Saudi Arabia, and Egypt

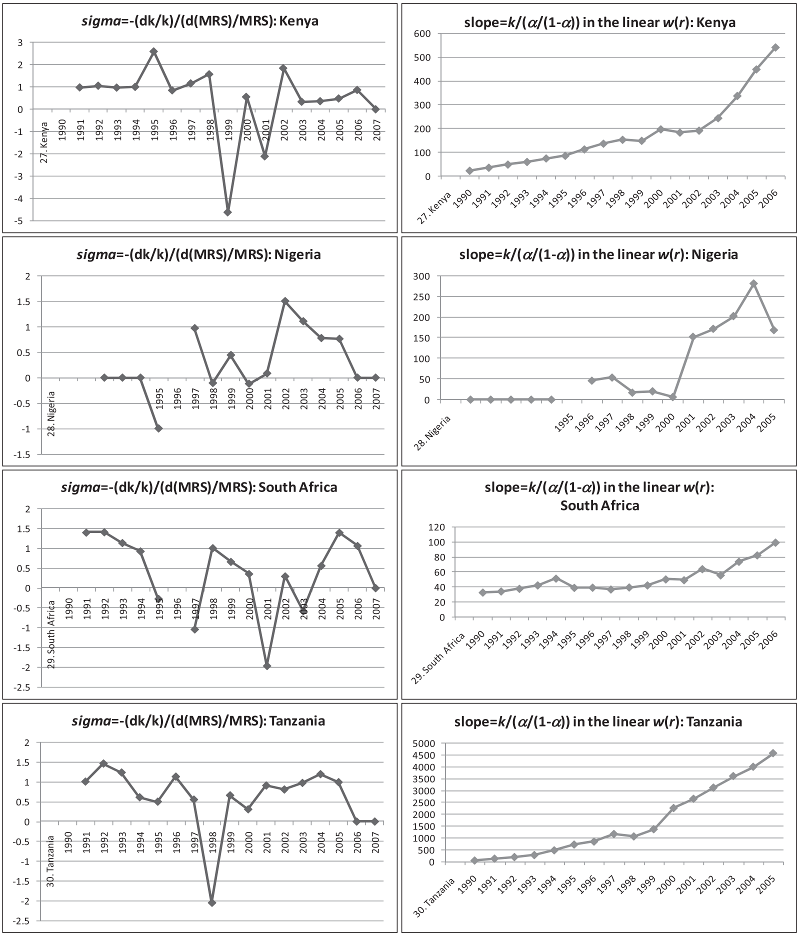


Figure 10 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Kenya, Nigeria, and Tanzania

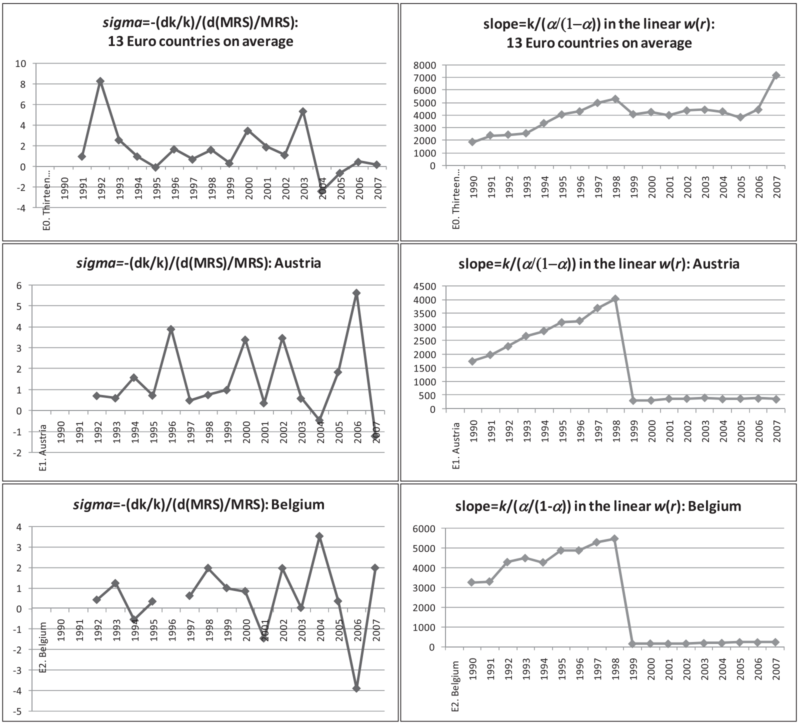


Figure 11 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: 13 Euro currency countries, Austria, and Belgium

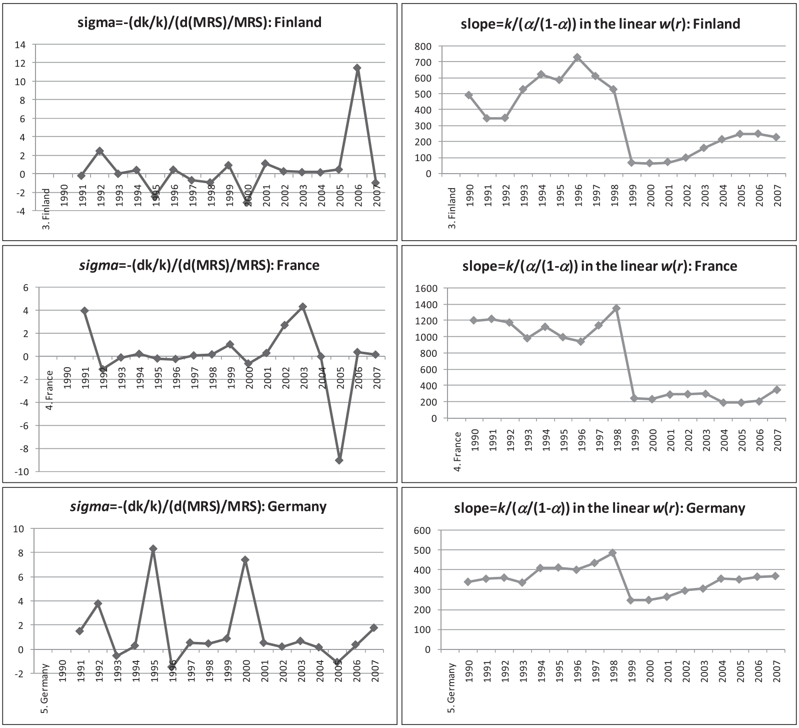


Figure 12　Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Finland, France, and Germany

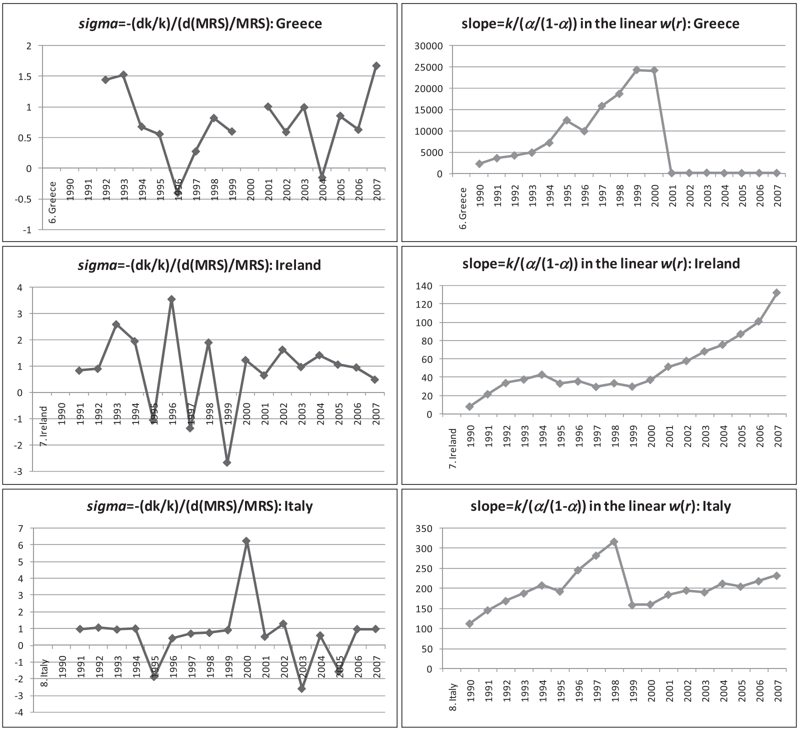


Figure 13 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Greece, Ireland, and Italy

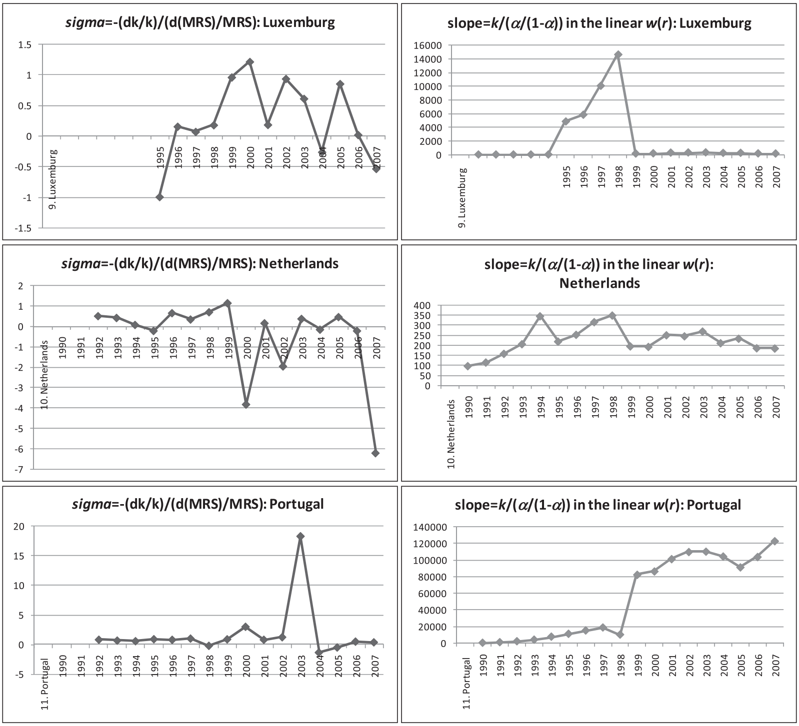


Figure 14 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Luxemburg, Netherlands, and Portugal

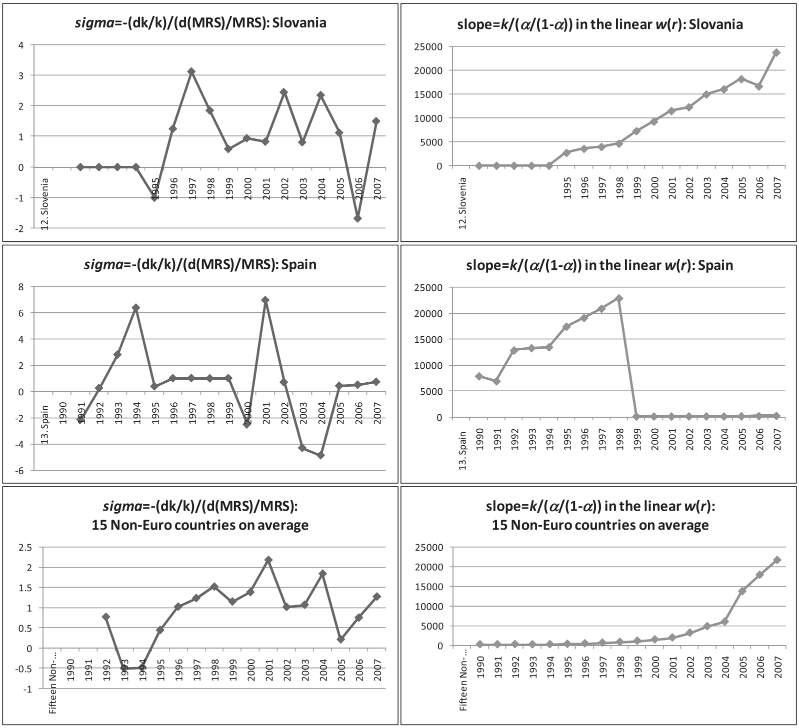


Figure 15 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Slovenia, Spain, and 15 Non-Euro countries on average

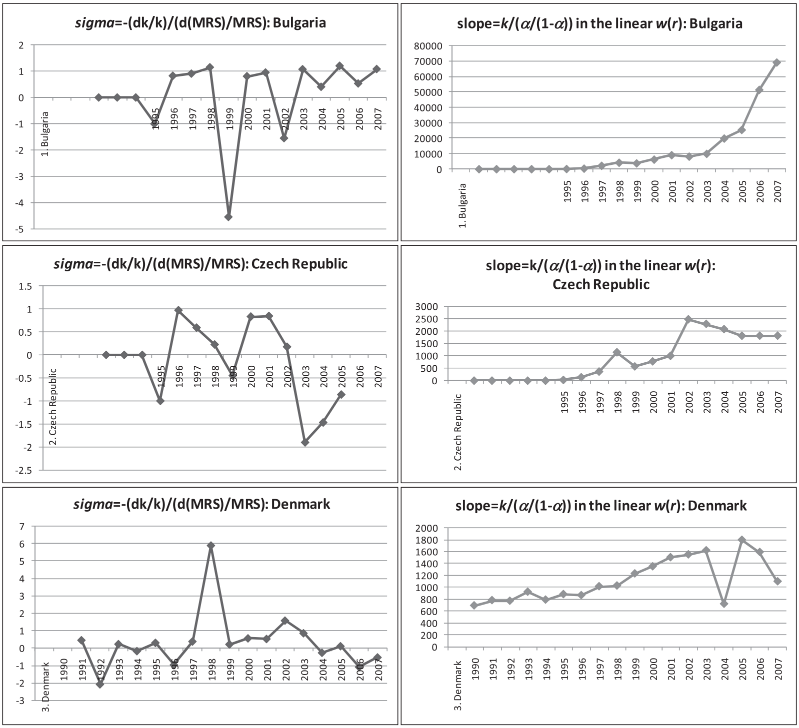


Figure 16 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Bulgaria, Czech Republic and Denmark

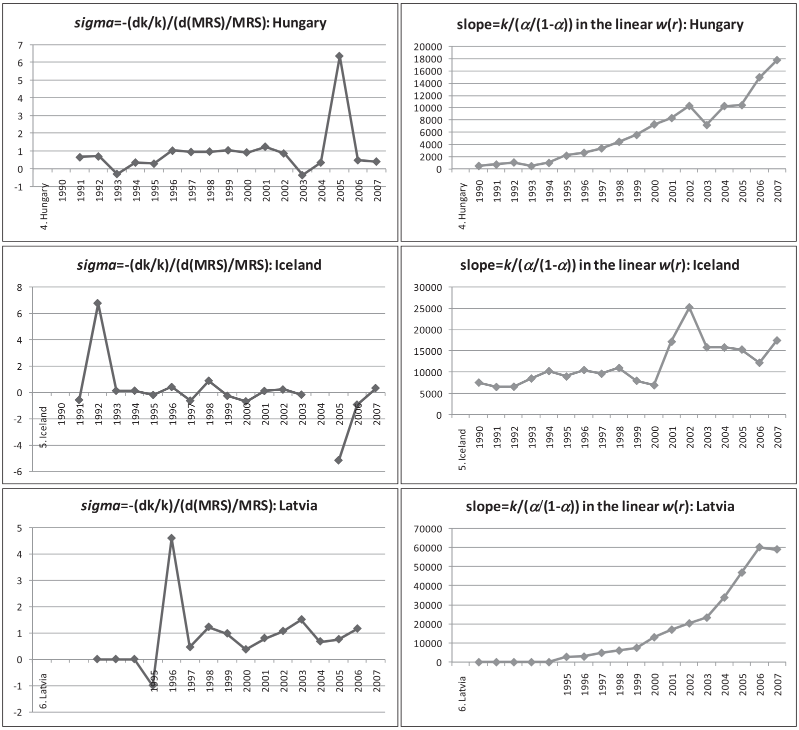


Figure 17 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Hungary, Ireland, and Latvia

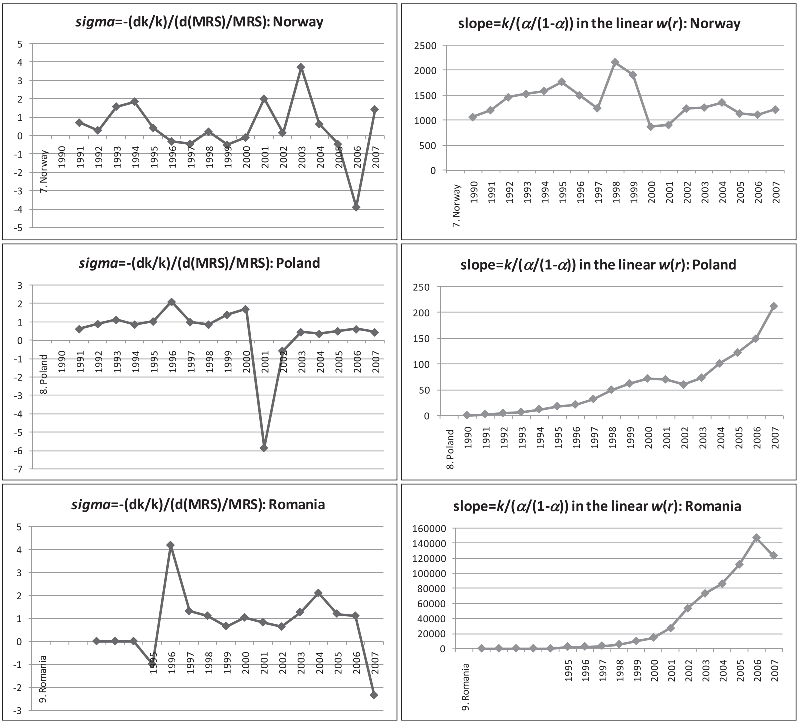


Figure 18 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Norway, Poland, and Romania

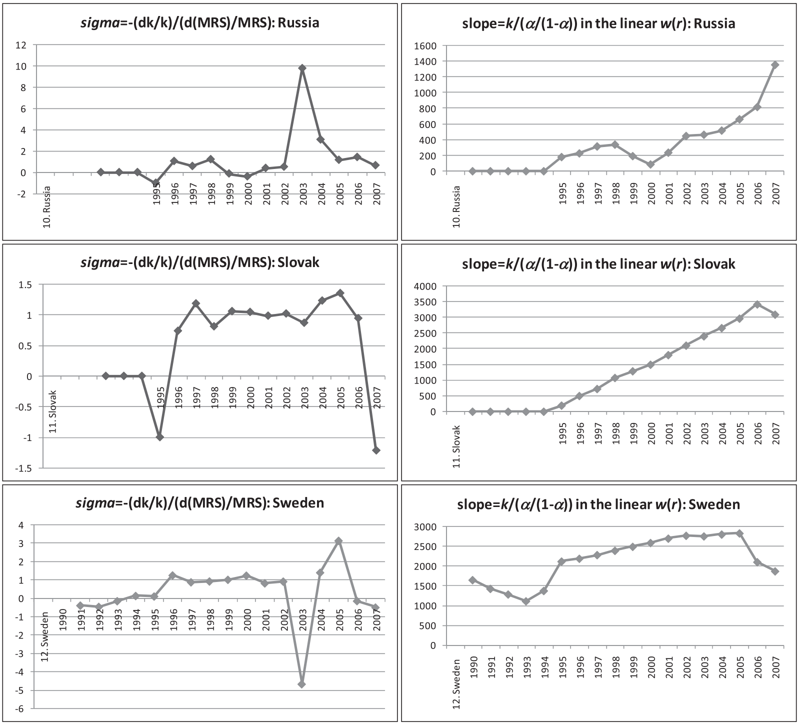


Figure 19 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Russia, Slovak, and Sweden

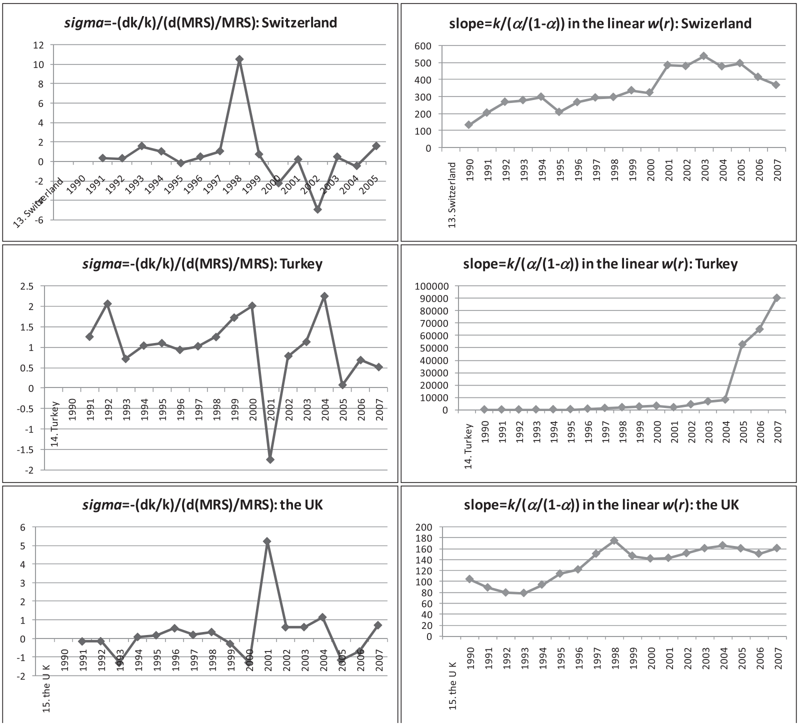


Figure 20 Elasticity of substitution, *sigma*, and marginal rate of substitution, *MRS*: Switzerland, Turkey, and the UK

Figure 21 Proof of elasticity of subs

	A recursive programming using the US, 2005, below							
	ES	ET					EY	EZ
	For elasticity, each denominator must be 'average of t & t – 1.						The marginal rate of	
	N	AK	AN	AM	AI	AJ	The elasticity of subst	
time, *t*	k	r/w	K	L	r	w	Δk/k	Δ(r/w)
0	75.33	0.00286	22464	298	0.0860	30.06		
1	77.20	0.00279	23273	301	0.0845	30.27	0.02485	(0.0243)
2	79.07	0.00273	24095	305	0.0831	30.49	0.02417	(0.0236)
3	80.93	0.00266	24930	308	0.0818	30.71	0.02352	(0.0230)
4	82.78	0.00260	25778	311	0.0805	30.93	0.0229	(0.0224)
5	84.63	0.00255	26640	315	0.0793	31.15	0.02232	(0.0218)
6	86.47	0.00249	27516	318	0.0782	31.38	0.02176	(0.0213)
7	88.31	0.00244	28406	322	0.0771	31.61	0.02124	(0.0208)
8	90.14	0.00239	29311	325	0.0761	31.85	0.02074	(0.0203)
9	91.96	0.00234	30230	329	0.0752	32.09	0.02026	(0.0199)
10	93.79	0.00230	31165	332	0.0743	32.33	0.01981	(0.0194)
11	95.60	0.00225	32115	336	0.0734	32.58	0.01939	(0.0190)
12	97.42	0.00221	33080	340	0.0726	32.83	0.01898	(0.0186)
13	99.23	0.00217	34062	343	0.0719	33.09	0.01859	(0.0183)
14	101.04	0.00213	35060	347	0.0711	33.36	0.01822	(0.0179)
15	102.84	0.00210	36075	351	0.0705	33.63	0.01788	(0.0176)
16	104.65	0.00206	37107	355	0.0698	33.91	0.01754	(0.0172)
17	106.45	0.00202	38157	358	0.0692	34.19	0.01723	(0.0169)
18	108.25	0.00199	39226	362	0.0686	34.49	0.01693	(0.0166)
19	110.06	0.00196	40312	366	0.0681	34.79	0.01665	(0.0164)
20	111.86	0.00193	41419	370	0.0676	35.10	0.01638	(0.0161)
21	113.66	0.00190	42544	374	0.0671	35.42	0.01613	(0.0159)
22	115.47	0.00187	43691	378	0.0667	35.75	0.01588	(0.0156)
23	117.28	0.00184	44858	382	0.0663	36.09	0.01566	(0.0154)
24	119.09	0.00181	46046	387	0.0659	36.44	0.01544	(0.0152)
25	120.90	0.00178	47257	391	0.0656	36.80	0.01524	(0.0150)
26	122.72	0.00176	48490	395	0.0653	37.17	0.01505	(0.0148)
27	124.55	0.00173	49747	399	0.0650	37.56	0.01488	(0.0147)
28	126.38	0.00170	51028	404	0.0647	37.96	0.01471	(0.0145)
29	128.22	0.00168	52334	408	0.0645	38.37	0.01455	(0.0143)
30	130.07	0.00166	53665	413	0.0643	38.79	0.01441	(0.0142)
31	131.92	0.00163	55024	417	0.0641	39.23	0.01428	(0.0141)
32	133.79	0.00161	56410	422	0.0639	39.69	0.01415	(0.0140)

titution, *sigma* = 1.0, in recursive programming

	FF = (ES12 − ES11)/((ES11 + ES12)/2)				**FG** = (ET12 − ET11)/((ET11 + ET12)/2)			
FA	FB	FC	FD	FE	**FF**	**FG**	**FH**	
substitution, *MRS* = *Δ(r/w)/(r/w)*					Speed of convergence is 55 times/years.			
itution, σ = − (Δk/k)/(Δ(r/w)/(r/w)					beta*	0.79281	delta	0.46075
σ 1	Δk/k	Δ(r/w)	**σ 2**	**σ 1 + 2**	Δk/k	Δ(r/w)	**σ 3**	time, *t*
σ1 = − EY12/EZ12			**σ2** = − FB12/FC12				**σ3** = − FF12/FG12	
1.02485	0.02425	(0.0249)	0.9757	2.0006	0.0245	(0.0245)	**1.0000**	1
1.02417	0.02360	(0.0242)	0.9764	2.0006	0.0239	(0.0239)	**1.0000**	2
1.02352	0.02298	(0.0235)	0.9770	2.0005	0.0232	(0.0232)	**1.0000**	3
1.0229	0.02239	(0.0229)	0.9776	2.0005	0.0226	(0.0226)	**1.0000**	4
1.02232	0.02183	(0.0223)	0.9782	2.0005	0.0221	(0.0221)	**1.0000**	5
1.02176	0.02130	(0.0218)	0.9787	2.0005	0.0215	(0.0215)	**1.0000**	6
1.02124	0.02080	(0.0212)	0.9792	2.0004	0.0210	(0.0210)	**1.0000**	7
1.02074	0.02032	(0.0207)	0.9797	2.0004	0.0205	(0.0205)	**1.0000**	8
1.02026	0.01986	(0.0203)	0.9801	2.0004	0.0201	(0.0201)	**1.0000**	9
1.01981	0.01943	(0.0198)	0.9806	2.0004	0.0196	(0.0196)	**1.0000**	10
1.01939	0.01902	(0.0194)	0.9810	2.0004	0.0192	(0.0192)	**1.0000**	11
1.01898	0.01863	(0.0190)	0.9814	2.0004	0.0188	(0.0188)	**1.0000**	12
1.01859	0.01825	(0.0186)	0.9817	2.0003	0.0184	(0.0184)	**1.0000**	13
1.01822	0.01790	(0.0182)	0.9821	2.0003	0.0181	(0.0181)	**1.0000**	14
1.01788	0.01756	(0.0179)	0.9824	2.0003	0.0177	(0.0177)	**1.0000**	15
1.01754	0.01724	(0.0175)	0.9828	2.0003	0.0174	(0.0174)	**1.0000**	16
1.01723	0.01694	(0.0172)	0.9831	2.0003	0.0171	(0.0171)	**1.0000**	17
1.01693	0.01665	(0.0169)	0.9834	2.0003	0.0168	(0.0168)	**1.0000**	18
1.01665	0.01637	(0.0166)	0.9836	2.0003	0.0165	(0.0165)	**1.0000**	19
1.01638	0.01612	(0.0164)	0.9839	2.0003	0.0162	(0.0162)	**1.0000**	20
1.01613	0.01587	(0.0161)	0.9841	2.0003	0.0160	(0.0160)	**1.0000**	21
1.01588	0.01564	(0.0159)	0.9844	2.0002	0.0158	(0.0158)	**1.0000**	22
1.01566	0.01542	(0.0157)	0.9846	2.0002	0.0155	(0.0155)	**1.0000**	23
1.01544	0.01521	(0.0154)	0.9848	2.0002	0.0153	(0.0153)	**1.0000**	24
1.01524	0.01501	(0.0152)	0.9850	2.0002	0.0151	(0.0151)	**1.0000**	25
1.01505	0.01483	(0.0151)	0.9852	2.0002	0.0149	(0.0149)	**1.0000**	26
1.01488	0.01466	(0.0149)	0.9853	2.0002	0.0148	(0.0148)	**1.0000**	27
1.01471	0.01450	(0.0147)	0.9855	2.0002	0.0146	(0.0146)	**1.0000**	28
1.01455	0.01435	(0.0146)	0.9857	2.0002	0.0144	(0.0144)	**1.0000**	29
1.01441	0.01420	(0.0144)	0.9858	2.0002	0.0143	(0.0143)	**1.0000**	30
1.01428	0.01407	(0.0143)	0.9859	2.0002	0.0142	(0.0142)	**1.0000**	31
1.01415	0.01396	(0.0142)	0.9860	2.0002	0.0141	(0.0141)	**1.0000**	32