Forms of 24 Hyperbola Equations with Graphic Expositions

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Introduction

The purpose of this paper is to present useful graphs in 24 rectangular hyperbola equations, with respective characteristics and wholly expose the mechanics of hyperbola equations. The data comes from the author's endogenous growth model and its KEWT 4.10 data-sets by country, sector, and year. 24 equations are divided into two: (1) twelve basic equations as formulated at Appendix of *Journal of Economic Sciences* 14 (Sep), 2010 and (2) twelve specified equations as formulated in detail at Appendix of this paper. In 24 equations, the 'inverse' equations are included: for example, *speed* (*i*) versus *i*(*speed*) and, *speed*(*n*) versus *n*(*speed*). As a result, six basic equations and six specified equations exist after excluding respective inverse equations. The inverse equations are useful when policy-makers set up alternative and priority policies. Six basic equations and six specified equations each has its own form such as $y=\dfrac{1}{ax+b}$, $y=\dfrac{cx+d}{ax}$, $y=\dfrac{cx+d}{b}$, $y=\dfrac{cx}{ax+b}$, $y=\dfrac{cx}{ax+b}$, and $y=\dfrac{cx+d}{ax+b}$. These forms start with the standard form, $y=\dfrac{cx+d}{ax+b}$, and exist after setting zero one or two values of a, b, c, and d. Despite the differences of forms, each hyperbola equation shows a common hyperbola, $\left(y-\dfrac{c}{a}\right)\left(x+\dfrac{b}{a}\right)=\dfrac{f}{a}$. Thus, it is possible to commonly draw each hyperbola graph, as shown in this paper.

The author examined each resultant graph of selected equations by country and sector: The author realized that the differences of graphic results by equation remained partial and that anyone could wholly integrate and interpret various graphs. The author first intended to classify graphic characteristics by equation but, the author finally found that it was not meaningful to classify these graphic characteristics by equation, country, sector, and year. This implies that differences of graphic characteristics come from each country's policy-making by year and these characteristics are respectively shown at a point of view. A problem is how to find the differences of policy-making by country, sector, and year. Some reasons are: (1) Each country has its own unique real asset characteristics as results of unique policy-making. (2) Each country shows its economic stage, measured by the upper limit of the horizontal asymptote of the capital-output ratio equation of the ratio of net investment to endogenous output/income, $i = I / Y$: Ω^{*} (i) in equilibrium. (3) Each country pursues its own policies yet, these policies significantly differ by country beyond the differences of the economic stages. Globalization does not influence national taste and preferences in some countries (as in Singapore) and, strongly influences national taste and preferences in other countries (as in Japan). Each country's policy-makers have respective ideas and philosophy so that partial classification of characteristics by equation is not meaningful. Common criterion, nevertheless, exist in some equations.

Let the author raise a typical example; the use of the rate of return equation of $i = I / Y$: r^{*} (i). The endogenous rate of inflation/deflation is measured by its horizontal asymptote and significantly differs by country. As a result, the higher the growth rate of output the higher the endogenous inflation is. The real rate of return in equilibrium, as a result, stays at inflation (plus) or deflation (minus), although the nominal rate of return always remains above zero. Starting with the relationship between nominal and real, the nominal cost of capital is compared with the real cost of capital, where the cost of capital is the rate of return less the growth rate of output. This paper does not deepen the issue of the real cost of capital, yet, it is important for policy-makers to perceive that the real cost of capital by country does differ slightly, despite of much differences of policies by country. This is a unique new finding. This implies that the real cost of capital constitutes a common criterion globally in the world economy.

Is this a result of globalization in recent years? No, it is not. The similar level of the real cost of capital comes from the essence of related hyperbola equations. Repeating, the higher the growth rate of output in equilibrium the higher the endogenous inflation rate is. Some researchers indicate that the globalization leads to similar economic results in many respects. The author adversely advocates that these similar results do not come from the globalization but from weak or obedient philosophy of policy-makers at certain countries. For example, look at graphic results of Singapore. Singapore had entered into a developed country much earlier than other countries in Asia. If policy-makers in Singapore had just followed global ideas and philosophy, graphic results must be different from what it shows today, falling into common difficulties that all developed countries have suffered from (this is soon discussed in a separate paper).

A hyperbola equation is expressed as an assembly of hyperbola curve, the circle, and the rectangular equilateral triangle, where the circle touches the hyperbola and the top of the triangle. The radius of the circle equals the hypotenuse of the triangle. The hypotenuse is the inverse number of curvature. The Width is the base of the triangle. Hyperbola equation remains two dimensions in the real world due to the above assembly. When the hypotenuse is plus ($f / a > 0$), the graph remains to stay at the 1st quadrant. When the hypotenuse is minus ($f / a < 0$), the graph extends to locate at the 4st quadrant.

Without the 4th quadrant, the endogenous rate of deflation could not be proved. Figure 1 shows the above plus and minus hypotenuses.

Hyperbola equations have horizontal and/or vertical asymptotes. There are two criteria: (1) the upper limit of the capital-output ratio as its horizontal asymptote and (2) endogenous inflation/deflation rate of the rate of return as its horizontal asymptote, each in equilibrium. These are illustrated by hyperbola graphs. Furthermore, there are several measures for sustainable robustness, which are each measured by the Width of the triangle that supports hyperbola curve. Weakened developed countries have much smaller rectangular equilateral triangles or the Widths. The above two criteria and several Widths of key hyperbola equations clarify the future of each country's current economic situations. This paper presents key graphic expositions by country, thanking for space allowance. For graphical presentations, the author got exclusive software from Katsuhisa Tomoda, Osaka Education University, as the founder of general type Grapes since the 1990s.

Details of graphic expositions

Hyperbola equations uses seven endogenous parameters in equilibrium by sector (the G and PRI sectors): λ^*, Ω^*, α, β^*, δ_0 and, n, i.[1] At the government sector (the G sector), λ^*_G, Ω^*_G, α_G, β^*_G, $\delta_{0\ G}$ and, n_G, i_G, are used. At the private sector (the PRI sector), λ^*_{PRI}, Ω^*_{PRI}, α_{PRI}, β^*_{PRI}, $\delta_{0\ PRI}$ and, n_{PRI}, i_{PRI}, are used. Basic variables in equilibrium are the rate of return, r^*, the growth rate of output, g^*_Y, the cost of capital, $r^*-g^*_Y$, and the valuation ratio, $v = r^*/(r^*-g^*_Y) = V/K$. The data-sets of KEWT 4.10, without assumption, have the

1) For the measurement of the total economy in KEWT data-sets, nine parameters are required: the above seven plus two: λ^*, Ω^*, α, β^*, δ_0 and, n, i, and, n_G, i_G, to get endogenous equilibrium by sector. This equilibrium is measured by the speed years, as the inverse number of the convergence coefficient, λ^*.

relative price level = 1.0 by fiscal year and in its recursive programming by fiscal year. Thus, nominal and real are distinguished in the data-sets. The data-sets also have $MPK = r$ and $MPL = w$, without assumption. The use of the discrete Cobb-Douglas production function in the data-sets makes it possible to measure nominal and real.

Graphic results mostly stay at the 1st quadrant. This is because most endogenous parameters must be plus in equilibrium such as net investment to output $i = I / Y$, the capital-output ratio $\Omega^* = K / Y$, the speed years, and the diminishing returns to capital (DRC) coefficient $delta_0$. Other parameters and variables are able to show minus such as the growth rate of population n and $\alpha = \Pi / Y$. As a result, the 1st quadrant cooperates with the 2nd quadrant in some equations: a typical case is $speed(n)$, $beta(n)$, $Omega(n)$, and $r^*(n)$, where the rate of change in population in equilibrium, n_E, is able to have minus, as a cushion to manipulate endogenous equilibrium.

The author divides graphic details/contents into seven, I to VII: I. Classification of forms; II. Differences of forms by equation, using a, b, c, and/or d, with f or f/a (excluding the inverse cases); III. Final form common to twenty four hyperbola equations, IV; Findings in 24 hyperbola equations, V; Forms of 24 hyperbola equations by function, VI; Characteristics of hyperbola equations, with the Excel cell addresses, and VII; Key graphic results formulated by Katsuhiko Tomoda's exclusive software.

Twelve specified hyperbolic equations are summarized in detail at Appendix at the end. These equations are: $i(n)$, $n(i)$, $\Omega^*(\beta^*)$, $\beta^*(\Omega^*)$, $\beta^*(n)$, $n(\beta^*)$, and $\widetilde{\beta^*}(n)$, $n(\widetilde{\beta^*})$, $\beta^*(i)$, $\widetilde{\beta^*}(i)$, $\alpha(i)$, $\tilde{\alpha}(i)$, where β^* versus $\widetilde{\beta^*} = 1 - \beta^*$, and α versus $\tilde{\alpha} = 1 - \alpha$, are special cases. The final form common to all equations is shown by $\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{f}}{\mathrm{a}}$, where horizontal asymptote (HA) is given by $\frac{\mathrm{c}}{\mathrm{a}}$, and vertical asymptote (VA) is given by $\frac{-\mathrm{b}}{\mathrm{a}}$. When the values of $\frac{\mathrm{f}}{\mathrm{a}}$

changes repeatedly during 1990–2008, it implies that policy-makers endeavour to adjust the combination of real (including fiscal) and financial/market policies using actual data. The financial and market assets in the long-term (such as *M2*, ten year debt yield, and the exchange rate) are neutral to the real assets, and fairly evaluate the real assets.

I. *Classification of forms*:

(1) $y = \dfrac{1}{ax+b}$. 1–1 *peed*(i): and 1–3 *peed*(n). For the speed, only using 1: 1/100.

(2) $y = \dfrac{cx+d}{ax}$. 2–1 $r^*(i)$. 6–1 $\beta^*(i)$.

(3) $y = \dfrac{cx+d}{b}$. 2–3 $r^*(n)$.

(4) $y = \dfrac{cx}{ax+b}$. 3–1 $\Omega^*(i)$. 4–1 $i(n)$. 4–3 $\Omega^*(\beta^*)$.

(5) $y = \dfrac{d}{ax+b}$. 3–3 $\Omega^*(n)$.

(6) $y = \dfrac{cx+d}{ax+b}$. 5–1 $\beta^*(n)$. 5–3 $\widetilde{\beta^*}(n)$. 6–3 $\alpha(i)$.

II. Differences of forms by equation, using a, b, c, and/or d, with f or f/a (excluding the inverse cases):

For speed, $\dfrac{1}{\lambda^*}$:

1–1 ***peed(i)***: $y = \dfrac{1}{ax+b}$. $x = i$. y = speed. $a = (1-\beta^*)(1-\delta_0)$. $b = n(1-\alpha)$. $d = f = 1$.

1–3 ***peed(n)***: $y = \dfrac{1}{ax+b}$. $x = n$. y = speed. $a = (1-\alpha)$. $b = i(1-\beta^*)(1-\delta_0)$. $d = f = 1$.

For r^*:

2–1 $\boldsymbol{r^*(i)}$: $y = \dfrac{cx+d}{ax}$. $a = \beta^*(1-\alpha)$. $c = \alpha(1-\beta^*)(1+n)$. $d = f = \alpha \cdot n(1-\alpha)$.

$$\frac{\mathrm{f}}{\mathrm{a}}=\frac{a\cdot n}{\beta^*}=\frac{a\cdot n(1-\alpha)}{\beta^*(1-\alpha)}.$$

2–3 $\boldsymbol{r^*(n)}$: $\mathrm{y}=\dfrac{\mathrm{cx+d}}{\mathrm{b}}$. $\mathrm{b}=\mathrm{i}\cdot\beta^*(1-\alpha)$. $\mathrm{c}=\mathrm{i}\cdot\alpha(1-\beta^*)+\alpha(1-\alpha)$.
$\mathrm{d}=\mathrm{f}=\mathrm{i}\cdot\alpha(1-\beta^*)$.

For Ω^*:

3–1 $\boldsymbol{\Omega^*(i)}$: $\mathrm{y}=\dfrac{\mathrm{cx}}{\mathrm{ax+b}}$. $\mathrm{a}=(1-\beta^*)(1+\mathrm{n})$. $\mathrm{b}=n(1-\alpha)$. $\mathrm{c}=\beta^*(1-\alpha)$.
$\mathrm{f}=\dfrac{-\beta^*(1-\alpha)n(1-\alpha)}{(1-\beta^*)(1+n)}$.

3–3 $\boldsymbol{\Omega^*(n)}$: $\mathrm{y}=\dfrac{\mathrm{d}}{\mathrm{ax+b}}$. $\mathrm{a}=i(1-\beta^*)+(1-\alpha)$. $\mathrm{b}=i\cdot(1-\beta^*)$.
$\mathrm{d}=\mathrm{f}=\beta^*\cdot i(1-\alpha)$.

For $i(n)$ and $\Omega^*(\beta^*)$:

4–1 $\boldsymbol{i(n)}$: $\mathrm{y}=\dfrac{\mathrm{cx}}{\mathrm{ax+b}}$. $\mathrm{a}=(1-\beta^*)\Omega^*$, $\mathrm{b}=(1-\beta^*)\,\Omega^*-\beta^*\,(1-\alpha)$.
$\mathrm{c}=-(1-\alpha)\,\Omega^*$. $\mathrm{f}=\dfrac{-\left(\beta^*(1-\alpha)-(1-\beta^*)\Omega^*\right)\cdot(1-\alpha)\Omega^*}{(1-\beta^*)\Omega^*}=\dfrac{\mathrm{b\cdot c}}{\mathrm{a}}$.

4–3 $\boldsymbol{\Omega^*\ (\beta^*)}$: $\mathrm{y}=\dfrac{\mathrm{cx}}{\mathrm{ax+b}}$. $\mathrm{a}=i(1+\mathrm{n})$. $\mathrm{b}=-(i(1+n)+n(1-\alpha))$.
$\mathrm{c}=-i(1-\alpha)$. $\mathrm{f}=\dfrac{-\left(i(1+n)+n(1-\alpha)\right)\cdot i(1-\alpha)}{i(1+n)}=\dfrac{\mathrm{b\cdot c}}{\mathrm{a}}$.

For $\beta^*(\mathrm{n})$ and $\widetilde{\beta^*}(n)$:

5–1 $\boldsymbol{\beta^*(n)}$: $\mathrm{y}=\dfrac{\mathrm{cx+d}}{\mathrm{ax+b}}$. $\mathrm{a}=\Omega^*\cdot i$. $\mathrm{b}=i(1-\alpha+\Omega^*)$. $\mathrm{c}=\Omega^*\,(i+(1-\alpha))$.
$\mathrm{d}=\Omega^*\cdot i$. and $\mathrm{f}=\dfrac{1-\Omega^*(1-\alpha+i)i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}=\dfrac{1-\mathrm{b\cdot c}}{\mathrm{a}}$.

5–3 $\boldsymbol{\widetilde{\beta^*}\ (n)}$: $\mathrm{y}=\dfrac{\mathrm{cx+d}}{\mathrm{ax+b}}$. $\mathrm{a}=\Omega^*\cdot i$. $\mathrm{b}=i(1-\alpha+\Omega^*)$.
$\mathrm{c}=\Omega^*\cdot i-\Omega^*(i+(1-\alpha))$. $\mathrm{d}=i(1-\alpha)$.
$\mathrm{f}=(1-\alpha)\{i+(1-\alpha+\Omega^*)\}$.

For $\beta^*(i)$ and $\alpha(i)$:

6–1 $\boldsymbol{\widetilde{\beta^*}\ (i)}$: $\mathrm{y}=\dfrac{\mathrm{cx+d}}{\mathrm{ax}}$. $\mathrm{a}=(1-\alpha)+\Omega^*\,(1+\mathrm{n})$. $\mathrm{c}=(1+\mathrm{n})\Omega^*$.
$\mathrm{d}=\mathrm{f}=(1-\alpha)\,\Omega^*\cdot\mathrm{n}$.

6–3 $\boldsymbol{\alpha(i)}$: $y = \dfrac{cx+d}{ax+b}$. $a = \beta^*$. $b = -\Omega^* \cdot n$. $c = -\{(1+n)(1-\beta^*)\,\Omega^* - \beta^*\}$.

$d = -\Omega^* \cdot n$.

$$f = -\Omega^* \cdot\ n - \frac{\Omega^* \cdot n\{(1+n)(1-\beta^*)\Omega^* - \beta^*\}}{\beta^*} = -\left(d + \frac{b \cdot c}{a}\right).$$

III. Final form common to twenty four hyperbola equations

Examples: final form of 24 hyperbola equations

(Case of the US, 2007, using KEWT 4.10, 1990-2008)

For *speed*(i): $(y - 0)\ (x + 0.05557852) = 6.56530$.

For i(*speed*): $(y + 0.05557852)\ (x+0) = 6.56530$.

For *speed*(n): $(y - 0)\ (x+0.017366348) = 1.1481903$.

For n(*speed*): $(y + 0.0173663480)\ (x + 0) = 1.1481903$.

For r^*(i), $(y - 0.0516375)\ (x + 0) = 0.00168776$.

For $i(r^*)$, $(y - 0)\ (x - 0.051647739) = 0.00168776$.

For $r^*(n)$: $y = 1.79976276x + 0.051140425$, where $y = 0.0686$ when $x = 0.00972$.

For $n(r^*)$: $y = 0.55562878x - 0.028415092$, where $y = 0.00972$ when $x = 0.0686$.

For $\Omega^*(i)$: $(y - 2.50012)\ (x + 0.032684829) = -0.0817012$.

For $i(\Omega^*)$:$(y + 0.032684829)\ (x - 2.50012) = -0.0817012$.

For $\Omega^*(n)$: $(y - 0)\ (x+0.028415093) = 0.071735923$.

For $n(\Omega^*)$: $(y + 0.028415093)\ (x + 0) = 0.071735923$.

For $i(n)$: $(y + 3.395322)\ (x - 0.3419923) = -1.161174$.

For $n(i)$: $(y - 0.3419923)(x + 3.395322) = -1.161174$.

For $\Omega^*(\beta^*)$: $(y + 0.862516341)(x - 1.084427581) = -0.935336465$.

For $\beta^*(\Omega^*)$: $(y - 1.084427581)(x + 0.862516341) = -0.935336465$.

For $\beta^*(n)$: $(y - 9.77039275)(x + 1.46297379) = -13.2938285$.

For $n(\beta^*)$: $(y + 1.46297379)(x - 9.77039275) = -13.2938285$.

Setting $\widetilde{\beta^*} = 1 - \beta^*$,

For $\widetilde{\beta^*}(n)$: $(y + 8.77039)(x + 1.46297379) = 13.2938266$.

For $n(\widetilde{\beta^*})$: $(y - 1.46297379)(x + 8.77039) = -13.2938266$.

Setting $\tilde{\alpha} = 1 - \alpha$,

For $\beta^*(i)$: $(y - 0.68562793)(x + 0) = 0.00574807$.

For $\widetilde{\beta^*}(i)$: $(y - 0.31432938)(x + 0) = -0.00574807$.

For $\alpha(i)$: $(y - 0.34473158)(x - 0.024592) = -0.016114484$.

For $\tilde{\alpha}(i)$: $(y - 0.655268338)(x - 0.024592) = 0.016114484$.

IV. Findings in 24 hyperbola equations

Finding 1: Inverse function: e.g., $i = i(\Omega^*)$ is the inverse function of $\Omega^* = \Omega^*(i)$

For Ω^*(i): (y – 2.50012) (x + 0.032684829) = –0.0817012.

For i(Ω^*): (y + 0.032684829) (x – 2.50012) = –0.0817012.

1. The c/a value of $\Omega^*(i)$ is the same as the b/a value of $i(\Omega^*)$.
2. The b/a value of $\Omega^*(i)$ is the same as the c/a value of $i(\Omega^*)$. At 1. and 2., related values cross each other.
3. Each f/a value of the RHD is the same, with the same sign. ‘Downward’ to the right.

Finding 2: Relationship between $\beta^*(i)$ & $\widetilde{\beta^*}(i)$ and, $\alpha(i)$ & $\tilde{\alpha}(i)$, each in

$$\left(y-\frac{c}{a}\right)\left(x+\frac{b}{a}\right)=\frac{f}{a}$$

For $\beta^{*}(i)$:(y – 0.68562793) (x + 0) = 0.00574807.

For $\widetilde{\beta^{*}}(i)$: (y – 0.31432938) (x + 0) = –0.00574807, where $\widetilde{\beta^{*}} = 1 - \beta^{*}$.

1. The sum of c/a values of $\beta^{*}(i)$ and $\widetilde{\beta^{*}}(i)$, at the first term of the LHS, is equal to 1.0.
2. Each b/a value is the same at the second term of the LHD; under the same $i = I/Y$.
3. The two f/a values at the RHD are the same but, each sign differs. 'Upward' to the right (the less $i = I/Y$ the more β^{*}) versus downward to the right (the less. $i = I/Y$ the less $\widetilde{\beta^{*}}$).

For $\alpha(i)$: (y – 0.34473158) (x – 0.024592) = –0.016114484.

For $\tilde{\alpha}(i)$: (y – 0.655268338) (x – 0.024592) = 0.016114484, where $\tilde{\alpha} = 1 - \alpha$.

1. The sum of c/a values of $\alpha(i)$ and $\tilde{\alpha}(i)$, at the first term of the LHS, is equal to 1.0.
2. Each b/a value is the same at the second term of the LHD; under the same $i = I/Y$.
3. The two f/a values at the RHD are the same but, each sign differs. 'Downward' to the right (the less $i = I/Y$ the less α) versus upward to the right (the less $i = I/Y$ the more $\tilde{\alpha}$).

V. Forms of 24 hyperbola equations by function

For speed: $y=\frac{1}{ax+b}$ at 1–1 and 1–3. $y=\frac{cx+1}{ax}$ at 1–2 and 1–4.

For r^{*} and Ω^{*}: $y=\frac{cx+d}{ax}$ at 2.1 and 3–4. $y=\frac{d}{ax+b}$ at 2–2 and 3.3.

$y=\frac{cx+d}{b}$ at 2–3 and 2.4 (linear). $y=\frac{cx}{ax+b}$ at 3–1 and 3–2.

For $i(n)$, $n(i)$, $\Omega^*(\beta^*)$, $\beta^*(\Omega^*)$: $y = \dfrac{cx}{ax+b}$ at 4–1, 4–2, 4–3, and 4–4.

For $\beta^*(n)$, $n(\beta^*)$, $\widetilde{\beta^*}(n)$, $n(\widetilde{\beta^*})$: $y = \dfrac{cx+d}{ax+b}$ at 5–1, 5–2, 5–3, and 5–4.

For $\beta^*(i)$, $\widetilde{\beta^*}(i)$, $\alpha(i)$: $y = \dfrac{cx+d}{ax}$ at 6–1, 6–2, and 6–3.

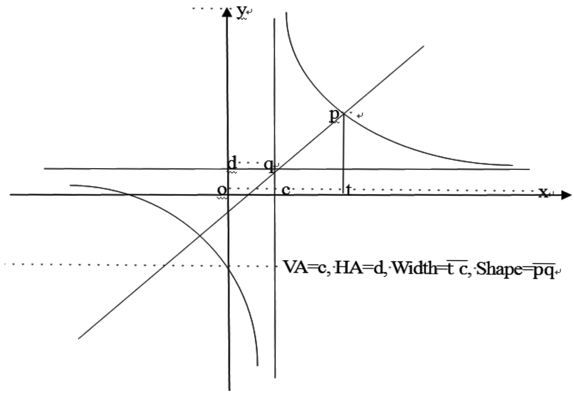
For $\tilde{\alpha}(i)$: $y = \dfrac{cx}{ax+b}$ at 6–4.

VI. Characteristics of hyperbola equations, with the Excel cell addresses

cell of Excel	y axis by sector		H. A.	V. A.	sign of f/a	quadrants
PS	*speed*(i)	+	0	yes (–)	+ or –	1, 2
KD, PY	*speed*(n)	+	0	yes (–)	+	1, 2
JS; PV	$r^*(n)$	+ or –	yes (+ or –)	0	+ or –	1 or 4
KG	$r^*(n)$	+ or –	0	0	grad. & interce	1 or 2
QX	$\Omega^*(n)$	+	yes (+)	yes (–or+)	+ or –	1, 2, 4
WI	$\Omega^*(n)$	+	0	yes (–)	+	1, 2
VG	$\beta^*(n)$	+	yes (+)	yes (–)	–	1, 2, 4
VR		+	yes (–)	yes (–)	+	1, 2
WU	$i(n)$	+	yes (–)	yes (–)	–	1, 2, 4
XE	$\Omega^*(\beta^*)$	+	yes (–)	yes (+)	–	1
XO	$\beta^*(i)$	+	yes (+)	0	+	1
XY	$\alpha(i)$	+	yes (+)	yes (+)	–	1

Note: Sign (+, –) of f/a determines quadrants, where 2f/a is the hypotenuse (as the principal axis) of the hyperbola. If this sign changes repeatedly in 1990–2008, it indicates that policies by government work for adjustment by year (even without relying on endogenous data). Some of endogenous parameters and variables (*i*, *Omega*, $delta_0$, and *the speed*) cannot be minus, while others such as the real rate of return and the real cost of capital be minus at the 4th quadrant. Market values in the long run do not interrupt but evaluate the real assets, as shown in *International Advances in Economic Research* 16 (3): 282–296. If hyperbola equations are sound and robust, any fund for short-sighted speculations cannot spread its work in the World.

When *f/a* is plus, the diagonal is upward to the right



When *F/A* is minus, the diagonal is downward to the right

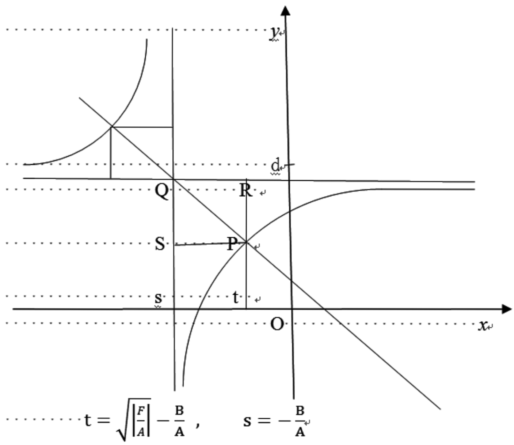


Figure 1 Relationship between the rectangular hyperbola and the rectangular equilateral triangle: *f/a > 0* versus *F/A < 0*

VII. Key graphic expositions, using K. Tomoda's exclusive 'Grapes' for hyperbola equations

This section shows sixteen expositions of hyperbola equations. For these graphs, the author uses Katsuhisa Tomoda's Grapes.[2)] His graphic software was rearranged for the author's purpose, where the author shows 'made by Kamiryo and Tomoda' at each below of graphs when the author have discussions as done at Shanghai on 17th of Sep 2010. In each figure below, the author compares six countries, China, India, the US, Japan, phillipines, and Sigapore. Repeating, the larger the base of rectangular equilateral triangle or the square the more sustainable robustness is regardless of the differences of economic stages (see Singapore).

Figure 1 Hyperbola equation of $speed(i)$: related to endogenous equilibrium
Figure 2 Hyperbola equation of $speed(n)$: related to endogenous equilibrium
Figure 3 Hyperbola equation of $r^*(n)$: related to endogenous unemployment
Figure 4 Hyperbola equation of $r_G\ (n_G)$: related to endogenous unemployment
Figure 5 Hyperbola equation of $r_{PRI}\ (n_{PRI})$: related to endogenous unemployment
Figure 6 Hyperbola equation of $\alpha\,(i)$: inflation countries versus deflation country, Japan
Figure 7 Hyperbola equation of $r^*(i)$: inflation countries versus deflation country, Japan
Figure 8 Hyperbola equation of $\Omega^*(i)$: difference and characteristics of economic stages
Figure 9 Hyperbola equation of $\beta^*(i)$: characteristics of technology and net investment
Figure 10 Hyperbola equation of $\beta^*(n)$: characteristics between technology and population
Figure 11 Hyperbola equation of $r_G^*(i_G)$: related to endogenous inflation/deflation at the G sector

2) Katsuhisa Tomoda has worked for the formulations of various functions and equations for the last twenty years or more. This time, he set up a series of 'exclusive' Grapes of hyperbola equations. His 'general' Grapes has been open to schools and researchers for many years. Graphics to hyperbola equations is one of most difficult task, for which I am grateful to Katsuhisa Tomoda. The author believes that hyperbola graphics reveals required macro-policies rigidly and useful to policy-makers by country and in the World.

A postscript on 26 Dec 2010:

The author formulated KEWT 5.11 on 23 Dec 2010, by adding 2009 data to KEWT 4.10; since IMF yearbook was renewed earlier than usual. KEWT 5.11 presents 65 countries by sector with three averages, 14 Euro currency countries, 15 European countries except for Euro currency, and 17 Pacific and Asian countries. The remainders are 19 for Latin America, Near East, and Africa, without their averge due to some reasons.

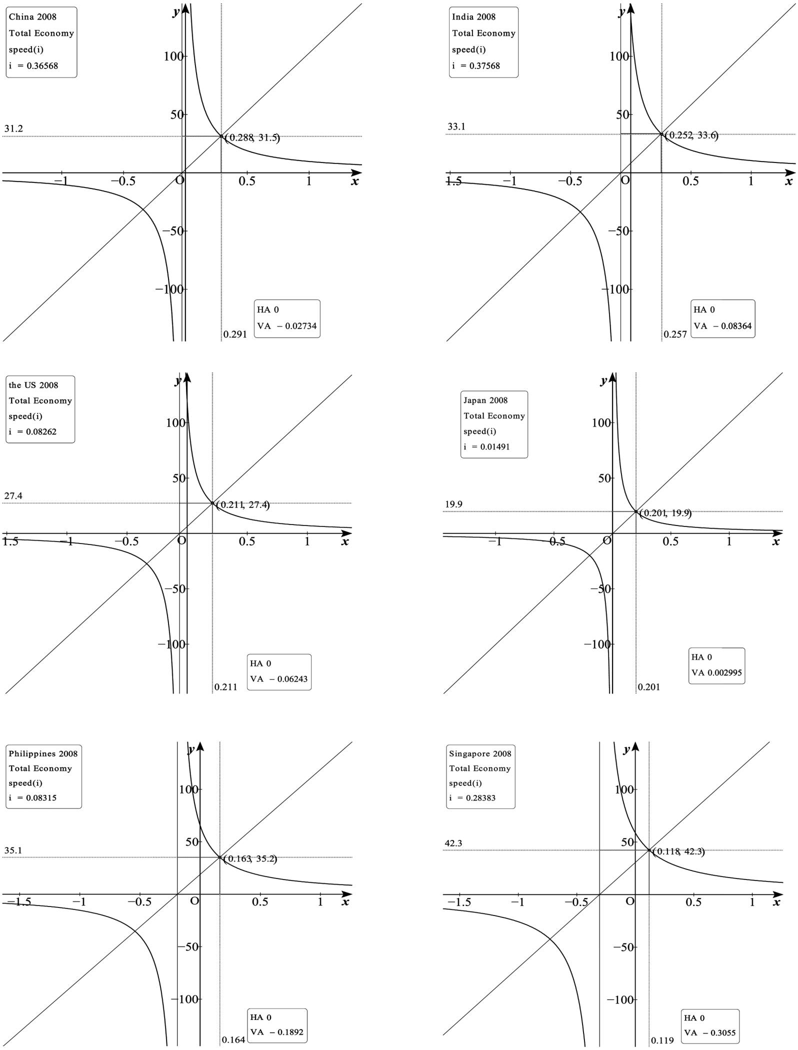


Figure 1 Hyperbola equation of *speed*(*i*): related to endogenous equilibrium

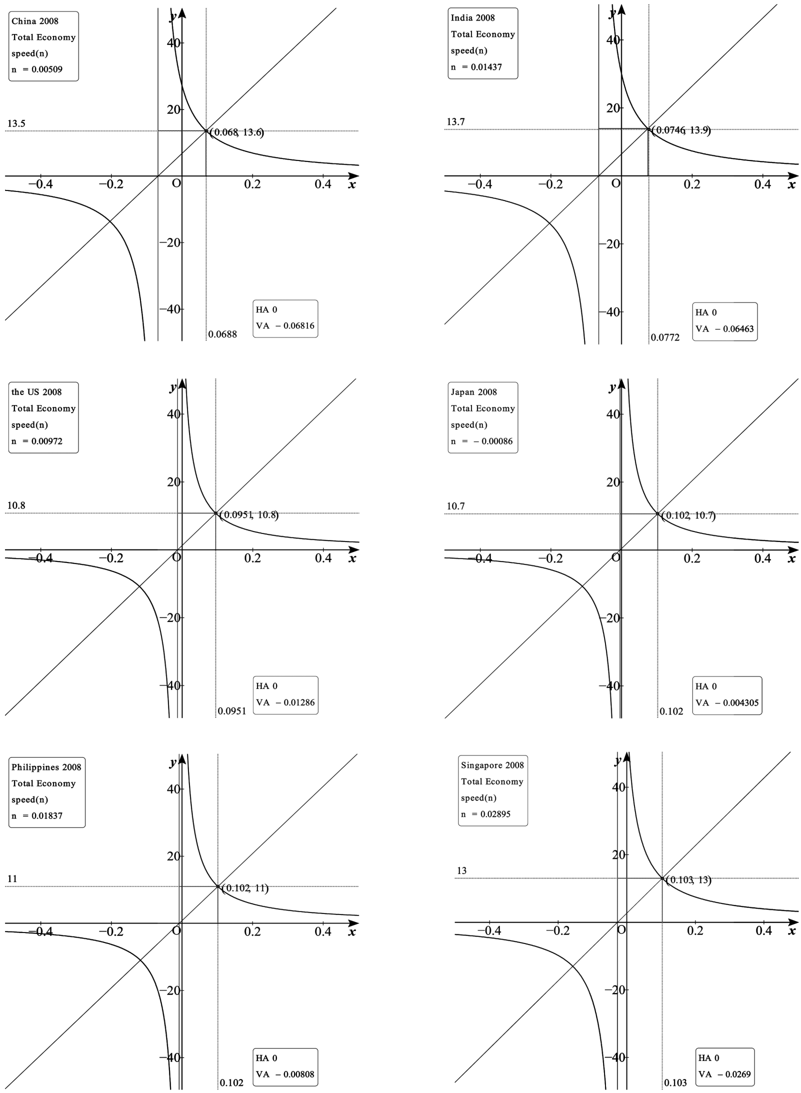


Figure 2　Hyperbola equation of *speed*(*n*): related to endogenous equilibrium

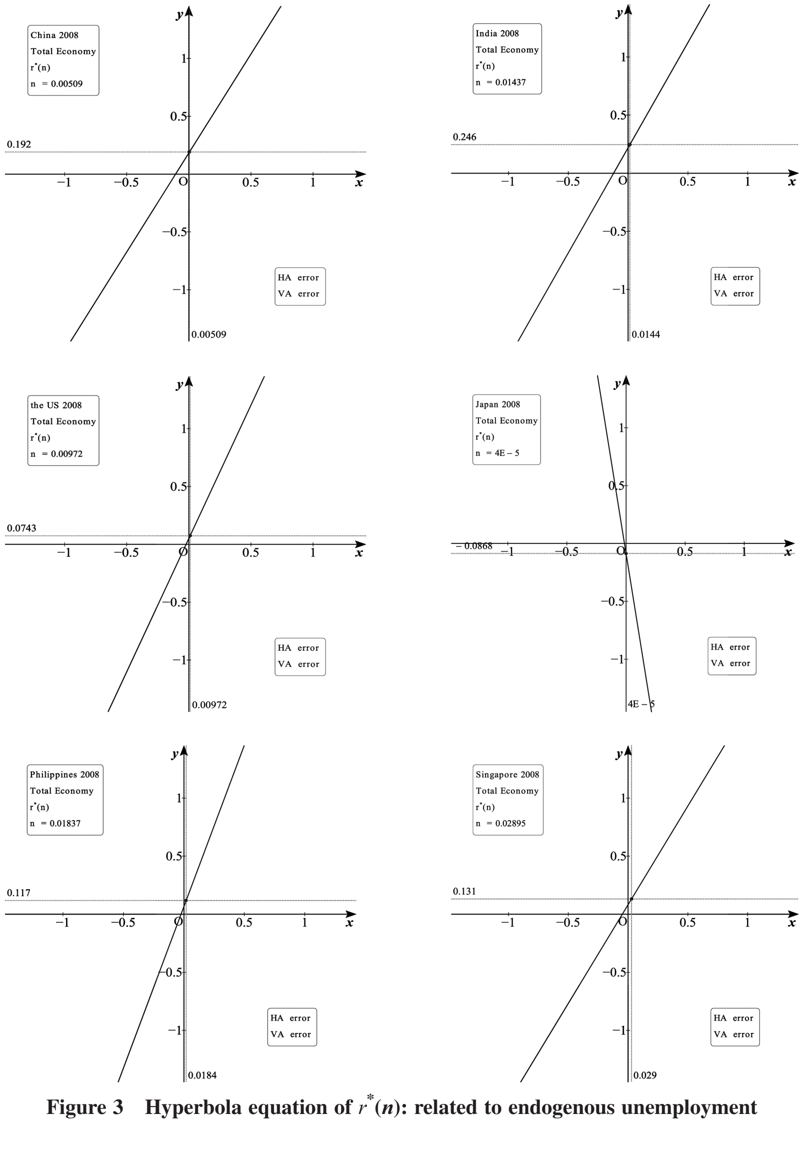


Figure 3 Hyperbola equation of $r^*(n)$: related to endogenous unemployment

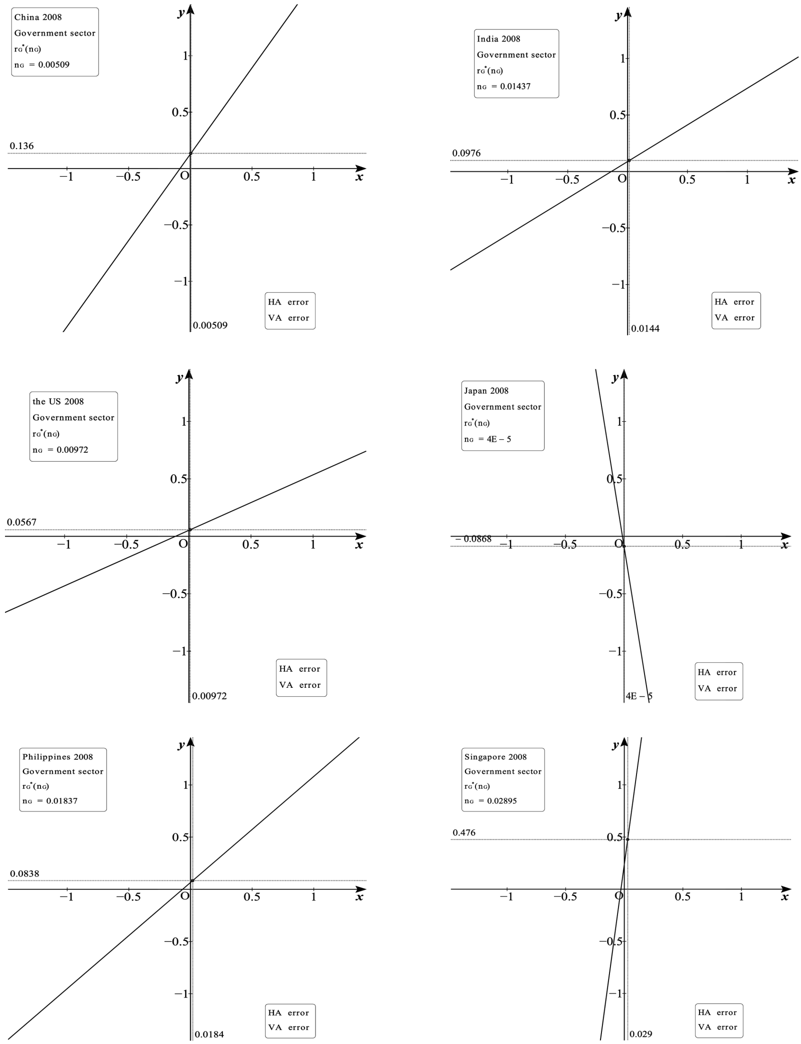


Figure 4 Hyperbola equation of r_G (n_G): related to endogenous unemployment

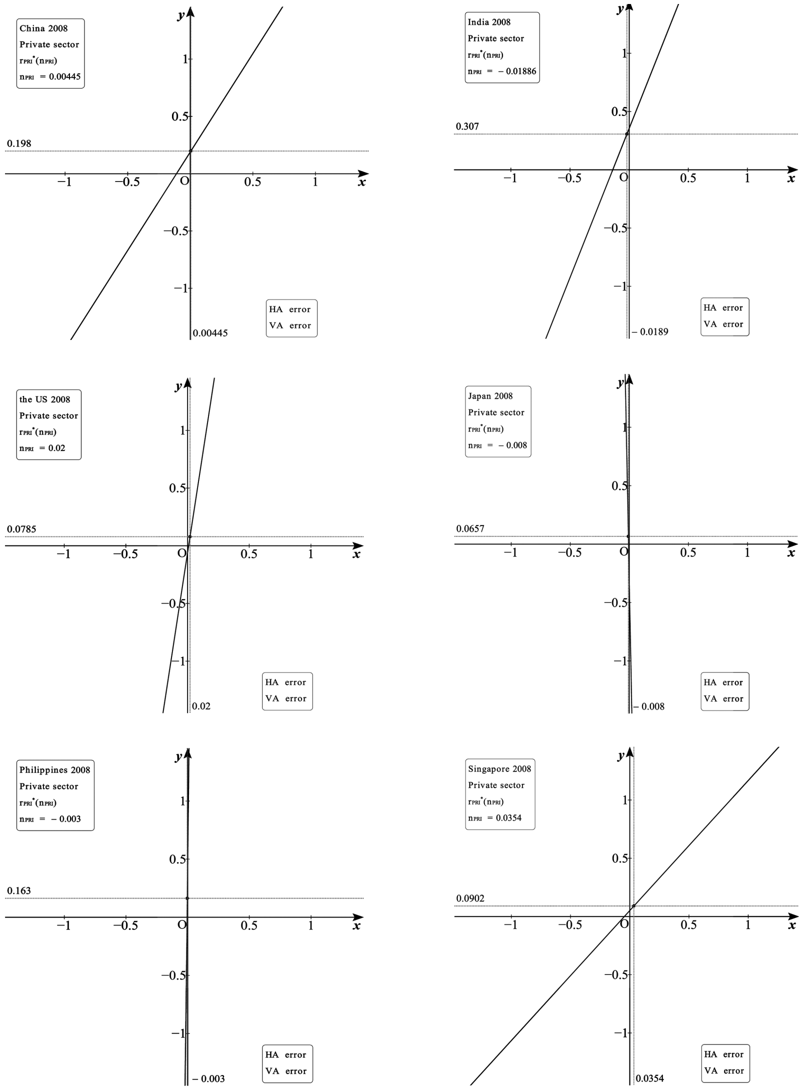


Figure 5 Hyperbola equation of r^*_{PRI} (n_{PRI}): related to endogenous unemployment

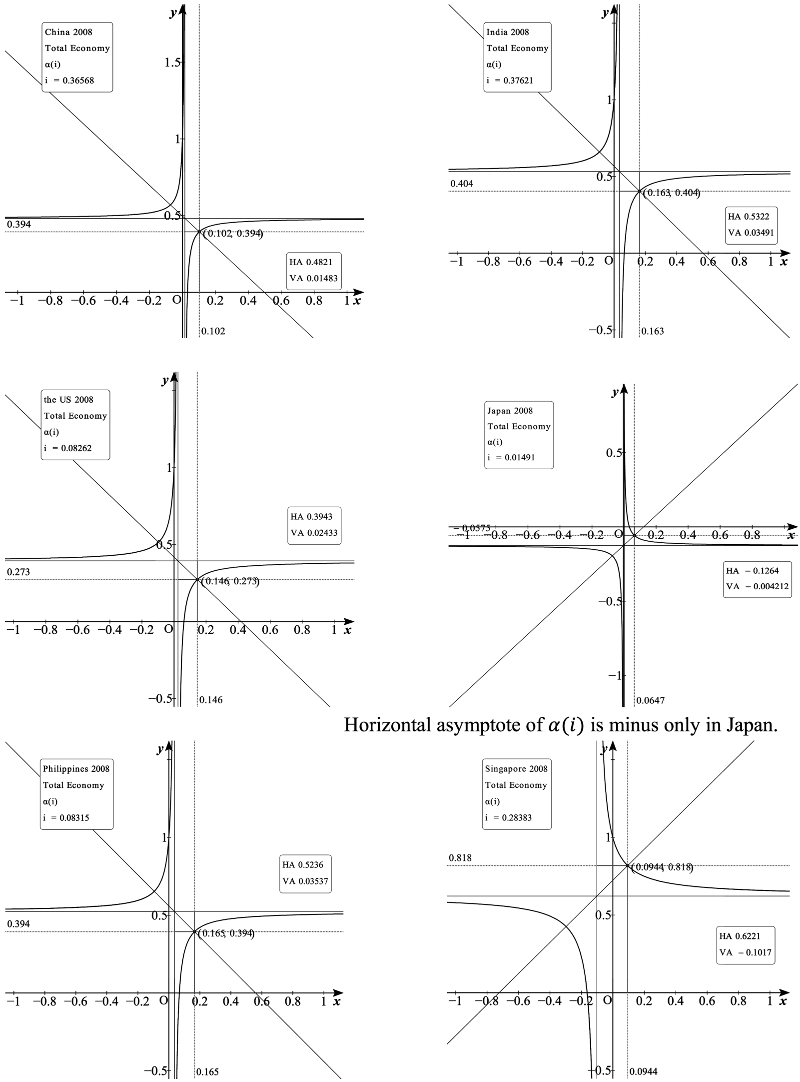
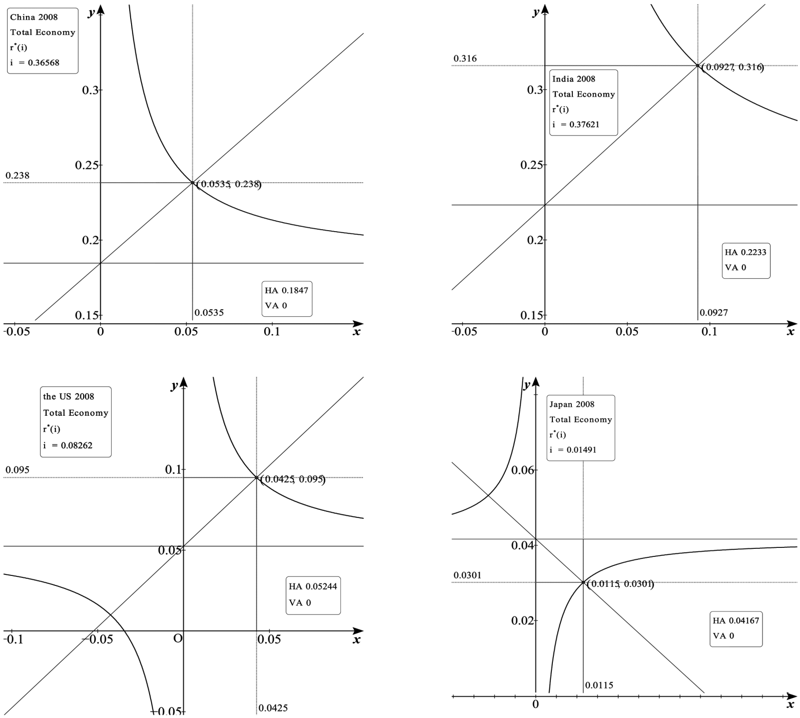


Figure 6 Hyperbola equation of $\alpha(i)$: inflation countries versus deflation country, Japan



Japan 2008: shrinks to the extreme due to deficit.

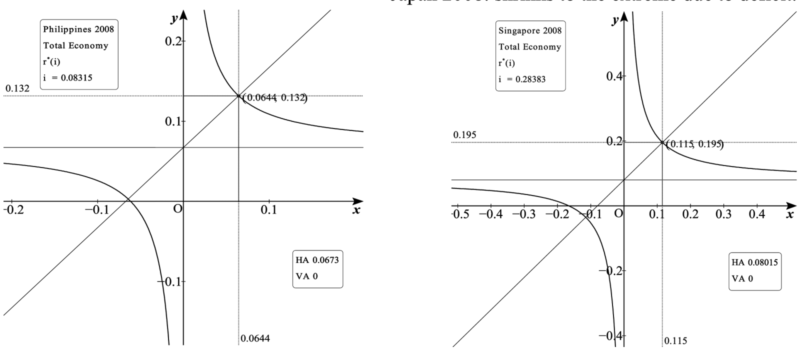


Figure 7 Hyperbola equation of $r^*(i)$: inflation countries versus deflation country, Japan

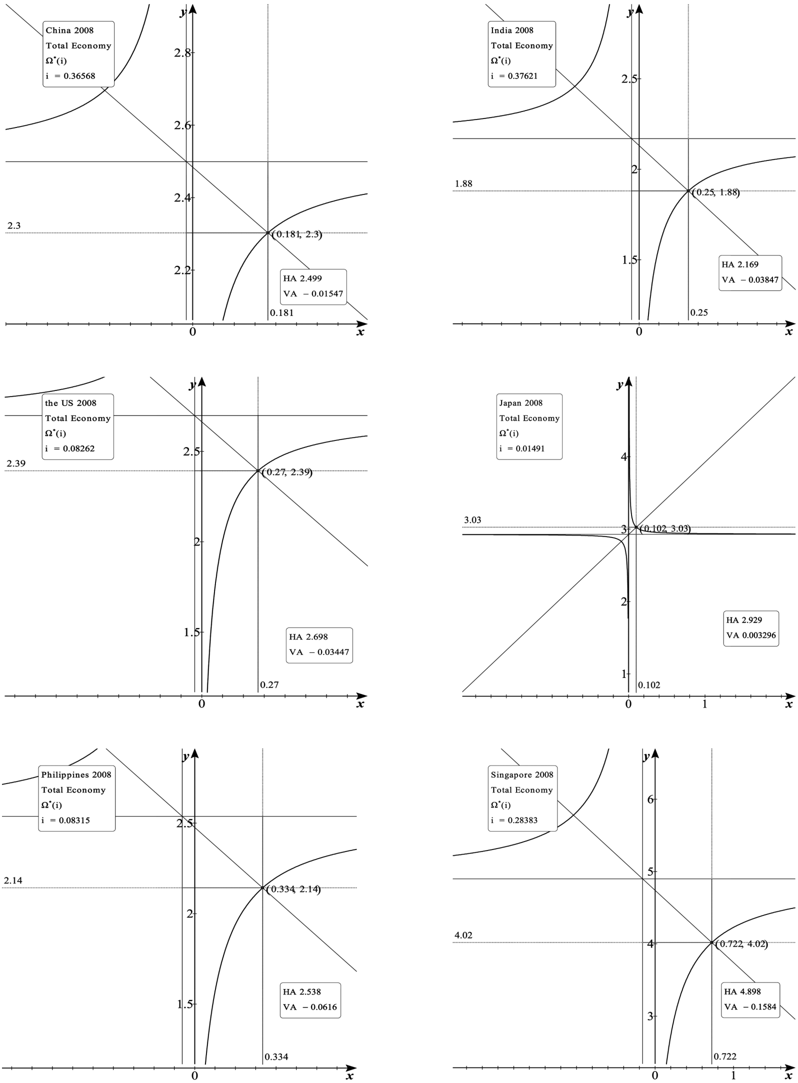


Figure 8 Hyperbola equation of $\Omega^{*}(i)$: difference and characteristics of economic stages

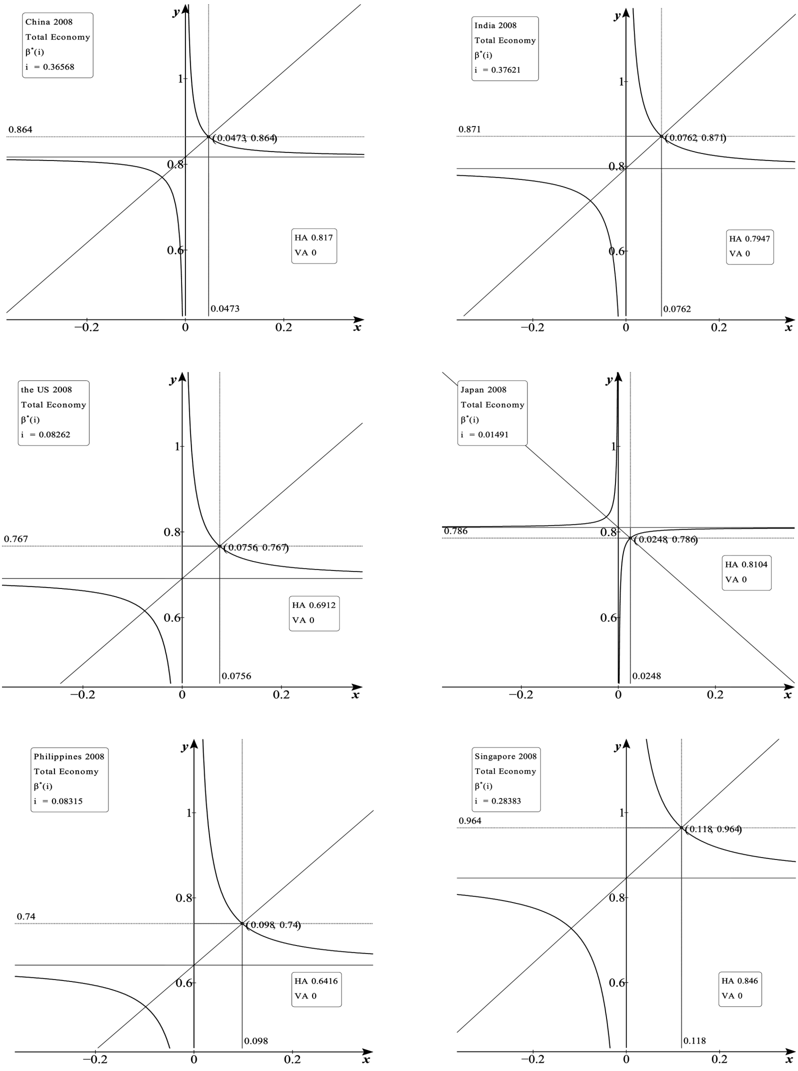


Figure 9　Hyperbola equation of $\beta^*(i)$: characteristics of technology and net investment

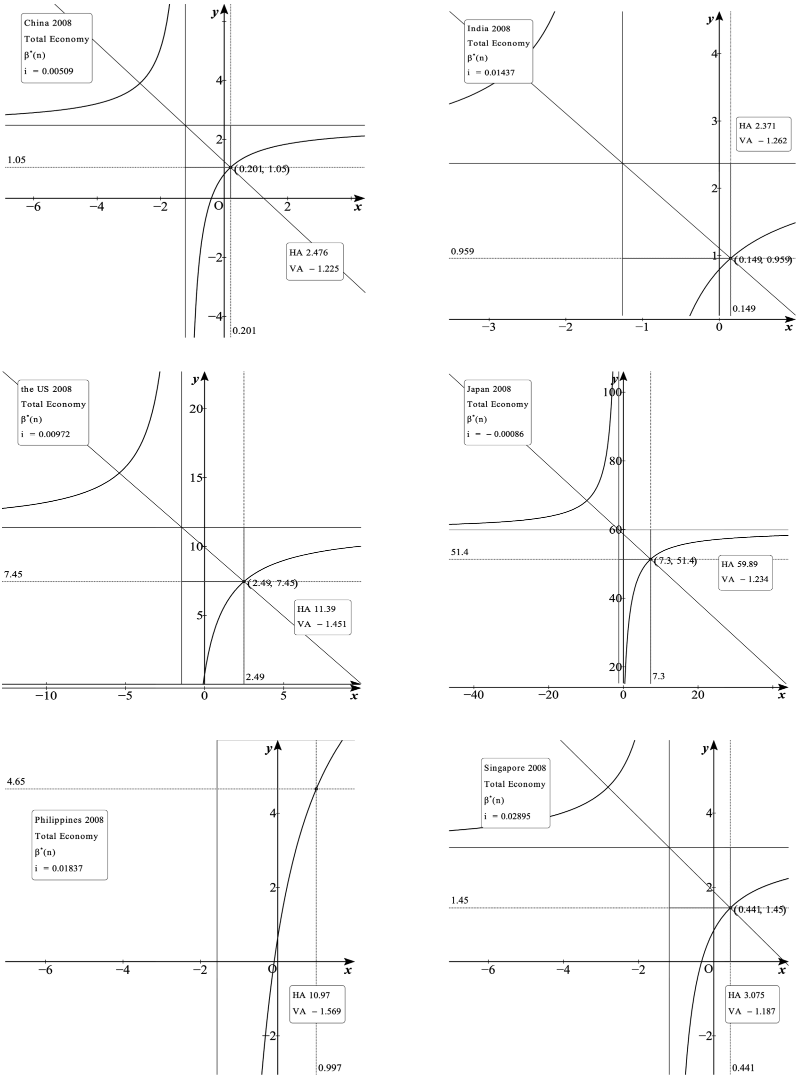


Figure 10 Hyperbola equation of $\beta^*(n)$: characteristics between technology and population

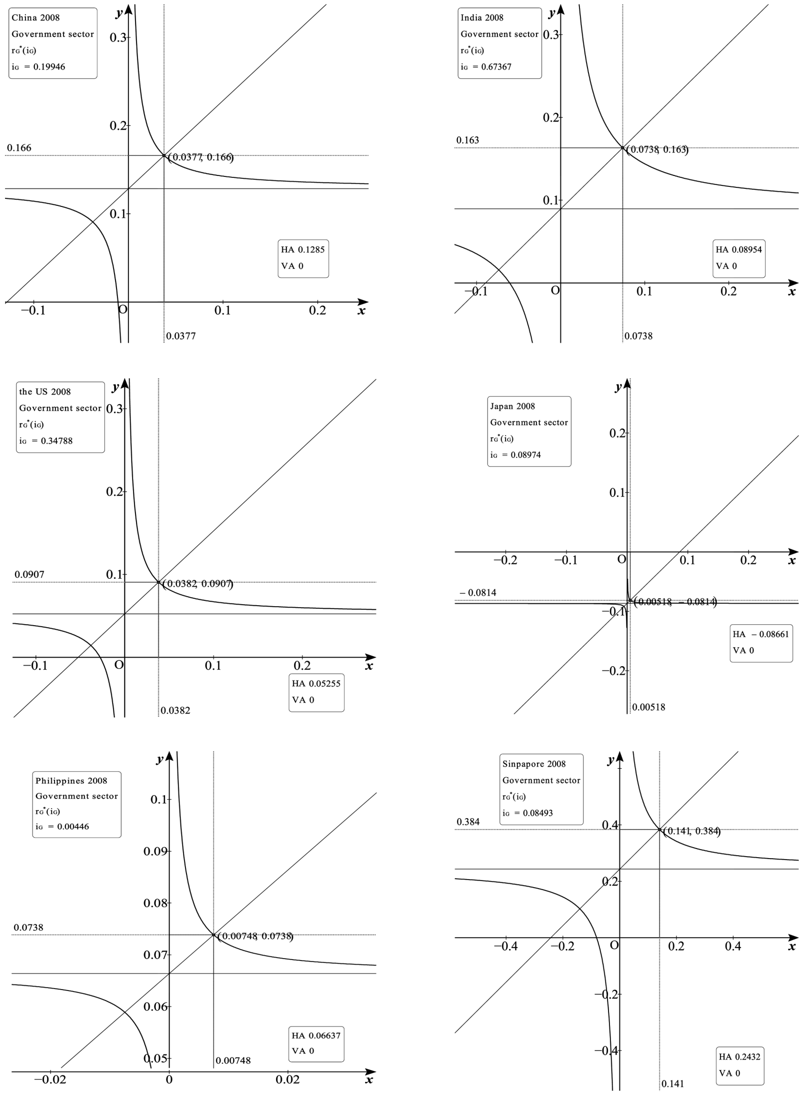


Figure 11 Hyperbola equation of r_G^* (i_G): related to endogenous inflation/deflation at the G sector

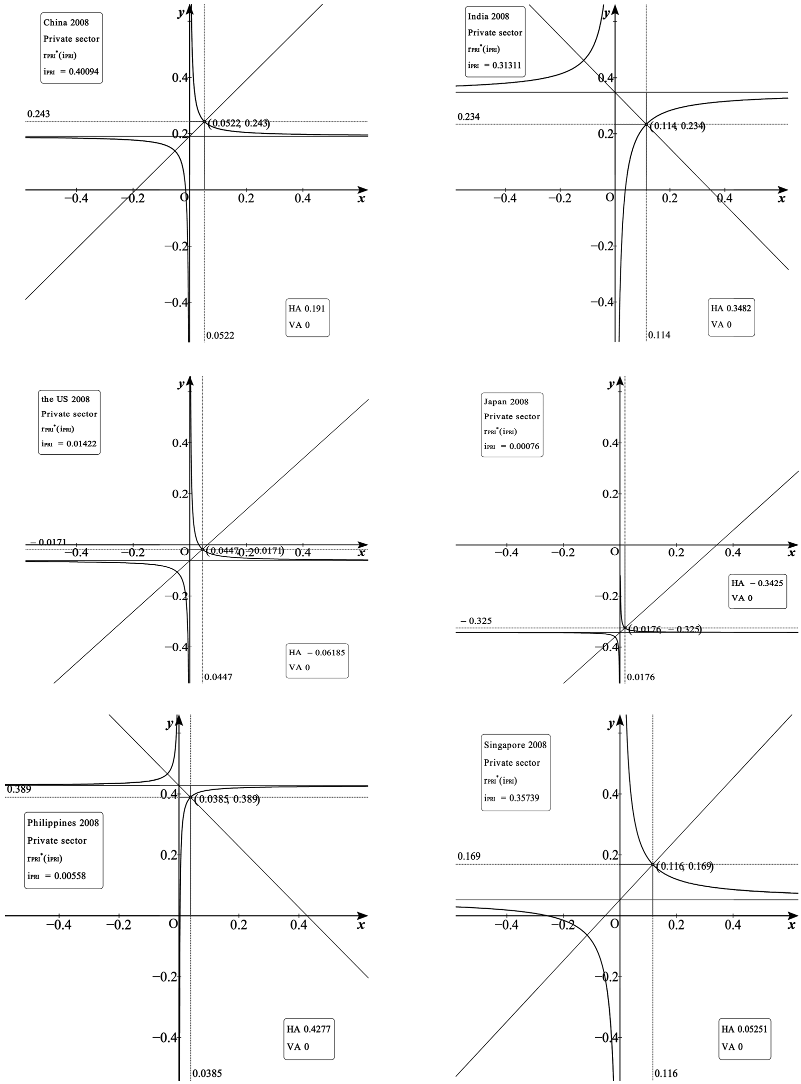


Figure 12　Hyperbola equation of r^*_{PRI} (i_{PRI}): related to endogenous inflation/deflation at the PRI sector

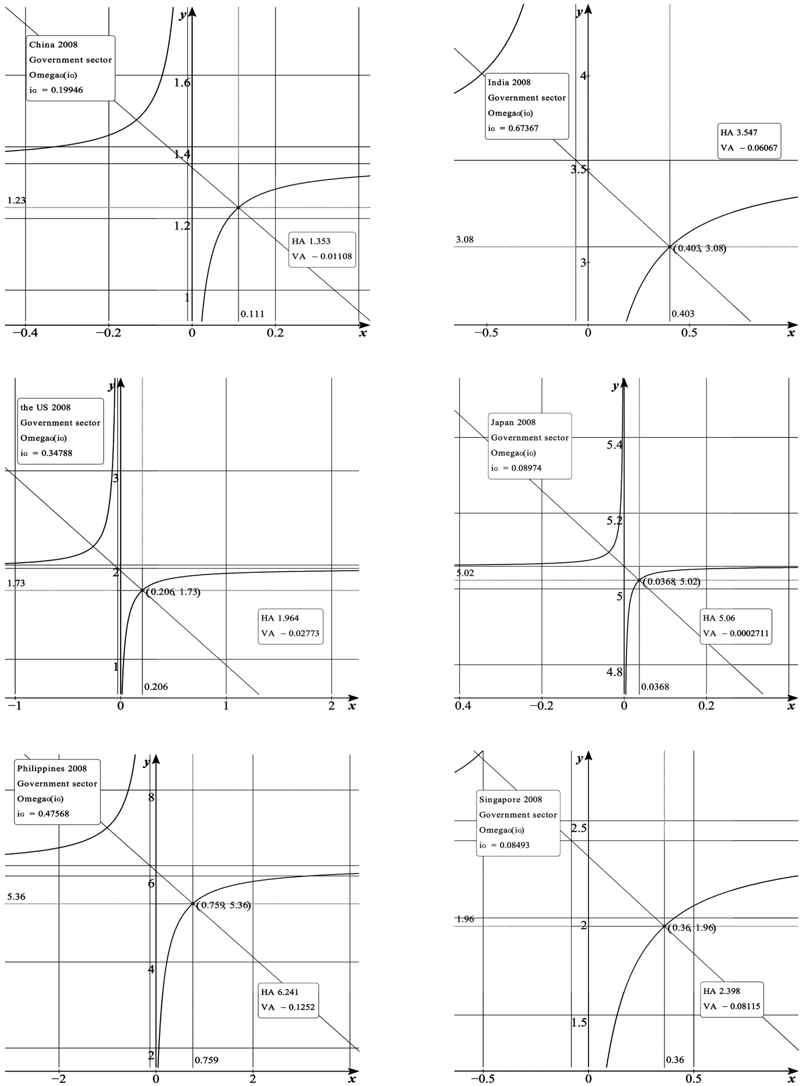
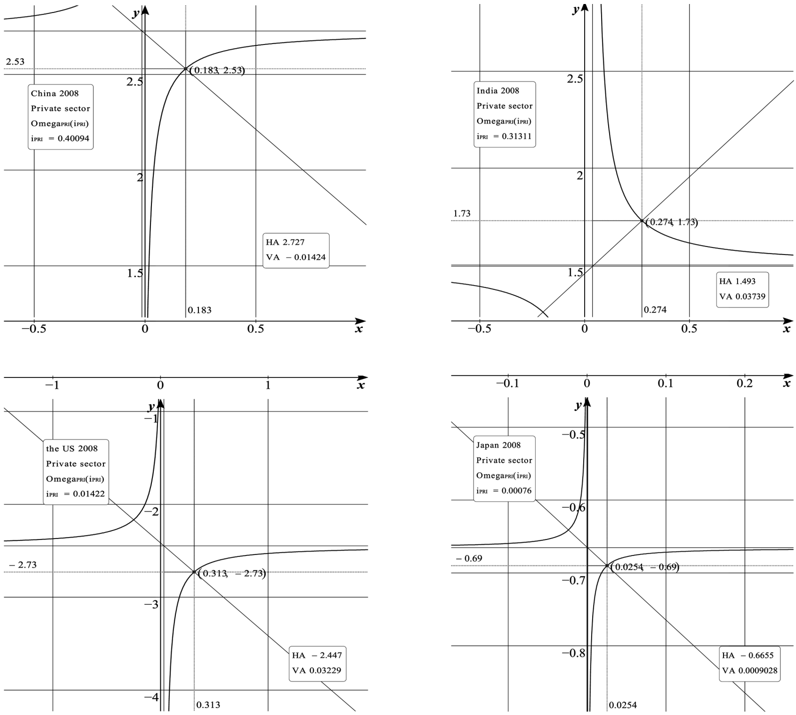


Figure 13 Hyperbola equation of Ω_G (i_G): related to the upper limit of economic stages at the G sector



The private sector of Japan 2008 has fallen into disequilibrium.

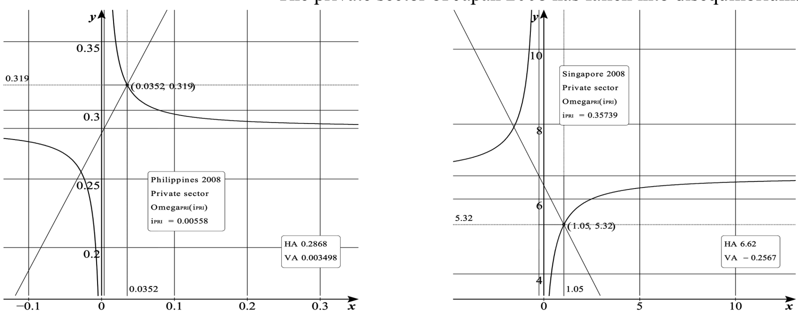


Figure 14 Hyperbola equation of Ω_{PRI} (i_{PRI}): related to the upper limit of economic stages at the PRI sector

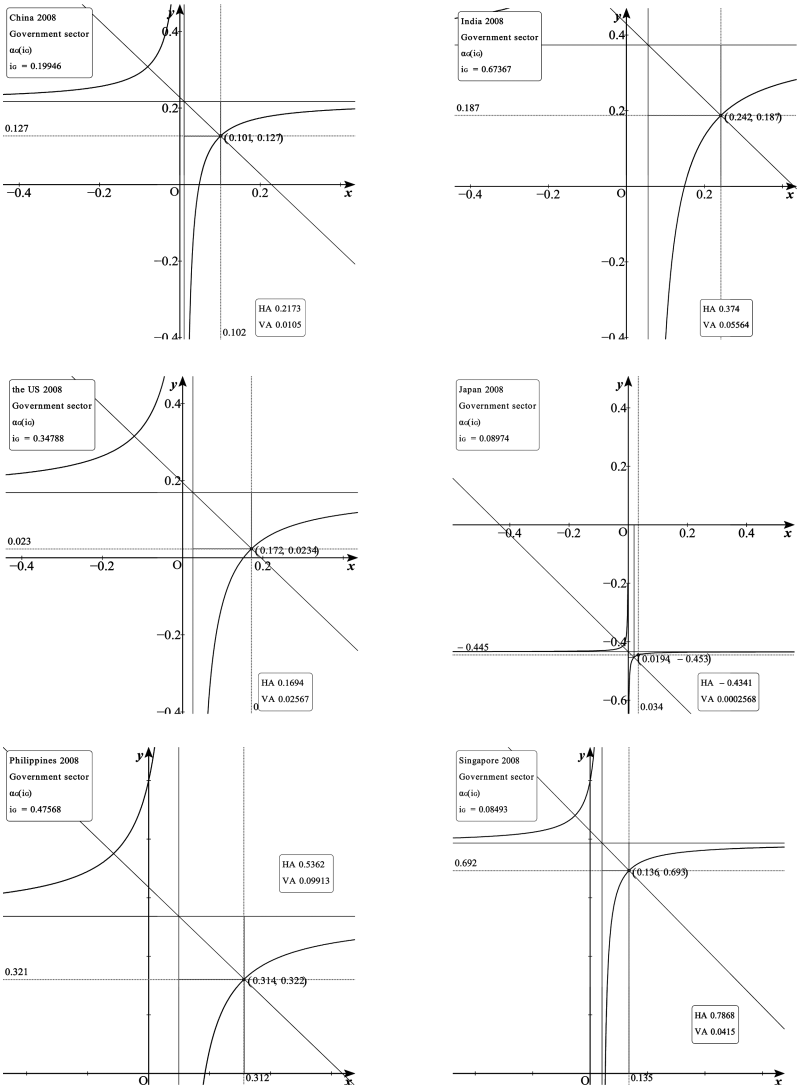


Figure 15 Hyperbola equation of α_G (i_G): related to stop macro-inequality at the G sector

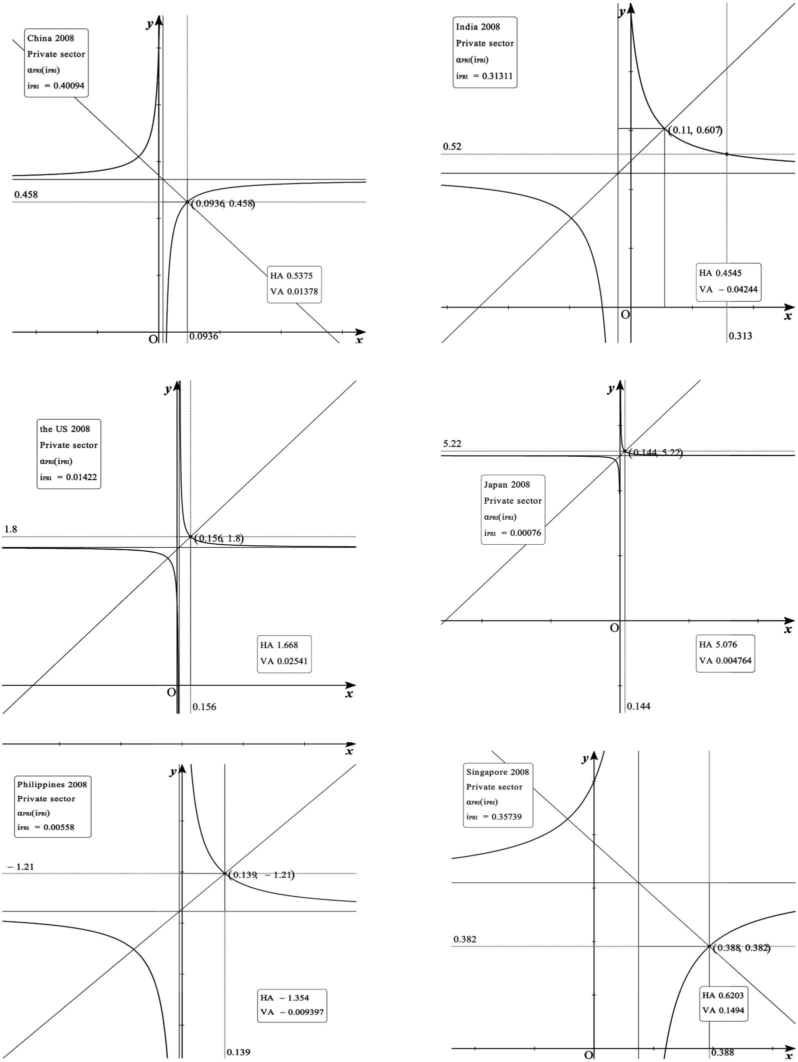


Figure 16　Hyperbola equation of α_{PRI} (i_{PRI}): related to stop macro-inequality at the PRI sector

Appendix Twelve specified hyperbola equations, with calculation processes

24 hyperbola equations are composed of 12 basic and 12 specified equations. 12 basic equations were already summarized in *JES* 14 (1, Sep), 2010. The followings are 12 specified equations.

4–1 $i(n)$:

$$i(n)=\frac{-(1-\alpha)\Omega^{*}\cdot n}{(1-\beta^{*})\Omega^{*}\cdot n+\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}}.$$

Here starting with $\beta^{*}=\dfrac{(1+n)\Omega^{*}\cdot i+(1-\alpha)\Omega^{*}\cdot n}{i\left((1-\alpha)+\Omega^{*}(1+n)\right)}$ and using $\beta^{*}i\left((1-\alpha)+\Omega^{*}(1+n)\right)$ = $(1+n)\Omega^{*}\cdot i+(1-\alpha)\,\Omega^{*}\cdot$ n.

$$\mathrm{y}=\frac{\mathrm{cx}}{\mathrm{ax+b}}\ \text{and}\ \mathrm{y}=\frac{\mathrm{c}}{\mathrm{a}}+\frac{-\frac{\mathrm{b\cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}.$$

a = $(1-\beta^{*})\Omega^{*}$, b = $(1-\beta^{*})\,\Omega^{*}-\beta^{*}\,(1-\alpha)$, and c = $-(1-\alpha)\,\Omega^{*}$.

$$\mathrm{f}=\frac{-\left(\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}\right)\cdot(1-\alpha)\Omega^{*}}{(1-\beta^{*})\Omega^{*}}=\frac{\mathrm{b\cdot c}}{\mathrm{a}}.$$

$$\frac{\mathrm{f}}{\mathrm{a}}=\frac{-\left(\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}\right)\cdot(1-\alpha)\Omega^{*}}{\left((1-\beta^{*})\Omega^{*}\right)^{2}}=\frac{\mathrm{b\cdot c}}{\mathrm{a}^{2}}.$$

$$VA_{i(n)}=\frac{(1-\beta^{*})\Omega^{*}-\beta^{*}(1-\alpha)}{(1-\beta^{*})\Omega^{*}}=\frac{-\mathrm{b}}{\mathrm{a}}.\quad HA_{i(n)}=\frac{-(1-\alpha)}{(1-\beta^{*})}=-\frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{i(n)}=\sqrt{\left|\frac{\left(\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}\right)\cdot(1-\alpha)\Omega^{*}}{-\left((1-\beta^{*})\Omega^{*}\right)^{2}}\right|}.$$

$$Shape_{i(n)}=\sqrt{2\left|\frac{\left(\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}\right)\cdot(1-\alpha)\Omega^{*}}{-\left((1-\beta^{*})\Omega^{*}\right)^{2}}\right|}.$$

$$Curvature_{i(n)}=1/\sqrt{2\left|\frac{\left(\beta^{*}(1-\alpha)-(1-\beta^{*})\Omega^{*}\right)\cdot(1-\alpha)\Omega^{*}}{\left((1-\beta^{*})\Omega^{*}\right)^{2}}\right|}.$$

$i(n) = \dfrac{-1.63825n}{0.48250215n + 0.165012}$, where a = 0.48250215, b = 0.165012, c = –1.63825, $b \cdot c$ = –0.270330909. f = –0.560268817 = $\dfrac{-bc}{a}$, and $\dfrac{f}{a}$ = –1.16117372.

$VA_{i(n)}$ = –0.3419923 = $\dfrac{-b}{a}$. $HA_{i(n)}$ = –3.395322 = $\dfrac{c}{a}$.

$Width_{i(n)}$ = 1.0775777 = $\sqrt{1.16117372}$. $Shape_{i(n)}$ = 1.523925 = $\sqrt{2.32234744}$.

$Curvature_{i(n)}$ = 0.6562 = $1/\sqrt{2.32234744}$.

$\left(y - \dfrac{c}{a}\right)\left(x + \dfrac{b}{a}\right) = \dfrac{f}{a}$. For $i(n)$: $(y + 3.395322)(x - 0.3419923) = -1.161174$.

4–2 $n(i)$:

$$n(i) = \frac{-\left((1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right)i}{(1-\beta^*)\Omega^* \cdot i + \Omega^*(1-\alpha)}.$$

Here starting with $\beta^* = \dfrac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{i\left((1-\alpha) + \Omega^*(1+n)\right)}$ and using $\beta^* i\left((1-\alpha) + \Omega^*(1+n)\right) = (1+n)\Omega^* \cdot i + (1-\alpha)\,\Omega^* \cdot n$.

$$y = \frac{cx}{ax+b} \text{ and } y = \frac{c}{a} + \frac{-\frac{b \cdot c}{a}}{ax+b}.$$

a = $(1-\beta^*)\Omega^*$, b = $\Omega^*(1-\alpha)$, $c = -\left((1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right)$,

$$\frac{f}{a} = \frac{-\Omega^*(1-\alpha)\left\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right\}}{\left((1-\beta^*)\Omega^*\right)^2}.$$

$$VA_{n(i)} = \frac{-\Omega^*(1-\alpha)}{(1-\beta^*)\Omega^*} = \frac{-b}{a}. \quad HA_{n(i)} = \frac{-\left((1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right)}{(1-\beta^*)\Omega^*} = \frac{c}{a}.$$

$$Width_{n(i)} = \sqrt{\left|\frac{-\Omega^*(1-\alpha)\left\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right\}}{\left((1-\beta^*)\Omega^*\right)^2}\right|}.$$

$$Shape_{n(i)} = \sqrt{2\left|\frac{-\Omega^*(1-\alpha)\left\{(1-\beta^*)\Omega^* - \beta^*(1-\alpha)\right\}}{\left((1-\beta^*)\Omega^*\right)^2}\right|}.$$

$$Curvature_{n(i)}=1/\sqrt{2\left|\frac{-\Omega^*(1-\alpha)\left\{(1-\beta^*)\Omega^*-\beta^*(1-\alpha)\right\}}{\left((1-\beta^*)\Omega^*\right)^2}\right|}.$$

$n(i)=\dfrac{-0.165012i}{0.48250215i+1.63825}$, where a = 0.48250215, b = 1.63825, c = –0.165012, $b\cdot c$ = –0.270330909. f = –0.560268817 = $\dfrac{-b\cdot c}{a}$, and $\dfrac{f}{a}$ = –1.16117372.

$VA_{n(i)}$ = –3.395322 = $\dfrac{-b}{a}$. $HA_{n(i)}$ = –0.341992258 = $\dfrac{c}{a}$.

$Width_{n(i)}$ = 1.0775777= $\sqrt{|-1.16117372|}$. $Shape_{n(i)}$ = 1.523925 = $\sqrt{|-2.32234744|}$.

$Curvature_{n(i)}$ = 0.6562 = $1/\sqrt{|-2.32234744|}$.

$\left(y+\dfrac{c}{a}\right)\left(x+\dfrac{b}{a}\right)=\dfrac{f}{a}$. For $n(i)$: $(y-0.3419923)(x+3.395322)=-1.161174$.

4–3 $\Omega^*(\beta^*)$:

$\Omega^*(\beta^*)=\dfrac{-i(1-\alpha)\beta^*}{i(1+n)\beta^*-\left(i(1+n)+n(1-\alpha)\right)}$, using. $\Omega^*=\dfrac{\beta^*\cdot i(1-\alpha)}{i(1-\beta^*)(1+n)+n(1-\alpha)}$.

$y=\dfrac{cx}{ax+b}$ and $y=\dfrac{c}{a}+\dfrac{-\frac{b\cdot c}{a}}{ax+b}$. a = $i(1+n)$. b = $-(i(1+n)+n(1-\alpha))$. c = $-i(1-\alpha)$.

$VA_{\Omega^*(\beta^*)}=\dfrac{i(1+n)+n(1-\alpha)}{i(1+n)}$. $HA_{\Omega^*(\beta^*)}=\dfrac{-i(1-\alpha)}{i(1+n)}$.

$\dfrac{f}{a}=\dfrac{-\left(i(1+n)+n(1-\alpha)\right)\cdot i(1-\alpha)}{\left(i(1+n)\right)^2}=\dfrac{-bc}{a^2}$.

$\Omega^*=\dfrac{-0.08648037\beta^*}{0.100265196\beta^*-0.108730344}$. $\dfrac{f}{a}$ = –0.935336465.

$VA_{\Omega^*(\beta^*)}=1.084427581=\dfrac{-b}{a}$. $HA_{\Omega^*(\beta^*)}=-0.862516341=\dfrac{c}{a}$.

$Width_{\Omega^*(\beta^*)}$ = 0.967127946 = $\sqrt{|-0.935336465|}$.

$Shape_{\Omega^*(\beta^*)}$ = 1.367725458 = $\sqrt{|-1.87067293|}$.

$Curvature_{\Omega^*(\beta^*)}$ = 0.731140883 = $1/\sqrt{|1.87067293|}$.

$\left(y-\frac{c}{a}\right)\left(x+\frac{b}{a}\right)=\frac{f}{a}$. For $\Omega^*(\beta^*)$: $(y + 0.862516341)(x - 1.084427581) = -0.935336465$.

4–4 $\beta^*(\Omega^*)$:

$\beta^*(\Omega^*)=\dfrac{\big(i(1+n)+n(1-\alpha)\big)\Omega^*}{i(1+n)\Omega^*+i(1-\alpha)}$, using $\beta^*=\dfrac{\Omega^*\big(i+i\cdot n+n(1-\alpha)\big)}{i\big((1-\alpha)+\Omega^*(1+n)\big)}$.

$y=\dfrac{cx}{ax+b}$ and $y=\dfrac{c}{a}+\dfrac{-\frac{b\cdot c}{a}}{ax+b}$. $a = i(1 + n)$. $b = i(1-\alpha)$. $c = i(1 + n) + n(1-\alpha)$.

$\dfrac{f}{a}=\dfrac{i(1-\alpha)\big(i+(1+n)+n(1-\alpha)\big)}{\big(i(1+n)\big)^2}=\dfrac{-b\cdot c}{a^2}$.

$\beta^*=\dfrac{0.108730344\Omega^*}{0.100265196\Omega^*+0.08648037}$. $\dfrac{f}{a}=-0.935336465$.

$VA_{\beta^*(\Omega^*)}=-0.862516341=\dfrac{-b}{a}$. $HA_{\beta^*(\Omega^*)}=1.084427581=\dfrac{c}{a}$.

$Width_{\beta^*(\Omega^*)} = 0.967127946 = \sqrt{|-0.935336465|}$.

$Shape_{\beta^*(\Omega^*)} = 1.367725458 = \sqrt{|-1.87067293|}$.

$Curvature_{\beta^*(\Omega^*)} = 0.731140883 = 1/\sqrt{|-1.87067293|}$.

$\left(y-\frac{c}{a}\right)\left(x+\frac{b}{a}\right)=\frac{f}{a}$ For $\beta^*(\Omega^*)$: $(y - 1.084427581)(x + 0.862516341) = -0.935336465$.

5–1 $\beta^*(n)$:

$\beta^*(n)=\dfrac{\Omega^*\cdot n\big(i+(1-\alpha)\big)+\Omega^*\cdot i}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$ from $\beta^*=\dfrac{\Omega^*\big(i+i\cdot n+n(1-\alpha)\big)}{i\big((1-\alpha)+\Omega^*(1+n)\big)}$.

$y=\dfrac{cx+d}{ax+b}$ and $y=\dfrac{c}{a}+\dfrac{d-\frac{b\cdot c}{a}}{ax+b}$,

where $a = \Omega^*\cdot i$, $b = i(1-\alpha+\Omega^*)$, $c = \Omega^*(i + (1-\alpha))$, $d = \Omega^*\cdot i$,

$e = \dfrac{\Omega^*\big(i+(1-\alpha)\big)}{\Omega^*\cdot i}$, and $\dfrac{f}{a}=\dfrac{1-\Omega^*(1-\alpha+i)i(1-\alpha+\Omega^*)}{(\Omega^*\cdot i)^2}=\dfrac{1-b\cdot c}{a^2}$, where the Width as a base of the rectangle equilateral triangle is $\sqrt{\left|\frac{f}{a}\right|}$.

$$VA_{\beta^*(n)} = \frac{-b}{a} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^* \cdot i}. \quad HA_{\beta^*(n)} = \frac{c}{a} = \frac{\Omega^*\left(i+(1-\alpha)\right)}{\Omega^* \cdot i}.$$

$$Width_{\beta^*(n)} = \sqrt{\left|(\Omega^* \cdot i)\left(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^* + i)\right)\right|}.$$

$$Shape_{\beta^*(n)} = \sqrt{2\left|(\Omega^* \cdot i)\left(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^* + i)\right)\right|}.$$

$$Curvature_{\beta^*(n)} = 1/\sqrt{2\left|(\Omega^* \cdot i)\left(\Omega^* \cdot \alpha + i \cdot \alpha - (1+\Omega^* + i)\right)\right|}.$$

$$\beta^*(n) = \frac{1.8251n + 0.18679}{0.18679n + 0.27327}.$$

E.g., a = 0.18679323 = 1.8811 × 0.0993, b = 0.2732736 = 0.0993 × 2.752, c = 1.8251 = 1.8811 × (0.0993 + 0.8709), d = 0.18679323 = 1.8811 × 0.0993, where $x = n = 0.00972$,

f = –2.48328 = 0.18679323 – $\frac{0.2732736 \times 1.8251}{0.18679323} = d - \frac{b \cdot c}{a}$.

$$\frac{f}{a} = -13.2938285 = \frac{-2.483197165}{0.18679323}$$

$c \cdot n + d$ = 0.2045332 = $1.8251 \times 0.00972 + 0.18679323$ and $a \cdot n + b$ = 0.27508923 = $0.18679323 \times 0.00972 + 0.2732736$, and thus, $beta^*$ = y = 0.7435 = 0.2045332 ÷ 0.27508923.

–2.483197 = $0.18679323 \times (1.8811 \times 0.1291 + 0.0993 \times 0.1291 - (2.8811 + 0.0993))$ when f = $(\Omega^* \cdot i)(\Omega^* \cdot \alpha + i \cdot \alpha - (1 + \Omega^* + i))$ is used.

$$VA_{\beta^*(n)} = -1.46297379 = \frac{-0.2732736}{0.18679323} = \frac{-b}{a}.$$

$$HA_{\beta^*(n)} = 9.77039275 = \frac{1.8251}{0.18679323} = \frac{c}{a}.$$

$$Width_{\beta^*(n)} = 3.64607 = \sqrt{|-13.2942732|}.$$

$\left(y - \frac{c}{a}\right)\left(x + \frac{b}{a}\right) = \frac{f}{a}$. For $\beta^*(n)$: $(y - 9.77039275)(x + 1.46297379) = -13.2938285$.

5–2 $n(\beta^*)$:

$n(\beta^*) = \dfrac{-i(1-\alpha+\Omega^*)\beta^*+\Omega^*\cdot i}{\Omega^*\cdot i\cdot\beta^*-\Omega^*(1-\alpha+i)}$ from $n = \dfrac{\beta^*\cdot i(1-\alpha+\Omega^*)-\Omega^*\cdot i}{\Omega^*(1-\alpha+i-\beta^*\cdot i)}$.

$y=\dfrac{Cx+D}{Ax+B}$ and $y=\dfrac{C}{A}+\dfrac{D-\dfrac{B\cdot C}{A}}{Ax+B}$,

where A = $\Omega^*\cdot i$, B = $-\Omega^*(1-\alpha+i)$, C = $-i(1-\alpha+\Omega^*)$, D = $\Omega^*\cdot i$,

$\dfrac{C}{A}=\dfrac{-i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}$, and $\dfrac{F}{A}=\dfrac{-\Omega^*\cdot i-\dfrac{\Omega^*(1-\alpha+i)i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}}{\Omega^*\cdot i}$,

$Width_{n(\beta^*)}=\sqrt{\left|\dfrac{F}{A}\right|}$.

$VA_{n(\beta^*)}=\dfrac{\Omega^*(1-\alpha+i)}{\Omega^*\cdot i}=\dfrac{-B}{A}$. $HA_{n(\beta^*)}=\dfrac{-i(1-\alpha+\Omega^*)}{\Omega^*\cdot i}=\dfrac{C}{A}$.

$Width_{n(\beta^*)}=\sqrt{\left|(\Omega^*\cdot i)\left(\Omega^*\cdot\alpha+i\cdot\alpha-(1+\Omega^*+i)\right)\right|}$.

$Shape_{n(\beta^*)}=\sqrt{2\left|(\Omega^*\cdot i)\left(\Omega^*\cdot\alpha+i\cdot\alpha-(1+\Omega^*+i)\right)\right|}$.

$Curvature_{n(\beta^*)}=1/\sqrt{2\left|(\Omega^*\cdot i)\left(\Omega^*\cdot\alpha+i\cdot\alpha-(1+\Omega^*+i)\right)\right|}$.

A = 0.18679323. B = –1.82504322. C = –0.2732736. D = 0.18679323. C/A = –1.46297379.

–B/A = 9.77039275. F = –2.483197165 = 0.18679323 – $\dfrac{0.49873613}{0.18679323}=D-\dfrac{B\cdot C}{A}$.

$\left(y-\dfrac{C}{A}\right)\left(x+\dfrac{B}{A}\right)=\dfrac{F}{A}$. For $n(\beta^*)$: $(y+1.46297379)(x-9.77039275)$ = –13.2938285.

5–3 $\widetilde{\beta^*}(n)$:

$\widetilde{\beta^*}(n)=\dfrac{(\Omega^*\cdot i-\Omega^*\left(i-(1-\alpha)\right))\cdot n+i(1-\alpha)}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$ from

$\widetilde{\beta^*}(n)=1-\dfrac{\Omega^*\cdot n\left(i+(1-\alpha)\right)+\Omega^*\cdot i}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$,

setting $\widetilde{\beta^*}$ = 1 – β^* and starting with $\beta^*(n)$ = $\dfrac{\Omega^*\cdot n\left(i+(1-\alpha)\right)+\Omega^*\cdot i}{\Omega^*\cdot i\cdot n+i(1-\alpha+\Omega^*)}$.

$$f = i(1-\alpha) - \frac{i(1-\alpha+\Omega^*)\left(\Omega^* i - \Omega^*(i-(1-\alpha))\right)}{\Omega^* i}.$$

And, $f = i(1-\alpha) - \dfrac{i(1-\alpha+\Omega^*)\left(\Omega^*(-(1-\alpha))\right)}{\Omega^* i} = (1-\alpha)(i + (1-\alpha+\Omega^*)$.

$$\mathrm{y} = \frac{\mathrm{cx}+\mathrm{d}}{\mathrm{ax}+\mathrm{b}}. \quad \mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b}\cdot\mathrm{c}}{\mathrm{a}}}{\mathrm{ax}+\mathrm{b}}.$$

a $= \Omega^* \cdot i$, b $= i(1-\alpha+\Omega^*)$, c $= \Omega^* \cdot i - \Omega^*(i+(1-\alpha))$, d $= i(1-\alpha)$, $\mathrm{e} = \dfrac{\Omega^* \cdot i - \Omega^*\left(i+(1-\alpha)\right)}{\Omega^* \cdot i}$, and

$$\mathrm{f} = i(1-\alpha) - \frac{i(1-\alpha+\Omega^*)\left\{\Omega^* \cdot i - \Omega^*\left(i-(1-\alpha)\right)\right\}}{\Omega^* \cdot i} = \mathrm{d} - \frac{\mathrm{b}\cdot\mathrm{c}}{\mathrm{a}},$$

f = 2.4832 = 0.8709 × (0.0993 + 0.8709 + 1.8811).

And, using f $= (1-\alpha)\{i+(1-\alpha+\Omega^*)\}$, $\dfrac{\mathrm{f}}{\mathrm{a}} = \dfrac{(1-\alpha)\left\{i+(1-\alpha+\Omega^*)\right\}}{\Omega^* \cdot i}$, where

$$Width_{\widetilde{\beta^*}(n)} = \sqrt{\left|\frac{\mathrm{f}}{\mathrm{a}}\right|}.$$

$$VA_{\widetilde{\beta^*}(n)} = \frac{-i(1-\alpha+\Omega^*)}{\Omega^* \cdot i} = \frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\widetilde{\beta^*}(n)} = \frac{\Omega^* \cdot i - \Omega^*\left(i+(1-\alpha)\right)}{\Omega^* \cdot i} = \frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\widetilde{\beta^*}(n)} = \sqrt{\left|\frac{(1-\alpha)\left\{i+(1-\alpha+\Omega^*)\right\}}{\Omega^* \cdot i}\right|}. \quad Shape_{\widetilde{\beta^*}(n)} = \sqrt{2\left|\frac{(1-\alpha)\left\{i+(1-\alpha+\Omega^*)\right\}}{\Omega^* \cdot i}\right|}.$$

$$Curvature_{\widetilde{\beta^*}(n)} = 1/\sqrt{2\left|\frac{(1-\alpha)\left\{i+(1-\alpha+\Omega^*)\right\}}{\Omega^* \cdot i}\right|}.$$

E.g., a = 0.18679323 = 1.8811 × 0.0993, b = 0.2732736 = 0.0993 × 2.752, c = –1.63825 = 0.18679323 – 1.8811 × (0.0993 + 0.8709), d = 0.08648 = 0.0993 × 0.8709, where x = n = 0.00972, then,

c $\cdot n +$ d = 0.070505 = $-1.63825 \times 0.00972 + 0.08648$ and a $\cdot n +$ b = 0.27508923 = 0.18679323 × 0.00972 + 0.2732736, and thus, $\widetilde{\beta^*}$ = 0.2565 = 0.070556 ÷ 0.27508923.

$$\mathrm{f} = \left(\mathrm{d} - \frac{\mathrm{bc}}{\mathrm{a}}\right) = 2.48319681 = 0.08648 - \frac{0.2732736 \times -1.63825}{0.18679323}.$$

$\frac{\mathrm{f}}{\mathrm{a}} = 13.2938266 = \frac{2.48319681}{0.18679323}$.

And, using f = $(1-\alpha)\{i + (1-\alpha + \Omega^*)\}$, $\frac{\mathrm{f}}{\mathrm{a}} = 13.26585428 = \frac{2.47797177}{0.18679323}$ (no error in the Excel).

$$\widetilde{\beta^*} = 0.2565 = \frac{-1.63825}{0.18679323} + \frac{2.48319681}{0.27508923} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}.$$

$$VA_{\widetilde{\beta^*}(n)} = -1.46297379 = \frac{-0.2732736}{0.18679323} = \frac{-\mathrm{b}}{\mathrm{a}}.\quad HA_{\widetilde{\beta^*}(n)} = -8.77039 = \frac{-1.63825}{0.18679323} = \frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\widetilde{\beta^*}(n)} = 3.64607 = \sqrt{13.2938266}.$$

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\widetilde{\beta^*}(n)$: $(y + 8.77039)(x + 1.46297379) = 13.2938266$.

5–4 $n(\widetilde{\beta^*})$:

$$n(\widetilde{\beta^*}) = \frac{\{i(1-\alpha+\Omega^*)\}(1-\beta^*) - i(1-\alpha)}{\Omega^* \cdot i(1-\beta^*) + \Omega^*(1-\alpha)}.$$

Here starting with $\beta^*(n) = \frac{\Omega^* \cdot n\left(i + (1-\alpha)\right) + \Omega^* \cdot i}{\Omega^* \cdot i \cdot n + i(1-\alpha+\Omega^*)}$ and using

$n(\beta^*) = \frac{i(1-\alpha+\Omega^*)\beta^* - \Omega^* \cdot i}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)}$, where

$$n(\widetilde{\beta^*}) = \frac{\Omega^* \cdot i(1-\beta^*) + \Omega^*(1-\alpha) - i(1-\alpha+\Omega^*)\beta^*}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)} = 1 - \frac{i(1-\alpha+\Omega^*)\beta^* - \Omega^* \cdot i}{-\Omega^* \cdot i \cdot \beta^* + \Omega^*(1-\alpha+i)}.$$

$\mathrm{y} = \frac{\mathrm{cx+d}}{\mathrm{ax+b}}$ and $\mathrm{y} = \frac{\mathrm{c}}{\mathrm{a}} + \frac{\mathrm{d} - \frac{\mathrm{b \cdot c}}{\mathrm{a}}}{\mathrm{ax+b}}$.

a = $\Omega^* \cdot i$. b = $\Omega^*(1-\alpha)$. c = $i(1-\alpha+\Omega^*)$. d = $-i(1-\alpha)$.

a = 0.18679323. b = 1.63825. c = 0.2732736 = 0.0993 × 2.752. d = –0.08648.

c/a = 1.46297379. –b/a = –8.7703928. f = –2.48319681 = –0.08648 – $\frac{0.447690475}{0.18679323}$ = d – $\frac{\mathrm{b \cdot c}}{\mathrm{a}}$. f/a = –13.2938266.

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $n(\widetilde{\beta^*})$: $(y - 1.46297379)(x + 8.77039) = -13.2938266$.

Special cases: β^* versus $\widetilde{\beta^*} = 1 - \beta^*$, and a versus $\tilde{\alpha} = 1 - \alpha$

6–1 $\beta^*(i)$:

$$\beta^*(i) = \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{\left\{(1-\alpha) + \Omega^*(1+n)\right\} i}.$$

Here using $\beta^* = \dfrac{\Omega^*\left(i + i \cdot n + n(1-\alpha)\right)}{i\left((1-\alpha) + \Omega^*(1+n)\right)}$.

$\mathrm{y} = \dfrac{\mathrm{cx+d}}{\mathrm{ax}}$ and $\mathrm{y} = \dfrac{\mathrm{c}}{\mathrm{a}} + \dfrac{\mathrm{d}}{\mathrm{ax}}$.

a = $(1 - \alpha) + \Omega^*(1 + n)$. b = 0. c = $(1 + n)\Omega^*$. d = $(1 - \alpha)\,\Omega^* \cdot n$.

$$\frac{\mathrm{f}}{\mathrm{a}} = \frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha) + \Omega^*(1+n)}.$$

$$VA_{\beta^*(i)} = 0 = \frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\beta^*(i)} = \frac{(1+n)\Omega^*}{(1-\alpha) + \Omega^*(1+n)} = \frac{\mathrm{c}}{\mathrm{a}}.$$

a = 2.7702843 = 0.8709 + 1.8811 × 1.00972, b = 0 c = 1.899384292, f = d = 0.015923789 = 0.8709 × 1.8811 × 0.00972. f/a = 0.00574807 = 0.015923789 ÷ 2.7702843.

$$VA_{\beta^*(i)} = 0 = \frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\beta^*(i)} = 0.68562793 = \frac{\mathrm{c}}{\mathrm{a}}. \quad Width_{\beta^*(i)} = \sqrt{\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha) + \Omega^*(1+n)}}.$$

$$Shape_{\beta^*(i)} = \sqrt{2\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha) + \Omega^*(1+n)}}. \quad Curvature_{\beta^*(i)} = 1 \Big/ \sqrt{2\frac{(1-\alpha)\Omega^* \cdot n}{(1-\alpha) + \Omega^*(1+n)}}.$$

$\left(\mathrm{y} - \dfrac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \dfrac{\mathrm{b}}{\mathrm{a}}\right) = \dfrac{\mathrm{f}}{\mathrm{a}}$. For $\beta^*(i)$: $(y - 0.68562793)(x + 0) = 0.00574807$.

6–2 $\widetilde{\beta^*}(i)$:

$\widetilde{\beta^*}(i) = \dfrac{(1-\alpha)i - (1-\alpha)\Omega^* \cdot n}{\left((1-\alpha) + \Omega^*(1+n)\right) i}$, where $\widetilde{\beta^*} = 1 - \beta^*$.

Here starting with $\beta^*(i) = \dfrac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{\left\{(1-\alpha) + \Omega^*(1+n)\right\} i}$ and, using

$$\widetilde{\beta^*}(i) = \frac{\left((1-\alpha) + \Omega^*(1+n)\right)i - (1+n)\Omega^* \cdot i - (1-\alpha)\Omega^* \cdot n}{\left((1-\alpha) + \Omega^*(1+n)\right) i}$$

$$= 1 - \frac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{\left((1-\alpha) + \Omega^*(1+n)\right) i}.$$

$$\mathrm{y}=\frac{\mathrm{cx}+\mathrm{d}}{\mathrm{ax}} \text{ and } \mathrm{y}=\frac{\mathrm{c}}{\mathrm{a}}+\frac{\mathrm{d}}{\mathrm{ax}}, \text{ where f = d.}$$

$$VA_{\widetilde{\beta^*}(i)}=0=\frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\widetilde{\beta^*}(i)}=\frac{1-\alpha}{(1-\alpha)+\Omega^*(1+n)}=\frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\widetilde{\beta^*}(i)}=\sqrt{\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}. \quad Shape_{\widetilde{\beta^*}(i)}=\sqrt{2\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}.$$

$$Curvature_{\widetilde{\beta^*}(i)}=1/\sqrt{2\left|\frac{-(1-\alpha)\Omega^*\cdot n}{(1-\alpha)+\Omega^*(1+n)}\right|}.$$

a = 2.7702843 = 0.8709 + 1.8811 × 1.00972, b = 0 c = 0.8709, f = d = –0.015923789 = –0.8709 × 1.8811 × 0.00972. c/a = 0.31432938. f/a = –0.00574807 = 0.015923789 ÷ 2.7702843.

$$Width_{\widetilde{\beta^*}(n)}= 0.0758160=\sqrt{\left|\frac{\mathrm{f}}{\mathrm{a}}\right|}.$$

$\left(\mathrm{y}-\frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x}+\frac{\mathrm{b}}{\mathrm{a}}\right)=\frac{\mathrm{f}}{\mathrm{a}}$. For $\widetilde{\beta^*}$ (i): (y – 0.31432938)(x + 0) = –0.00574807.

6–3 $\alpha(i)$:

$$\alpha(i)=\frac{-\left\{(1+n)\Omega^*-\beta^*-\beta^*\Omega^*(1+n)\right\}i-\Omega^*\cdot n}{\beta^*\cdot i-\Omega^*\cdot n}, \text{ from}$$

$$\alpha(i)=\frac{\left\{(1+n)\Omega^*-\beta^*-\beta^*\Omega^*(1+n)\right\}i+\Omega^*\cdot n}{-\beta^*\cdot i+\Omega^*\cdot n}.$$

Here starting with $\beta^*=\frac{(1+n)\Omega^*\cdot i+(1-\alpha)\Omega^*\cdot n}{i\left((1-\alpha)+\Omega^*(1+n)\right)}$.

$$\mathrm{y}=\frac{\mathrm{cx}+\mathrm{d}}{\mathrm{ax}+\mathrm{b}} \text{ and } \mathrm{y}=\frac{\mathrm{c}}{\mathrm{a}}+\frac{\mathrm{d}-\frac{\mathrm{b}\cdot\mathrm{c}}{\mathrm{a}}}{\mathrm{ax}+\mathrm{b}}.$$

a = β^*, b = $-\Omega^*\cdot n$, c = $-\{(1+n)(1-\beta^*)\Omega^*-\beta^*\}$, d = $-\Omega^*\cdot$ n.

$$\mathrm{f}=-\Omega^*\cdot n-\frac{\Omega^*\cdot n\left\{(1+n)(1-\beta^*)\Omega^*-\beta^*\right\}}{\beta^*}=-\left(\mathrm{d}+\frac{\mathrm{b}\cdot\mathrm{c}}{\mathrm{a}}\right).$$

$$\frac{\mathrm{f}}{\mathrm{a}}=\frac{-\Omega^*\cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{\beta^{*2}}.$$

$$VA_{\alpha(i)}=\frac{\Omega^*\cdot n}{\beta^*}=\frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\alpha(i)}=\frac{-\left\{(1+n)(1-\beta^*)\Omega^*-\beta^*\right\}}{\beta^*}=\frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\alpha(i)} = \sqrt{\left|\frac{-\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{\beta^{*2}}\right|}.$$

$$Shape_{\alpha(i)} = \sqrt{2\left|\frac{-\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{\beta^{*2}}\right|}.$$

$$Curvature_{\alpha(i)} = 1/\sqrt{2\left|\frac{-\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{\beta^{*2}}\right|}.$$

$$\alpha(i) = \frac{-0.25630793i - 0.018284292}{0.7435i - 0.018284292}.$$

a = 0.7435 = β^*. b = –0.018284292. c = –0.25630793. d = –0.018284292.

f = –0.011981118 = –0.01828429– $\dfrac{-0.004686409}{0.7435}$.

$\dfrac{f}{a} = -0.0161145 = \dfrac{-0.011981118}{0.7435}$.

$VA_{\alpha(i)}$ = 0.024592 = $\dfrac{-b}{a}$ $HA_{\alpha(i)}$ = 0.34473158 = $\dfrac{-0.25630793}{-0.7435} = \dfrac{c}{a}$.

$Width_{\alpha(i)}$ = 0.126942928 = $\sqrt{|-0.0161145|}$.

$Shape_{\alpha(i)}$ = 0.17952437 = $\sqrt{2|-0.0161145|}$.

$Curvature_{\alpha(i)}$ = 5.570274386 = $1/\sqrt{2|-0.0161145|}$.

$\left(y-\dfrac{c}{a}\right)\left(x+\dfrac{b}{a}\right)=\dfrac{f}{a}$. For $\alpha(i)$: $(y - 0.34473158)(x - 0.024592) = -0.016114484$.

6–4 $\tilde{\alpha}(i)$:

$$\tilde{\alpha}(i) = \frac{(1+n)(1-\beta^*)\Omega^* \cdot i}{\beta^* \cdot i - \Omega^* \cdot n},$$

setting $\tilde{\alpha}$ = 1 – α, and starting with $\beta^* = \dfrac{(1+n)\Omega^* \cdot i + (1-\alpha)\Omega^* \cdot n}{i\left((1-\alpha)+\Omega^*(1+n)\right)}$.

$$y = \frac{cx}{ax+b} \quad \text{and} \quad y = \frac{c}{a} + \frac{-\frac{b \cdot c}{a}}{ax+b}.$$

a = β^*, b = $-\Omega^* \cdot$n, c = $(1 + n)(1-\beta^*)\Omega^*$, $\dfrac{f}{a} = \dfrac{\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{(\beta^*)^2}$.

$$VA_{\tilde{\alpha}(i)} = \frac{\Omega^* \cdot n}{\beta^*} = \frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\tilde{\alpha}(i)} = \frac{(1+n)(1-\beta^*)\Omega^*}{\beta^*} = \frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\tilde{\alpha}(i)} = \sqrt{\left|\frac{\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{(\beta^*)^2}\right|}.$$

$$Shape_{\tilde{\alpha}(i)} = \sqrt{2\left|\frac{\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{(\beta^*)^2}\right|}.$$

$$Curvature_{\tilde{\alpha}(i)} = 1/\sqrt{2\left|\frac{\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{(\beta^*)^2}\right|}.$$

$$\tilde{\alpha}(i) = \frac{0.48719207i}{0.7435i - 0.018284292}.$$

a = 0.7435. b = –0.018284292 = $-\Omega^* \cdot$ n. c = 0.48719207 = $(1+n)(1-\beta^*)\Omega^*$.

$$\mathrm{f} = 0.011981116 = \frac{0.00890796}{0.7435} = \frac{\Omega^* \cdot n\left\{(1+n)(1-\beta^*)\Omega^*\right\}}{\beta^*}.$$

$$\frac{\mathrm{f}}{\mathrm{a}} = 0.016114484 = \frac{0.00890796}{0.7435^2} = \frac{-\mathrm{bc}}{\mathrm{a}^2}.$$

$$VA_{\tilde{\alpha}(i)} = 0.024592 = \frac{-\mathrm{b}}{\mathrm{a}}. \quad HA_{\tilde{\alpha}(i)} = 0.655268338 = \frac{0.48719201}{0.7435} = \frac{\mathrm{c}}{\mathrm{a}}.$$

$$Width_{\tilde{\alpha}(i)} = 0.12694284 = \sqrt{0.016114484}.$$

$$Shape_{\tilde{\alpha}(i)} = 0.179524282 = \sqrt{2 \times 0.016114484}.$$

$$Curvature_{\tilde{\alpha}(i)} = 5.570277117 = 1/\sqrt{2 \times 0.016114484}.$$

$\left(\mathrm{y} - \frac{\mathrm{c}}{\mathrm{a}}\right)\left(\mathrm{x} + \frac{\mathrm{b}}{\mathrm{a}}\right) = \frac{\mathrm{f}}{\mathrm{a}}$. For $\tilde{\alpha}$ (i): (y – 0.655268338)(x – 0.024592) = 0.016114484.

For the formulation of endogenous equations, starting with **the first appearance**:

Kamiryo, Hideyuki. (2004). Basics of an Endogenous Growth Model: the Optimum CRC Situation and Conditional Convergence, *Journal of Economic Sciences* 7 (Feb, 2), 51–80. (First appearance to *beta* and *Omega*).

Kamiryo, Hideyuki. (2005). Empirics of A Consumption Function Consistent with A Technology-Golden Rule: Using Three Dimensional Graphs, *Papers of the Research Society of Commerce and Economics* 46 (1), 159–212. (First appearance to *national* *taste* which integrates utility with time preference at the macro level).

Kamiryo, Hideyuki. (2006). Towards the Relationship between Constant Returns to Scale and Diminishing /Increasing Returns to Scale Using Two Production Functions: with Recursive Programming, *Journal of Economic Sciences* 10 (1), 73–104. (First appearance to *delta* and *lambda;* after revised).

Kamiryo, Hideyuki. (2008). Structural Equations Formulated in the Endogenous Growth Model, In: *Papers of the Research Society of Commerce and Economics* 49 (Sep, 1): 133–178.

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Kamiryo, Hideyuki. (2010a). Changes in Elasticity of Substitution *sigma* in KEWT 3.09, with Seven Vital Aspects 58 Countries and Three Areas by Sector, 1990–2007, *Papers of the Research Society of Commerce and Economics* 50 (Feb, 2): 387–426.

Kamiryo, Hideyuki. (2010b). Endogenous I-S and External L-M Diagram in Equilibrium towards Policy-Making, *International Advances in Economic Research*, forthcoming.

Kamiryo, Hideyuki. (2010c). Policy-Oriented Endogenous Model and Its Data-Sets KEWT 4.10 in Equilibrium, 65p. for text, with xi and 63p. for figures; under review.

(see www@riee.tv, www.megaegg.ne.jp/˜kamiryo/, and http://ci.nii.ac.jp/; for related all the Excel files, please ask using the author's email address, kamiryo@ms3.megaegg. ne.jp).